Improved Total Focusing Method (ITFM) for ultrasonic imaging of thick walled welds based on ray-tracing algorithm
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ABSTRACT
Thick walled welds are widely applied in key components of nuclear reactors. Ultrasonic imaging is one of the most commonly used Non-Destructive Evaluation (NDE) technique for thick walled welds, which helps to identify and locate the defects. Total Focusing Method (TFM) is a useful technique to generate images based on a matrix of ultrasonic signals. The signal matrix can be acquired by a transducer array using Full Matrix Capture (FMC) technique. The application of traditional imaging methods often leads to deviations, as the materials are considered as isotropic ones. Improved Total Focusing Method (ITFM) is developed for ultrasonic imaging of thick walled welds, in order to have the anisotropy taken into consideration. The ultrasonic propagation is estimated by an efficient ray-tracing algorithm mainly derived from Dijkstra algorithm. The propagation paths can be acquired by minimizing the propagation time between two selected points, based on Fermat’s principle. In this way, the time-of-flight of each pixel in an ITFM image can be calculated.

To evaluate the imaging performance, a simulation model was established, consisting of the isotropic base material and the anisotropic weld. The ultrasonic field of the model was calculated by a multi-Gaussian beam modeling method, which was further utilized to generate the FMC signal matrix for imaging. TFM and ITFM images were generated and their imaging performances were compared and discussed.

INTRODUCTION
Thick walled welds are widely applied in key components of nuclear reactors, which are usually the critical elements for these large components’ structural integrity. Non-Destructive Evaluation (NDE) of the structural health is important for these components, both at the manufacturing stage and throughout the operational life. Ultrasonic imaging is one of the most commonly used NDE technique for thick walled welds, which helps to identify and locate the defects. However, ultrasonic propagation in thick walled welds is often complicated, due to ultrasonic waves’ scattering and deflection caused by the anisotropic coarse-grain microstructure \(^1\). The application of traditional imaging methods often leads to deviations, as the materials are assumed as isotropic ones.

Total Focusing Method (TFM) is a useful technique to generate images based on a matrix of ultrasonic signals. The signal matrix can be acquired by a transducer array using Full Matrix Capture (FMC) technique. Unlike other phased array technique, TFM can make full use of all the signals in the FMC matrix. In a TFM image, the intensity of each pixel depends on its time-of-flight according to the ultrasonic propagation \(^2\). However, the ultrasonic propagation is calculated with the assumption of isotropic materials in traditional TFM, and brings about deviations inevitably.

To have the anisotropy taken into consideration, we developed Improved Total Focusing Method (ITFM) suitable for imaging of thick walled welds. For anisotropic materials, it is important to estimate the ultrasonic propagation, which is usually referred to as the ray-tracing problem. Rather than the commonly used but time-consuming finite element methods, we prefer the semi-analytical ray-tracing methods. Based on the works of Ogilvy \(^3\), Nowers et al. \(^4\) and many other researchers, we have applied an efficient ray-tracing algorithm in ITFM, mainly derived from the Dijkstra algorithm \(^5\). The propagation paths can be acquired by minimizing the propagation time between two selected points based on Fermat’s principle. In this way, the time-of-flight of each pixel in an ITFM image can be calculated.

To evaluate the imaging performance, a simulation model was established, consisting of the isotropic base material and the anisotropic weld. The ultrasonic field of the model was calculated by a multi-Gaussian beam modeling method \(^6\) - \(^7\), which was further utilized to generate the FMC signal matrix for imaging. TFM and ITFM images were generated and their imaging performances were compared and discussed.
PRINCIPLES OF ITFM

ITFM contains two major algorithms, including the ray-tracing algorithm and the imaging algorithm. Both the algorithms are based on the propagation paths estimation in anisotropic welds. In this section, the principles of the algorithms will be described in detail.

Ultrasonic propagation in anisotropic welds

Although all metallic crystals are inherently anisotropic, the material can be considered as an isotropic one when the ultrasonic wavelength is much greater than the size of grains. However, in materials like thick-walled austenitic welds, the anisotropic effect can hardly be ignored, as the grains often grow large enough compared with the wavelength. In isotropic materials, ultrasonic waves propagate in two modes: longitudinal and shear. Propagating modes get more complicated in anisotropic materials. Three quasi-modes will appear, including a quasi-longitudinal mode \((qL)\) and two quasi-shear modes \((qS1)\) and \((qS2)\). When ultrasonic waves propagate in a certain material, the distribution of velocities contributes the most influence on the propagation paths. There are two kinds of velocities, including the phase velocity \(v^p\) that indicates the moving velocity of the wavefront, and the group velocity \(v^G\) at which the envelope of a wave packet propagates. In anisotropic materials, \(v^p\) and \(v^G\) are usually not the same, in both magnitude and direction.

The analysis of the wave propagation often starts with the material’s elastic behavior, which is described by Hooke’s law \(^5\) as

\[
\sigma_{ij} = [C_{ijkl}] \varepsilon_{kl} = \frac{1}{\rho} \left( \varepsilon_{ij} - \frac{1}{3} \varepsilon_{kk} \delta_{ij} \right)
\]

where \(\sigma_{ij}\) is the stress tensor, \(\varepsilon_{ij}\) is the strain tensor, \(C_{ijkl}\) is the elastic stiffness tensor, and \(\rho\) is the density of the material.

Consider Newton’s second law and the characters of \(\sigma_{ij}\) and \(\varepsilon_{ij}\), one can get

\[
\rho \left( \frac{\partial^2 u_i}{\partial t^2} - \frac{1}{\rho} \left( \varepsilon_{ij} - \frac{1}{3} \varepsilon_{kk} \delta_{ij} \right) u_i \right) = 0
\]

where \(u_i\) is the particle displacement, and \(n_i\) is the directional cosine of a certain direction.

Equation (3) can be written in the form known as the Christoffel Equation

\[
\Gamma_{ij} = \rho \left( \frac{\partial^2 u_i}{\partial t^2} - \frac{1}{\rho} \left( \varepsilon_{ij} - \frac{1}{3} \varepsilon_{kk} \delta_{ij} \right) u_i \right)
\]

where \(\Gamma_{ij}\) is Green-Christoffel acoustic tensor, given as

\[
\Gamma_{ij} = C_{ijkl} n_i n_j n_k
\]

The simplification of Equation (5) has been described in detail in the appendix of 8), referring to the components in Equation (2).

The Christoffel Equation (4) actually indicates an eigenvalue problem in a way of \(A x = \lambda x\). The solution yields three eigenvalues \(\lambda_M\) and corresponding eigenvectors \(x_M\), associated with the three propagation modes \(qL, qS1, qS2\). The phase velocity of each mode can be calculated as

\[
v^p_M = \sqrt{\frac{\lambda_M}{\rho}}; M = qL, qS1, qS2
\]

Based on the definition of group velocity, \(v^G\) of each mode can be calculated as

\[
(v^G)_M = \frac{C_{ijkl} n_i (x_M)_j (x_M)_k}{\rho v^p_M}; M = qL, qS1, qS2
\]

And the directional cosines of \(v^G\) are given by

\[
n^G_i = \frac{(v^G)_M}{\sqrt{(v^G)^2_i + (v^G)^2_j + (v^G)^2_k}}
\]

When considering the thick walled welds discussed in this research, the base material can be dealt as an isotropic material, while the austenitic weld is generally anisotropic. Fortunately, due to
its structural features, the weld can be considered as a transversely isotropic material, whose stiffness matrix can be simplified as:

\[
C = \begin{bmatrix}
    C_{11} & C_{12} & C_{13} \\
    C_{12} & C_{11} & C_{13} \\
    C_{13} & C_{13} & C_{33} \\
    C_{44} & C_{44} & C_{66} \\
    C_{14} & C_{14} & C_{66}
\end{bmatrix}
\]

\[C_{66} = C_{11} - C_{13} \frac{2}{2}\]  \hspace{1cm} (9)

Therefore, the ultrasonic testing problem of thick walled welds can be simplified to planar problem in \(x_1-x_3\) plane (\(x_2\) is defined as the direction along the welds).

**Ray-tracing algorithm**

Dijkstra algorithm is an efficient ray-tracing algorithm which can find the path between two arbitrary points with the minimum cost. Nowers et al. provide a practical procedure of Dijkstra algorithm’s application in the ray-tracing of ultrasonic propagation. As the foundation of Dijkstra algorithm, the cost between each two neighbor nodes should be defined clearly. Here, two nodes are regarded as “neighbors” when the distance between them is less than a given radius \(R_0\). As shown in Figure 1, nodes B and C are the neighbors of node A, while node D does not belong to the neighbor nodes. The ray-trace between A and B is also displayed in Figure 1. A random network of nodes is allocated when performing the ray-tracing algorithm, which can significantly increase the computing efficiency.

Based on Fermat’s principle, the ultrasonic energy always propagates along the path with the least time-of-flight. Thus, if we define the cost between two neighbor nodes as the time-of-flight between them, Dijkstra algorithm can be used to find the energy propagation paths. The cost between A and B is defined as

\[
\text{Cost}_\theta(A, B) = \frac{d_{AB}}{v_{GL}^\theta(\gamma)}; \gamma = \theta_{AB} - \beta_A
\]

where \(d_{AB}\) is the distance between A and B, \(\theta_{AB}\) is the angle between A and B, \(v_{GL}^\theta(\gamma)\) is the group velocity of \(qL\) mode, and \(\beta_A\) is the local anisotropic orientation at node A.

Based on Equations (4)-(8), the group velocities can be solved as a function of the directional angle, described as \(v^\theta(\gamma)\). The directional angle \(\gamma\) is the angle between the propagation direction and the local anisotropic orientation, calculated as \(\theta_{AB} - \beta_A\) in Equation (10). In fact, the velocity may be either that associated with node A, B, or a combination of the two. The choice was found to have a negligible effect on the results, so here we use \(v^\theta\) associated with node A.

Functionally, the cost of each node \(\text{Cost}(n)\) and the path information at each node \(\text{Path}(n)\) should be stored and calculated in Dijkstra algorithm, which obeys the following searching rules:
1. Choose a starting node according to the location of transducers, and set it as the current node \( n_0 \).

2. Assign every node an infinite cost except the starting node (zero cost). Set \( \text{Path}(n) = -1 \) for every node expect the starting node (set to zero).

3. Mark all nodes as unvisited ones.

4. Consider all the unvisited neighbor nodes \( n_i \) of \( n_0 \), and calculate the cost between \( n_i \) and \( n_0 \) as \( \text{Cost}_0(n_0, n_i) \), according to Equation (10).

5. If \( \text{Cost}(n_i) \) is larger than \( \text{Cost}(n_0) + \text{Cost}_0(n_0, n_i) \), update the information of node \( n_i \) as

\[
\text{Path}(n_i) = n_0; \quad \text{if Cost}(n_i) > \text{Cost}(n_0) + \text{Cost}_0(n_0, n_i)
\]

(11)

6. Once all neighbor nodes are considered, mark \( n_0 \) as a visited node.

7. Set the unvisited node with the least cost as the newest current node \( n_0 \), and repeat 4-6.

8. Repeat until all the nodes have been visited.

**Imaging algorithm**

Supposing that there are \( N \) elements in a linear array which are applied to acquire FMC data. As each element acts as a transmitter or receiver at each time, totally \( N^2 \) signals can be acquired as a signal matrix, described as

\[
S(t) = [S_{ij}(t)]; \quad i, j = 1, 2, ..., N
\]

(12)

where \( S_{ij}(t) \) represents the signal transmitted from element \( i \) and received by element \( j \).

Similar with TFM, ITFM imaging requires the time-of-flight of all the points in the imaging area, which has already been calculated by Dijkstra algorithm. The starting node should be chosen as the location of each element \( i \). Here we apply the reciprocity principle, so a ray transmitting from element \( i \) and received by element \( j \) is the same with a ray transmitting from \( j \) and received by \( i \). In this way, the Dijkstra algorithm should be repeated for only \( N \) times. The intensity of a certain point \( P \) in the ITFM image can be calculated as

\[
I_P = \sum_i \sum_j S_{ij}(t'_p + t'_j)
\]

(13)

where \( t'_p \) is the propagation time between point \( P \) and element \( i \), and \( \text{Cost}_i \) is the Dijkstra result with the starting point being element \( i \).

Unlike ray-tracing algorithm, the imaging method here is based on a regular node network, which can facilitate the calculating process of imaging. Therefore, a transformation from the random network to this regular network should be calculated.

**MODELLING AND SIMULATION**

We used the multi-Gaussian beam modelling method to establish the simulation model. This method models the transducer wave field with a superposition of Gaussian beams. A Gaussian beam can be computed efficiently even for complicated situations. With proper weighting factors given by Wen and Breazeale, multi-Gaussian beams can be superposed based on paraxial approximations, and this method can perfectly fit the actual wave field for the high frequency ultrasound near the axis and in the far field.

**Principle of multi-Gaussian beam modeling**

The principle of multi-Gaussian beam modeling is also based on Christoffel Equation (4) mentioned above, which has been described in detail in several references. In our research, the modelling of multilayered anisotropic media is the critical issue, which can be efficiently addressed through the use of an ABCD matrix approach. Specifically, only longitudinal waves or \( qL \) mode waves are considered in this research. After the ultrasonic wave has been transmitted/reflected at \( m \) interfaces between \( m+1 \) different anisotropic media, the normalized velocity wave field \( \psi_{v(r)}(x,0)/v_0 \) of a piston transducer can be described as
\[ \frac{v_{m+1}(x, \omega)}{v_0} = p \sum_{n=1}^{10} A_n \left( \prod_{k=1}^{n} T_{k,k+1} \prod_{k=1}^{n} \frac{\sqrt{\det[M_k(D_k)]}}{\sqrt{\det[M_k(0)]}} \right) \exp \left[ i \omega \left( \sum_{k=1}^{n} \frac{1}{2} Y^T [M_{n+1}(D_{n+1})]_{n} Y \right) \right] \]  

(14)

and

\[ [M_k(0)]_{n} = \frac{2iB_k}{\omega \alpha^2} \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \]

(15)

The various terms in Equations (14) and (15) are as follows: \( \omega \) is the circular frequency. \( D_k \) is the distance that the beam has travelled in media \( k \). \( p \) is the polarization unit vector for the wave in media \( m+1 \). \( T_{k,k+1} \) is the transmission coefficient for the wave propagating from media \( k \) into media \( k+1 \). The point \( x \) is a point in media \( m+1 \) at which the velocity is evaluated. \( Y=(y_1,y_2)^T \) is the first two coordinate values of point \( x \) in the local slowness coordinate (with \( y_3=0 \)). \( (A_n,B_n) \) are the weighting coefficients obtained by Wen and Breazeale \(^9\) as listed in Table 1.

| Table 1 - Weighting coefficients for multi-Gaussian beam modeling |
|-----------------|-----------------|
| \( n \) | \( A_n \) | \( B_n \) |
| 1 | 11.428+0.95175i | 4.0697+0.22726i |
| 2 | 0.06002-0.08013i | 1.1531-20.933i |
| 3 | 4.2743-8.5562i | 4.4608+5.1268i |
| 4 | 1.6576+2.7015i | 4.3521+14.997i |
| 5 | -5.0418+3.2488i | 4.5443+10.003i |
| 6 | 1.1227-0.68854i | 3.8478+20.078i |
| 7 | -1.0106-0.26955i | 2.5280-10.310i |
| 8 | -2.5974+3.2202i | 3.3197-4.8008i |
| 9 | -0.14840-0.31193i | 1.9092-15.820i |
| 10 | -0.20850-0.23851i | 2.6340+25.009i |

As described in Equation (15), the matrix \( M \) can be calculated by an ABCD approach. For the propagation in media \( k \), we have

\[ A_k^i = D_k^i \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}, B_k^p = D_k \begin{bmatrix} v_k^p & -2(C'_{12})_k \\ -2(C'_{12})_k & v_k^p \end{bmatrix}, C_k^p = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} \]

(16)

And for the transmission at the interface \( k \) (supposing all the interfaces are planar), we have

\[ A_k' = \begin{bmatrix} \cos(\theta)_k + (A_k')_{11} \sin(\theta)_k \\ \cos(\theta)_k + (A_k')_{11} \sin(\theta)_k \end{bmatrix}, B_k' = C_k' = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} \]

\[ D_k' = \begin{bmatrix} \cos(\theta)_k + (A_k')_{11} \sin(\theta)_k \\ \cos(\theta)_k + (A_k')_{11} \sin(\theta)_k \end{bmatrix} \]

(17)

In Equations (16) and (17), \( (\theta)_k \) and \( (\theta)_k \) are the incident and refraction angle at interface \( k \), and \( A_k', A_k', C_{11}', C_{12}', C_{22}' \) are the parameters relating to anisotropy. Particularly, for the transversely isotropic media as austenite welds, we have \( A'_{11} = C'_{11} = 0 \), and other parameters can be calculated by analyzing the slowness surface of the media \(^5\). Thus, all the ABCD matrixes become diagonal, and we can get that \( A_k' D_k' = I \). In this way, Equation (15) can be rewritten as
Combining Equations (14) and (18), the ultrasonic wave field of a transducer can be simulated analytically by the multi-Gaussian beam modeling method.

Establishment of simulation model

The simulation model used here consists of the isotropic base material and the anisotropic weld, which is shown in Figure 2. Some of the parameters applied for the model are listed in Table 2. The base material is isotropic with a longitudinal wave velocity as \( v_L = 5820 \text{m/s} \). The weld is anisotropic (transversely isotropic), and is divided into five layers. The stiffness constants of the weld are the same with that in 1), and the orientations of anisotropy in the layers are set according to Ogilvy’s work 3). The calculation results of \([v^G(\gamma)]_{\text{eq}}\) as a function of the directional angle are given in Figure 3, based on the parameters in Table 2.

The array of transducers is placed on the top of the model with an interval of 2mm. There are totally 61 transducer elements in this array, and their \(x\) coordinate values vary from -60mm to 60mm. Each transducer can act as a transmitter or a receiver, and all the combinations of a transducer and a receiver should be repeated to acquire FMC data. Practically, this FMC experiment can be achieved by using at least two transducers.

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![Figure 2 - Simulation model](image)

### Table 2 – Parameters of simulation model

<table>
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<th>geometric parameters</th>
<th>( W_0 )</th>
<th>( H_0 )</th>
<th>( W_f )</th>
<th>( W_B )</th>
<th>( W_{\gamma} )</th>
<th>( \Delta_a )</th>
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Simulated signals

Based on the above model, the ultrasonic wave field of each transducer was simulated (with a given incident angle) by the multi-Gaussian beam modeling method. The radius of each transducer is 3mm, and the main frequency is 1MHz. As an example, the wave field generated by element 11 (-40,0) with an incident angle of 55deg is shown in Figure 4.

The simulated wave field was further utilized to generate the time-domain signals in an FMC matrix $S_{ij}(t)$, based on the method described by Kim et al.\(^\text{10}\). Three side-drilled holes with diameter of 2mm were added to the simulation model, which are supposed to hardly affect the wave field. The locations of these man-made defects are respectively (0,30), (0,50) and (0,70). The ultrasonic signals reflected by these defects were included in $S_{ij}(t)$. Figure 5 displays some of these simulated signals.
IMAGING RESULTS

Based on the simulated signals $S_i(t)$, an ITFM image was generated. An example of ray-tracing results is shown in Figure 6. For comparison, a TFM image was also generated, in which the model is assumed to be isotropic with the same velocities in the whole testing zone. The imaging results are compared in Figure 7.

ITFM proposed in this paper appears to have better imaging performance than traditional TFM. In Figure 6(b), the locating deviations of defects are obvious as a result of the isotropic assumption. The ITFM image in Figure 6(a) performs a better result of defect locating. In both Figure 6(a) and 6(b), the defects which are closer to the transducers yield weaker signals but better locating results, while the further defects have a stronger signals with less locating accuracy. This seems to be an inherent feather of this kind of imaging methods.
CONCLUSIONS

ITFM has been developed for the ultrasonic imaging of thick walled welds, as an improved method for traditional TFM. A ray-tracing algorithm derived from Dijkstra algorithm is applied in ITFM, in order to estimate the ultrasonic propagation considering anisotropy. The multi-Gaussian beam modeling method was applied to generate simulated FMC signals for imaging. Imaging results have shown that ITFM has a better performance than traditional TFM.

REFERENCES

