Forward and Inverse Modeling in Acoustic Microscopy

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Focusing of ultrasound by an acoustic lens.

- Transducer sends plane wave down sapphire rod.
- As wave enters transmission fluid, it is focused towards line or point near surface of specimen.
- Reflected or transmitted field is detected either by the same probe, or by another similar probe.
- The wave is commonly in the form of a tone burst (narrow band) or a short pulse (broad band / time resolved). Frequency range MHz to low GHz.
- Probe moved up and down for V(z) measurements, or scanned to produce images.
Time-resolved Acoustic Microscopy (TRAM)

A-scan

B-scan

C-scan
Focused Probe Ultrasonic Scans of Single Crystal Superalloy Components

Scan over cylindrical section and ray model calculation (Every and Amulele).

Measurements carried out at the CSIR, Pretoria.

Calculations have to take into account the elastic anisotropy of the medium.
The elastic wave equation for an anisotropic solid is

\[ \rho \frac{\partial^2 \mathbf{u}}{\partial t^2} = C_{ijkl} \frac{\partial^2 \mathbf{u}}{\partial x_i \partial x_j}, \]  

(1)

where \( \rho = \) density and \( C_{ijkl} = \) elastic modulus tensor.

Equation (1) admits plane wave solutions

\[ u_i = U_i \exp \left( i (k \cdot x - \omega t) \right), \]  

(2)

Polarisation \( U_i \), wave vector \( k \), and angular frequency \( \omega \), are related by

\[ \{ C_{ijkl} \omega^2 \delta_{ij} - \rho \omega^2 \delta_{ij} \} U_i = 0. \]  

(3)

For given \( k \), there are three solutions \( \omega_j \), \( U^j \), corresponding to \( j = L, FT, ST \). The phase velocity, \( v = c_j / k = \sqrt{\rho \omega_j}, \) where \( n = k / k \) is the wave normal.

A wave packet travels at group velocity \( V = \nabla_{k} \omega(k) \).

In an anisotropic solid the phase and group velocities generally differ both in magnitude and direction, but \( V.n = v \).
Wave packet travels oblique distance $2d_\theta$ at group velocity $V_\theta$, taking time $t = 2d_\theta / V_\theta$

But $d_\theta \cos \theta = d$ and $V_\theta \cos \theta = v$, and so $t = 2d / v$, i.e. the phase velocity and actual thickness can be used in computing the pulse transit time.

An hybrid ray model has been developed for accounting for the intensities of the first and multi-pass echoes. Mode conversion is taken into account.
The jet engine turbine blade

- The blade was scanned on the high and low pressure sides at 4 heights above the platform section, each scan being defined by 20 points.
- The six rod-like projections shown are surface-normal vectors generated by the blade’s CAD model.
ONE OF THE TURBINE BLADE’S CROSS-SECTION.

SHOWN ARE THE:
EXTERNAL PROFILE AS MEASURED BY THE CMM (IN DOTS)
INTERNAL PROFILE ALSO MEASURED BY THE CMM (SPLINE FIT)
INTERNAL PROFILE AS INFERRED FROM THE ULTRASONIC DATA (IN DOTS)
Narrow Band Acoustic Microscopy

Scanning Acoustic Microscopy (SAM)

- For imaging purposes, the probe is raster scanned at fixed level above the surface of a sample, and an image generated from the reflected amplitude.

- Contrast is influenced by crystallographic orientation of grains, material elastic constants, surface topography, presence of cracks and other flaws, and many other factors.

AM amplitude image of quartz grain in granite at 0.3GHz. Ripples are caused by interference between reflection from surface and inclined grain boundary. They allow boundary to be profiled. (Ilett et al, 1984)
Phase Sensitive Acoustic Microscopy (PSAM)

- This technique (Grill et al.) retains both phase and amplitude information on the detected signal.
- Phase information allows height profiling, using the simple relation
  \[ d = \frac{\lambda \phi}{4 \pi} \], film thickness determination, etc.
- Since phase is only determined to modulo \( 2\pi \), extending the phase continuously, or “patching” is required to infer surface topography, etc.

(Left) Phase image of reflected signal from a 16mm metal ball bearing, at a frequency 362 MHz. (Right) Height variation derived from a central scan line.
Phase Singularities

- Patching breaks down at singular points where the amplitude is zero and the phase is indeterminate. See image below of rubber sample.
- In a circuit around a phase singularity, the phase changes by $+2\pi$ or $-2\pi$.

- Singularities can arise in many ways, e.g. at a step in a surface, where the step height is $\Delta h = (2n + 1) \lambda / 4$

- Knowledge of the origin of phase singularities is needed for inversion.
Experimental setup

- Modeling is based on the dynamic Green’s function
  \[ G_{33}(x, \omega) = B \int d\mathbf{k}^2 \left( \frac{A_{33}}{\nu_3} \exp \left[ i\omega x \cdot s_{33}(\mathbf{k}, \omega) \right] \right) \exp(ik_x x_x) \]
  which is evaluated using FFT’s.
- Phonon imaging is similar in principle, but uses instead very high frequency (near THz) incoherent phonons. There are no interference effects, and the intensity is given by the ray approximation.
Imaging with Thermal Phonons of \( f \approx 10^{11} \) Hz

(a) Principle of phonon imaging, (b) Typical ballistic heat pulse profile

Calculated (Every) and measured (Wolfe et al.) phonon image (acoustic intensity plot) of (001)-oriented Si crystal
Slowness surface of Si

Ray surface of GaAs, 3D view and plane sectioned
Focused pattern of GaAs(001)

Measured and calculated TAM amplitude images of a 4.8 mm thick, (100) oriented GaAs crystal for $f = 362$ MHz (Pluta, Grill & Every).
Calculated images for Si(001), for d=20mm, and f=10 MHz. The interference fringes broaden and merge as the frequency is lowered.
Ultrasound reconstruction of a Siemens’s star

FIG. 5. (a) Numerical image reconstruction of a $1 + \sin(10\phi)$ Siemens test star, as measured through a 15-mm-thick silicon crystal at 20 MHz and then reconstructed with back-propagation procedures, (b) using L waves and (c) using ST waves. (d) Modulation transfer function shown in spatial frequency space (line/mm), of the imaging process (propagation and reconstruction) for 20-MHz ST waves. The darkness of the gray scale in (d) is proportional to the amplitude.
Probing Surface Dynamics

Reflectivity

\[ R(\theta, \phi, \omega) \]

Liquid

\[ \bar{k}_{\text{f}}, \omega \quad \bar{k}_{\text{r}}, \omega \]

\[ \theta \quad \theta \]

Surface Green's Function

\[ u_i = G_{ij}(\bar{k}_{\parallel}, \omega) F_j \]

Vacuum

\[ \bar{F} \exp(i(\bar{k}_{\parallel} x - \omega t)) \]

Layer \( \rho', c'_{ijkl} \)

Substrate \( \rho, c_{ijkl} \)

\[ h \]

\[ x_1 \]

\[ x_2 \]

\[ x_3 \]
Measuring the Surface Response of Fluid-loaded Anisotropic Solids

Experimental setup for obtaining angle-time images of surface acoustic waves. (Vines, Tamura and Wolfe, PRL 74, 2729 (1995))

One focusing transducer delivers a point- or line-like impulse to the surface and the other transducer measures the displacement response or Green's function $G_{33}(t/s)$. The Fourier transform of the excitation and detection regions are limited by the angular spectrum of the lenses, and this in turn limits the $k$ of SAW structures that are observed.
Surface Displacement Response Functions $G_{33}(k, \omega)$ and $G_{33}(x,t)$ of a Semi-infinite Solid

Section of constant frequency surface of a cubic crystal. Outgoing solutions bold. Inhomogeneous solutions dashed

$G_{33}(k, \omega)$ is linear combination of 3 outgoing waves, whose amplitudes are determined by the boundary conditions, which form matrix $B$.

$$G_{33}(k, \omega) = \frac{i}{\omega} \sum_{\alpha=1}^{3} \frac{\text{adj}(B_x)^\alpha}{\det|B|} U_{33}^\alpha$$

Threshold points a, b and c correspond to surface skimming “lateral” waves. RW denotes the Rayleigh wave.

Space-time domain response obtained as follows:

$$G_{33}(x,t) = \frac{1}{2} \Re G_{33}(k \rightarrow t, \omega \rightarrow x)$$
\( G_{33}(k_y/\omega) \) for Cu(001)

\( \text{Im} G_{33}(k_y/\omega) \) for all \( k_y/\omega \) in Cu(001)

- The L, FT and ST thresholds show strong angular dependence.
- The RW degenerates with the ST threshold in the [110] direction.
- Out to about 25° on either side of the [110] direction is a pseudo surface wave (pSAW), for which \( \text{det} |\beta| \) is small but non-zero (the pole lies off the real axis).
Point focus lenses are used in the measurements (left) and the point force Green's function is shown (right).
The upper intense structures are associated with RW, the lower with pSAW. The sharp lines are due to lateral waves.
Comparison Between Measurement (Vines et al.) and Theory (Every and Briggs, Phys. Rev. B58, 1601 (1998)) for a Fiber Composite

Line focus lenses are used in the measurements (left) and the line force Green’s function is shown (right). The water cutoff at 0.65 and Scholte wave between 0.75 and 1.00 \( \mu \text{s/mm} \), lie beyond the angular spectrum of the lenses, and are not coupled to in this experiment.
The lens focuses an incident sinusoidal wave to line or point at distance $z$ from the surface of specimen. The reflected signal, $V(z)$, is called the acoustic materials signature. In negative focus it exhibits pronounced oscillations as $z$ is varied.

- Oscillations arise from interference between specular ray and ray that excites a SAW.

$$V(z) = \int_0^\phi P(\theta) \bar{R}(\theta) \exp(-2ikz \cos \theta) d\theta,$$

$P(\theta) =$ aperture function, $\bar{R}(\theta) =$ reflectivity.

For a point focus lens, $\bar{R}(\theta) = \frac{1}{2\pi} \int_0^{2\pi} R(\theta, \varphi) d\varphi =$ the complex mean reflectance function.

Point focus $V(z)$ yields extremal SAW velocities for anisotropic solids.
Fluid Loading of a Surface. Reflectivity

Reflectivity (a) \( R(\theta, \varphi = 0^\circ) \), (b) \( R(\theta, \varphi = 45^\circ) \) for the water-loaded Cu(001) surface.

The phase \( \Phi \) of \( R \) undergoes decrease of \( \sim 2\pi \) at angles corresponding to the RW (D) and pSAW (A). Smaller decreases in \( \Phi \) at lateral waves a,b,c,d,e.

\[ k_z(\theta) = \omega \sin \theta / v_{\text{fluid}} \]
Calculated and measured $V(z)$ for Si(001)[100] (Achenbach et al)

The main contributions to the integral are from the axial and the RW rays. The separation between the maxima of $V(z)$ is

$$\Delta z = \frac{v}{2f(1 - \cos \theta_R)}$$

- In case of a slow solid, or lens with small angular aperture such that $v_R < v / \sin \theta_{\text{max}}$, oscillations may still be observed in $V(z)$, with the role of the RW being taken over by the longitudinal lateral wave.
- Inversion is usually carried out with $|V(z)|$ only.
$V(z)$ for Cu(001), f=225MHz

FFT $F(k)$ of $V(z)$

There are four peaks in $F(k)$. SAW velocities

$$v_{\text{SAW}} = v_{\text{water}} \sqrt{1 - \left(1 - \frac{v_{\text{material}}}{2/\Delta z}\right)^2},$$

given in Table, are close to extremal RW and PSAW velocities.
Conclusions

- Many different versions of acoustic microscopy, using both line and point focus lenses
- Wide range of applications including measuring surface topography, sub-surface defects, thickness, thin films etc.
- Allow measurement of surface and bulk dynamics for elastic properties measurements
- It is a broad field and I have mainly concentrated on aspects I have personal experience with
- Original reprints available on request