ABSTRACT

In this contribution a simulation tool is developed to compute the energy skewing angles and energy coefficients for the reflected and refracted plane waves in following general cases: (1) reflection and refraction of plane elastic waves at an interface between isotropic and transversely isotropic solid, (2) reflection and refraction of plane elastic waves at an interface between transversely isotropic and isotropic solid, (3) reflection and refraction of plane elastic waves at an interface between two general transversely isotropic solid and (4) reflection of plane elastic waves from a stress free boundary of a transversely isotropic solid. Computational results for analytically evaluated acoustic wave energy skewing angles and energy reflection and transmission coefficients in acoustically anisotropic materials such as austenitic steel materials exhibiting columnar grain orientation are presented. The obtained results show that the acoustic energy skewing angles and coefficients in austenitic steel materials strongly depend upon the columnar grain orientation and are less influenced by the grain orientations which are parallel and perpendicular to the interface.

INTRODUCTION

Analytical solutions to the problem of reflection and refraction of plane elastic waves in an anisotropic material are of general interest for both theoretical understanding of propagation characteristics and experimental investigation of material behavior. Austenitic weld materials which are extensively used in nuclear power plants are acoustically transversely isotropic because of their columnar grain structure is rotationally symmetric. The reflection and refraction phenomena in austenitic weld materials are different as compared to the isotropic steel materials because of the directional dependency of the elastic properties [1]. Calculation of reflection parameters in ice, beryl and zinc with different orientations of the plane boundary are reported by Musgrave [2]. He used a graphical technique to determine the reflected vertical slowness components. Henneke [3] has discussed reflection phenomena for a quasi-longitudinal wave incident upon a stress free boundary of zinc, magnesium and cadmium metals. Jones and Henneke [4] have presented the numerical results of reflection coefficients for plane wave incidence upon a stress free boundary of quartz single crystals. It is well known that the energy flow direction in an anisotropic material does not coincide with the direction of the wave normal. The problem of reflection of plane elastic waves at a free surface of an anisotropic solid was discussed by Nagy et.al [5]. Munikoti and Neumann [6] have presented the reflection and transmission energy coefficients in case of a plane wave incidence at the interface between austenitic base and weld metal, assuming sound propagation takes place in symmetric meridian plane.

The aim of this paper is twofold. At first, a review is made of the technique outlined by [7, 8] and energy velocities are derived in terms of energy flux vector (Poynting vector) and energy density. Secondly, the acoustic wave energy skewing angles and energy coefficients for the reflected and refracted modes for the four general cases are studied in context of the ultrasonic investigation of austenitic welds exhibiting different columnar grain orientations. Physical interpretations, given for the obtained computational results, enable the selection of valid domain of incident wave modes for the ultrasonic investigation of austenitic welds. The implemented algorithm is validated based on the fundamental reciprocity relations for the energy flow transformation coefficients.
THEORETICAL PROCEDURE

In order to set a desired plane of transversely isotropic solid as an incidence plane, we need to rotate the coordinate system by transforming the elastic stiffness matrix from crystallographic coordinate system to the calculated coordinate system. Bond matrix multiplication method [9] is used to obtain the stiffness matrix in the calculated coordinate system and is represented as

\[
[C'] = [M][C][M']
\]

(1)

where \([C]\) and \([C']\) are the matrices of stiffness constants in the old and new coordinate systems, respectively. \([M]\) stands for the Bond transformation matrix and \([M']\) represents its transposed pair.

The Christoffel equation of acoustic wave propagation in anisotropic solid is given as [9]

\[
k^2 \tau_{ij} - \rho \omega^2 \delta_{ij} [v_j] = 0.
\]

(2)

where \(\tau_{ij}\) is the Christoffel tensor, \(\rho\) is the density of the material, \(\omega\) is the angular frequency and \(v\) is the particle displacement velocity, \(k\) is the wave number. Equation (2) has non-trivial solutions for particle displacement velocity vectors \(v_j\), only if the determinant of the equation (2) equals zero. The eigenvalues of the determinant yields three phase velocity magnitudes which corresponds to the quasi longitudinal (qP), quasi shear vertical (qSV) and pure shear horizontal waves (SH). The particle displacement velocity vectors \(v_j\) can be obtained by solving the eigenvectors of the equation (2).

Let us consider a plane monochromatic quasi longitudinal wave with particle velocity displacement vector

\[
U^I = A^I a^I e^{-j \omega s^I r}
\]

(3)

incident from the upper medium onto a boundary between two general transversally isotropic austenitic columnar grained material (figure 1c), where \(A^I\) is the incidence amplitude, \(a^I\) is the incident particle polarization vector, \(s^I\) is the slowness vector of the incident wave and \(r\) is the position vector. In general, the incident wave transforms into three reflected and three transmitted waves. In the particular case of sound propagation in the austenitic columnar grained material, the horizontally polarized wave does not couple with qP and qSV waves. Here we have considered the columnar grain orientations in 2-D and thus the pure SH waves polarize along the y-direction. In case of anisotropic materials, the acoustic wave energy flux does not coincide with the wave vector direction and the incident group velocity and energy skewing angles are determined using the Poynting vector definitions [9]: The Poynting vector which defines the magnitude and direction of acoustic energy flow, is an important parameter to be analyzed in order to understand the energy flux carried by the wave for a particular angle of excitation.
Figure 1 - Illustration of wave reflection and refraction phenomena at a) the interface between anisotropic and isotropic media, b) the interface between isotropic and anisotropic media, c) the interface between two anisotropic solids and d) the free surface boundary of an anisotropic solid.
The average complex Poynting vector for the qP waves is represented as [9]

\[ P_{qP} = \frac{-v_j^* \cdot T_{ji}}{2}. \]  

(4)

The average stored energy for the qP waves in the transversely isotropic medium is expressed as

\[ U_{AVqP} = \frac{\rho (v_x^* v_x + v_y^* v_y)}{2}. \]

(5)

The energy velocities for the qP waves in an arbitrary rotated transversely isotropic medium is given by

\[ V_{qP} = \frac{P_{qP}}{U_{AVqP}}. \]

(6)

Moreover, the reflected and transmitted particle displacements are expressed as

\[ U_{Ra}^{R} = A_{Ra}^{R} a_{Ra}^{R} e^{-j\delta_{Ra}^{R}}, \]

\[ U_{Ta}^{T} = A_{Ta}^{T} a_{Ta}^{T} e^{-j\delta_{Ta}^{T}}, \]

(7)

(8)

where \( A_{Ra}^{R}, A_{Ta}^{T} \) and \( a_{Ra}^{R}, a_{Ta}^{T} \) are the amplitudes and particle polarization vectors for the reflected and refracted waves. \( s_{Ra}^{R} \) and \( s_{Ta}^{T} \) are the slowness vectors of the reflected and refracted waves, respectively.

According to Snell’s law all the projections of the slowness vector on the interface are equal to one another and which leads to the following relations:

\[ S_{x}^{R} = S_{x}^{T} = S_{y}^{R} = S_{y}^{T} = S_{z}^{R} = S_{z}^{T} = S_{x}. \]

(9)

(10)

The unknown vertical components \( (S_{y}) \) of reflected and refracted slowness vectors are determined using the modified Christoffel equation

\[ \begin{bmatrix} S_{x} & 0 & 0 & 0 & S_{z} & 0 \\ 0 & 0 & 0 & S_{z} & 0 & S_{x} \\ 0 & 0 & S_{z} & 0 & S_{x} & 0 \end{bmatrix} \left[ C^T \right] \begin{bmatrix} S_{x} & 0 & 0 & 0 & 0 & S_{z} \\ S_{x} & 0 & 0 & 0 & S_{z} & 0 \end{bmatrix} - \rho \delta_{ji} = 0. \]

(11)

The equation (11) is solved separately for the reflected and refracted medium which yields two fourth order polynomial equations in terms of normal component of reflected and refracted slowness vectors.
are the densities of the lower and the upper medium. The same procedure with appropriate changes is used for the case of beam incident from an isotropic medium to anisotropic solid and anisotropic medium to isotropic solid. In case of wave incident at a free surface boundary of transversely isotropic medium, the refraction coefficients will be equated to zero.
RESULTS AND DISCUSSION:

Material elastic constants used in the theoretical analysis [6] are as follows: For the transversely isotropic material XCrNi1811 the elastic stiffness constants are: \( C_{11} = 241.1 \text{ GPa} \), \( C_{13} = 138.03 \text{ GPa} \), \( C_{33} = 240.12 \text{ GPa} \), \( C_{44} = 122.29 \text{ GPa} \) and \( C_{66} = 72.092 \text{ GPa} \). The elastic constants for the isotropic ferritic steel are: \( C_{11} = 277 \text{ GPa} \), \( C_{44} = 82 \text{ GPa} \). The same density values used for the both the materials and are taken as \( \rho_1 = \rho_2 = 7820 \text{ kgm}^{-3} \).

Figure 2 - Normalized energy flux vector for a) the quasi pressure and b) the quasi shear vertical wave in homogeneous transversely isotropic austenitic stainless steel material exhibiting columnar grain orientation \( 0^0, 12.5^0, 25^0, 37.5^0, 50^0, 62.5^0, 75^0, 87.5^0 \). Figure 2a and 2b show the variation of normalized energy flux vector for the quasi pressure and quasi shear vertical waves with incident angle in a transversely isotropic austenitic stainless steel material for different columnar grain orientations \( 0^0, 12.5^0, 25^0, 37.5^0, 50^0, 62.5^0, 75^0 \) and \( 87.5^0 \).

Figure 3 - Energy skewing angles for the reflected a) quasi pressure and b) quasi shear vertical waves when an incident quasi pressure wave at an interface between transversely isotropic austenitic steel and isotropic ferritic steel.
The magnitude of the energy flux vector for the quasi pressure waves is less affected by the material anisotropic properties as compared to that of quasi shear waves. For the practical application of ultrasonic testing of transversely isotropic austenitic steel material it is preferred to select the angle of incidence at which the magnitude of energy flux is maximum and least deviated by the wave vector direction.

From figure 3a, it is apparent that the energy deviation angles are greatly influenced by the columnar grain orientation of the homogeneous transversely isotropic austenitic materials. The energy skewing angles for the reflected quasi pressure waves for an incident quasi pressure wave at an interface between transversely isotropic and isotropic steel materials vary around $13^\circ$. However, for the austenitic steel material with columnar grain orientations $62.5^\circ$ and $75^\circ$, the energy angles are more deviated from the wave vector angle.

Figure 3b shows the reflected quasi shear vertical wave energy skewing angles for an incident quasi pressure waves at an interface between transversely isotropic and isotropic ferritic steel. From the plot we observe that the skewing angles vary up to $37^\circ$. Again, figure 3b shows that for the columnar grain orientation $12.5^\circ$, for wide range of incident angles the energy skewing angles for the quasi shear vertical waves are negative, i.e., the energy of the wave is not in the reflected quadrant and the energy is directed in to the incident quadrant. However, for the columnar grain orientation $75^\circ$, the energy skewing angles are positive for a wide range of incident angles. The energy skewing angles for the transmitted quasi pressure and quasi shear vertical waves in case of an incident pressure waves at an interface between isotropic steel and transversely isotropic austenitic steel (see figure 4a and 4b) vary around $13.5^\circ$ and $36^\circ$, respectively.

![Figure 4 - Energy skewing angles for the transmitted a) quasi pressure and b) quasi shear vertical waves when an incident pressure wave at an interface between isotropic ferritic steel and transversely isotropic austenitic steel material](image)

Figure 5a shows the transmitted quasi pressure energy skewing angles for an incident quasi pressure waves at an interface between two general transversal isotropic austenitic steel materials. The columnar grains in the upper medium are assumed as perpendicular to the interface and the columnar grains of the lower medium are taken as $00, 12.50, 250, 37.50, 500, 62.50, 750$ and $87.50$. From the plot we observe that the energy skewing angles are vary up to $13.20$. The energy skewing angles for transmitted qSV waves (see figure 5b) vary up to $37.50$. 

![Figure 5a and 5b - Energy skewing angles](image)
Figure 5 - Energy skewing angles for the transmitted a) quasi pressure and b) quasi shear vertical waves when an incident quasi pressure wave at an interface between two general transversely isotropic austenitic steel materials. The columnar grain orientation of the upper medium is 0° and the lower medium exhibiting columnar grain orientations (0°, 12.5°, 25°, 37.5°, 50°, 62.5°, 75°, 87.5°).

Figure 6a shows the energy reflection and transmission coefficients for the quasi pressure wave incident at an interface between two transversely isotropic media where the upper medium has columnar grain orientations parallel to the interface and the lower medium has 45° columnar grain orientations. The critical angles for the transmitted quasi pressure waves are located at -71° and 71° incident angles. The critical angle in an anisotropic medium is defined as the incident angle at which the reflected or transmitted energy flux vector is parallel to the boundary [11]. Beyond the critical angle the incident energy is redistributed into other wave modes. The conversion coefficients (see figure 6b) for the reflected and transmitted quasi shear vertical waves are below 10% of the incident quasi pressure wave energy.

Figure 6 - a) Energy reflection and transmission coefficients and b) energy conversion coefficients for the reflection and transmission for a quasi-pressure wave incident at an interface between two transversely isotropic austenitic steel materials (X6CrNi1811). The columnar grain orientation of the upper medium is 0° and that of the lower medium is 45°.
Figure 7 - a) Energy reflection and transmission coefficients and b) energy conversion coefficients for the reflection and transmission for a quasi-pressure wave incident at an interface between two transversely isotropic austenitic steel materials (X6CrNi1811). The columnar grain orientation of the upper medium is 45° and that of the lower medium is 0°.

Figure 8 - Energy reflection coefficients for quasi pressure and quasi shear vertical waves for a) quasi pressure wave b) quasi shear vertical wave incident at a stress free boundary of a transversely isotropic austenitic steel materials (X6CrNi1811). The columnar grain orientation of the medium is 45°.

Reciprocity relations [8] can be used to verify the validity of the obtained reflection and transmission energy coefficients. In the first case, a quasi-pressure wave incident at an angle of 24° on a stress-free boundary of an austenitic steel material. The energy coefficient, associated to the reflected quasi shear vertical wave (see figure 8a), is 0.1321 and the corresponding reflected phase angle of qSV is 10.8°. In the second case, the quasi shear vertical wave incident at an angle of 10.8° on a stress free boundary of an austenitic steel material. Here the energy coefficient, associated with the reflected quasi pressure wave (see figure 8b), is 0.1321. Thus, the energy coefficients in both cases are the same and the corresponding reflected phase angle of qP is 24°. The reciprocity relations are valid for all the reflected and transmitted waves.
CONCLUSIONS

A complete analytical simulation tool is developed in order to evaluate the acoustic energy skewing angles and energy coefficients for the reflected and transmitted plane elastic waves, in the context of the ultrasonic wave propagation at a boundary between acoustically transversely isotropic austenitic steel materials. The computational results show that the acoustic energy skewing angles and reflection and transmission coefficients are greatly influenced by the columnar grain orientation of the austenitic steel materials. The computed results for the analytical solutions enable us to select the incident domain of angles for which the energy skewing is the minimum and the transmission factor is the maximum. The presented theory is extended to the optimization of experimental parameters, such as determining the optimal probe position where one can attain the maximum response from the flaw situated in the transversely isotropic austenitic weld materials. The experimental investigations are in progress in order to validate the simulation results.

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