FDR FOR NON DESTRUCTIVE EVALUATION: INSPECTION OF EXTERNAL POST-TENSIONED DUCTS AND MEASUREMENT OF WATER CONTENT IN CONCRETE

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Introduction

The water content in concrete (nuclear structures and nuclear waste repositories) is a major topic to understand and predict the behaviour at the end of the operating period. That is the reason why ANDRA and EDF are involved in research programs dedicated to concrete Thermo-Hydro-Mechanical (THM) modeling and to in situ water content assessment technologies [1].

Another example concerns the bridges which include “external” post-tensioned cables to reinforce these structures. These cables are not into the concrete material, hence potentially accessible for measurement. They are generally placed in High Density Poly-Ethylene (HDPE) ducts, where the residual space is filled under high pressure with a cement grout intended to prevent corrosion. Nevertheless, in some cases, the cables breaking occur in non-protected zones [2, 3] due to the presence of a “white paste” or grout voids.

To remote diagnosis anywhere and in real time of post tensioned ducts or to measure the water content in concrete, we propose a structural health monitoring method based on Frequency Domain Reflectometry (FDR). Today’s, advanced reflectometry methods provide an efficient solution for the fault-detection and for their diagnosis in electric transmission lines [4, 5].

This paper presents a direct model of the FDR method based on Telegrapher’s equations. An analysis of these signals, based on scattering theory, enables one to retrieve the impedance distribution of the electric line. The impedance distribution depends on damage into the duct or the water content in concrete. An inversion algorithm is realized with software ISTL provided by INRIA. FDR method has been applied to two real cases: measurement of the water content in concrete and the diagnostic of the external post tensioned duct.

Theoretical Background

The idea presented in this paper is based on examining the concrete walls and bridge cables thanks to the FDR method as they were a transmission line. The transmission parameters along an electric line (as velocity and attenuation) depend on the complex permittivity of the material that is placed between the wires of the electric line. It is well known that permittivity depends on the nature of the dielectric material which is function of water content, void, etc. The electric line is connected to a calibrated vector network analyser (VNA) which measures the reflection coefficient $S_{11}(\omega)$.

The FDR method is detailed through the diagnostic of the external post-tensioned cable. This first study allows us to give the way for measuring the water content in concrete.

For the post-stressed cable, we consider which is filled with cement, white paste and air. We characterize each material by the relative complex permittivity $\varepsilon_n = \varepsilon'_n - j\varepsilon''_n$ ($j=\sqrt{\text{−1}}$) where $n$ can be air, HDPE, cement, white paste or concrete which the relative permittivity is respectively 1, 2.5, 5, 63+j2.6/(2πε₀ω) and of the order of 10.

The height of each material $h_n$ in the transversal section depends on the position variable $z\epsilon$ [0,1] along the electric line and $d$ represents the diameter of the cable (see Figure 1).
In the post-tensioned duct diagnosis case, the principle of our method consists in adding two electrically conductive tapes of width $w$ on the upper and lower parts of the duct. Initially, we assume that cable model (Figure 1a) is equivalent to a coplanar geometry (Figure 1b). Practical micro-strip lines have width-to-height ratios $w < d$. The electric conductors and the ducts form an electrical line between tapes. Given a post-stressed cable, we can calculate an apparent permittivity $\varepsilon_{\text{app}}$ using the basic formula of a multi-layers planar medium’s permittivity [6]:

$$\frac{1}{\varepsilon_{\text{app}}(z)} = \frac{1}{d} \sum_{n} h_n(z)$$

In the water content measurement case (Figure 1c), the electric line is constituted by two parallel wires placed into a material (relative permittivity $\varepsilon_r$). The table 1 gives the different electric line parameters for the two electric lines used.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>For the strip-line</th>
<th>For the two parallel wires</th>
</tr>
</thead>
<tbody>
<tr>
<td>Permittivity $\varepsilon_{\text{rel}}(z)$</td>
<td>$\varepsilon_{\text{app}}(z) + 1 \frac{\varepsilon_{\text{app}}(z) - 1}{\sqrt{1 + 12d/w}}$</td>
<td>$\varepsilon_{\text{rel}}$ of the material</td>
</tr>
<tr>
<td>$L(z) = \mu \eta$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$C(z) = \varepsilon'(z) \eta$</td>
<td>with $\eta = \frac{1}{2\pi} \ln \left( \frac{8d}{w} + \frac{w}{4d} \right)$</td>
<td>$\eta = \frac{1}{\pi} \arcsin \left( \frac{D}{2a} \right)$</td>
</tr>
<tr>
<td>$G(z) = \omega C(z) \frac{\varepsilon''}{\varepsilon'}$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$R(z)$ with $\delta_r = \frac{1}{\sqrt{\mu \sigma f}}$</td>
<td>$\frac{2}{\sigma w \delta_r}$</td>
<td>$\frac{2}{\sigma \pi 2a \delta_r}$</td>
</tr>
</tbody>
</table>

The high-frequency impedance $Z_{\omega}(z)$, the loss terms $\frac{R(z)}{L(z)}$ and $\frac{G(z)}{C(z)}$ are related to the material permittivity $\varepsilon$. Finally, the high frequency impedance is given by [6]

$$Z_{\omega}(z) = \frac{R(z) + jL(z)\omega}{\sqrt{G(z) + jC(z)\omega}}$$
Electric model of the transmission line

The electric signal transmission is generally modeled using the "telegrapher's equation" and characterized by the line parameters R, L, C and G, representing, respectively, the resistance, the inductance, the capacitance and the shunt conductance. In the harmonic regime, telegrapher's equations are written as follow:

$$\frac{\partial V}{\partial z} - j \omega L \frac{\partial I}{\partial z} + R(z) I = 0$$
$$\frac{\partial I}{\partial z} - j \omega C V + G(z) V = 0$$

where the intensity $I(\omega, z)$ and the voltage $V(\omega, z)$ depend on the space position $z$ and the pulsation $\omega = 2\pi f$ where $f$ is the frequency in a range of frequencies from 1 MHz to 1GHz with the step $\Delta f = 1$ MHz.

The following boundary conditions are verified at both ends:

$$V(\omega, 0) - Z_S(\omega) I(\omega, 0) = V_S(\omega)$$
$$V(\omega, l) - Z_T(\omega) I(\omega, l) = 0$$

$Z_S(\omega)$ is the internal source impedance of the VNA connected at $z=0$ and $Z_T(\omega)$ is the terminal impedance connected at $z=l$. $V_S(\omega)$ represents the harmonic source generator.

Figure 2: Transmission line model with one source generator

Direct problem: Riccati Equation

Reflectometry technique consists in exciting a cable with a given signal and analyzing the returning signals, composed by all signals reflected by the heterogeneities of the line. Reflectometry experiments lead to observing voltages and currents as a function of the time at some position: only the traveling times and amplitudes of waves are accessible by such experiments. A fault can only be localized in terms of the traveling time of the reflected test wave starting from the test point. It becomes natural to work with the traveling time instead of the spatial coordinates. The traveling time is also called the electrical distance.

The Liouville transformation allows to transform the spatial coordinate $z$ into the electrical distance $x$, which writes as follow:

$$x(z) = \int_0^z \frac{ds}{v(s)}$$

$x(z)$ corresponds to the wave traveling time from position 0 to $z$.

The electrical distance $\tau = x(l)$ corresponds to the physical length $l$ and we will be able to write any function $g(x) \equiv g(x(z))$.

The wave velocity associated with the complex permittivity is defined as:

$$v(z) = \frac{c}{\sqrt{\varepsilon'(z) \left( \frac{1}{\varepsilon(z)} + \frac{1}{\varepsilon''(z)} \right)}}$$

where $\tan\delta(z)^2 = \frac{\varepsilon''(z)}{\varepsilon'(z)}$.

Reflection coefficient is modeled by a set of Riccati equations [7]. For a fixed frequency $f \in [1\text{MHz}, 1\text{GHz}]$, the corresponding Riccati equation, denoted by $R(f)$ shows the behavior of the reflection coefficient along the line $[0, \tau]$. We define the high-frequency impedance such as:
then the set of Riccati equations $R(f)$
$$\frac{d}{dx} Z_{app}(x, \omega) = \left(j\omega + \frac{G(x)}{C(x)}\right) Z^{-1}_\infty(x) Z^2_{app}(x, \omega) - \left(j\omega + \frac{R(x)}{L(x)}\right) Z_\infty(x)$$

$$Z_{app}(\tau, \omega) = Z_T(\omega)$$

where $\omega = 2\pi f$.
The reflection coefficient $S_{11}(\omega)$ is defined by
$$S_{11}(\omega) = \frac{Z_{app}(0, \omega) - Z_s}{Z_{app}(0, \omega) + Z_s}$$

where $Z_s$ is the internal impedance of source.

**Inverse problem: ISTL software**

The inversion method is processed by a software ISTL (Inverse Scattering Transform Lossless) developed at INRIA by the team SISYPHE. ISTL is based on an inverse scattering method applied to transmission lines [7].
The inverse scattering transform consists of the following steps for computing the profile of $L(x)/C(x) = Z_\infty^2(x)$ from the reflection coefficient $S_{11}(\omega)$ (see [2] for details).

1. Let the Fourier transform of the reflection coefficient $S_{11}(\omega)$ be
$$\rho(x) = \frac{1}{2\pi} \int_{-\infty}^{+\infty} S_{11}(\omega) e^{-j\omega x} d\omega$$

2. Solve the integral equations (known as Gelfand-Levitan-Marchenko equations) for its unknown kernels $A_1(x, y)$ and $A_2(x, y)$:
$$A_1(x, y) + \int_x^y A_2(x, s) \rho(y + s) ds = 0$$
$$A_2(x, y) + \rho(x + y) + \int_y^x A_1(x, s) \rho(y + s) ds = 0$$

3. Compute the heterogeneity function through
$$\frac{1}{Z_\infty(x)} \frac{d}{dx} Z_\infty(x) = 2A_2(x, x)$$

**Simulations and Validations**

In order to validate the proposed method for post-tensioned cable diagnosis, results of numerical simulation are presented in this section. To simulate reflectometry measurements, reflection coefficients are computed by solving the Riccati’s equation (2).

In the simulation, we consider a duct filled with cement, white paste and air. A material widths profile (Figure 3) is considered together with the permittivity’s values shown in Table 1.

For each excitation frequency in $\omega$ the reflection coefficient of a duct was calculated using (2) and (3). A $Z_\infty(z)$ profile is simulated in both $z$ and $x$ coordinates in Figure 4. The simulated reflection coefficient $S_{11}(f)$ (modulus and phase) is shown in Figure 5.

ISTL software was designed to retrieve the characteristic impedance for a lossless transmission line. Nevertheless, the algorithm is robust and it works also for lossy transmission line, that means it is capable of finding the impedance defined in (1). The $Z_\infty$ profile computed through ISTL (Figure 6) is compared to the original simulated profile in Figure 4: oscillations of retrieved impedance are due to the terminal impedance mismatch.

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We remind that the inverse scattering transform computes the high frequency as a function of \( x \). In practice it would be more useful to inspect the ratio as a function of \( z \), the true spatial coordinate of the transmission line. Like all reflectometry methods, the information obtained by observing incident and reflected waves is related to the wave propagation time \( x \).

**Experimental results**

Experiments were carried out to check the validity of the numerical simulations. Two mockup ducts of 6 m length and with a diameter of 90 mm have been considered. They are equipped with 12 steel cables of 15 mm diameter (Figure 7). These ducts were fixed on a frame reproducing the curvature of pre-tensioned ducts on a stack. The vents are positioned on either side of the top point, i.e., in practice each side of the deviator. Moreover, wooden wedges were inserted between the metal structures supporting the ducts. The cement was injected into duct A using a standard Superstresscem with a ratio Water/Cement 0.35. In duct B it has
been injected with cement together with a ChrysoGT adjuvant (Water/Cement 0.65) in order to promote the appearance of white paste.

![Figure 7: External post stressed duct (IFSTTAR laboratoires, France)](image)

$S_{11}$ measurements are performed using a VNA of Anritsu MS2026. Ducts are equipped with metal strips on the upper and lower parts of the ducts. The strip is composed of adhesive aluminum rubber (thickness 0.1mm and width 50mm). This pair of strips is connected to a coaxial cable through clips. The coaxial cable is linked to the VNA port. The measurements are performed on a frequency range of 1 MHz to 1 GHz with a frequency step of 1 MHz (1000 points).

The VNA coaxial cable is connected to the duct through clips and at the end of duct another pair of clips connects the metallic strips to known terminal impedance ($Z_T$).

On each cable, we performed three different experiences connected to three different terminal impedances branched at the end of the ducts (open circuits, short circuits and an impedance of 50 Ohms).

The ducts have been inspected by hammer auscultation. Figure 8 shows the location of void areas and vents on ducts. Cables have not been open so the exact profile of the heights of materials is unknown.

![Figure 8: Location of void areas (hatched) and vents (o) on ducts A and B](image)

**Data inversion**

$S_{11}(f)$ measurements of VNA has been fed to ISTL software. Results of the inversion software are in figure 9. Each figure shows a comparison between the three different experiences: the impedance profiles match until the end of the line where the terminal impedances are. ISTL is capable of detecting these three terminations. Both figures shows two bumps of impedance profile at both ends of the cable (around 5 ns and 40 ns): these variations of impedance correspond to the ruptures of impedance caused by the connection between the coaxial cable and tapes. On the other side, ISTL retrieve the impedance profile as a function of electrical distance. The physical end of the electric line is determined by adjusting the terminal impedance ($Z_T$).

From the impedance profile, we deduced the real part of the apparent permittivity and consequently lossless wave velocity. We were able to plot the impedance profiles as functions of physical distance. These inversion results have been compared to hammer auscultation (see Figure 9). The presence of void is detected clearly as an increment of the retrieved impedance: we can see a rise of impedance's values in the middle part of duct, where the void is supposed to be.

The oscillations at the beginning of the duct are also due to the ruptures of impedance caused by the connection. Moreover, the vents create a bump of impedance.
Conclusion and further direction

In this paper, we have presented a new non-destructive testing method based on FDR method. Associated to FDR measurement, we used the software Inverse Scattering Transmission Line (ISTL) as robust inversion algorithm. This method is validated to diagnose the external post-tensioned duct. This diagnostic method is able to detect the heterogeneities (void, white paste) of materials along a cable. We have developed an equivalent permittivity model taking the dispersion electromagnetic properties into account, allowing us a reflection coefficient simulation using the Telegrapher's model. Theoretical work, taking the properties of the materials and propagation phenomenon into account, enables one to predict the signal measured. Moreover, once the duct equipped with a strip line, the diagnosis can be done remotely to improve the safety of personnel in the event of a cable break. As the capacitive method, the method presented here, is able to distinguish a shrinkage and a void, which is not possible with classical inspection method using a hammer. Further research of the FDR method concerns the nuclear applications to measure the water content of the concrete of containment, cooling tower or also nuclear waste repositories. The idea is to take electromagnetic dispersion properties of the concrete into account to improve the measurement of a water content and to measure the moisture profile along the TDR probe.

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Bibliography