Frequency selective arrival time estimation with the MUSIC algorithm

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Arrival time estimation

Traditional ultrasonic arrival time estimation by the Hilbert transform.

1. Add imaginary part to waveform with Hilbert transform
2. Take complex magnitude
3. Find peak

$\text{Arrival time estimation}$

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High resolution estimation and MUSIC

High resolution estimation makes use of \textit{a-priori knowledge} and \textit{physical/mathematical models} of the measurement and noise to improve precision and accuracy.

The classic example is frequency estimation. MUSIC (MUltiple SIgnal Classification) was developed for this application and performs better than traditional alternatives such as finding the peak of the Fourier transform.

Gives high resolution frequency estimates based on relatively few samples.
Time-estimation MUSIC

Model: Sum of impulses + noise.

Need to find impulse times $t_i$

$$x(t) = \sum_{i=1}^{k} A_i \delta(t - t_i) + N(t) \quad \text{(time domain)}$$

$$X(f) = \sum_{i=1}^{k} A_i \exp(-j2\pi ft_i) + N(f) \quad \text{(freq. domain)}$$
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Time estimation problem is a frequency estimation problem in the frequency domain.
Application to nonlinear systems

- Nonlinear mixing creates sum and difference frequencies
- Need frequency selective arrival time estimator
- Time-estimation MUSIC can be used to identify arrival times for narrow frequency range
- Resolution much better for very narrow frequency bands than simple filtering.
Calculation

1. \[ x[t] \xrightarrow{FFT} X[f] \] (N samples)

2. Extraction of frequency domain samples (\(M \ll N\) samples over desired frequency range).

3. Calculate autocorrelation \(a_i\) of freq. domain samples.

4. Form Toeplitz matrix \(R\) from central \(L\) samples of autocorrelation.

5. Find eigenvectors and eigenvalues of \(R\).
Calculation (continued)

Model: \( k \) impulses (signal) plus noise

5. Find eigenvectors and eigenvalues of \( R \).

6. First \( k \) eigenvectors span the signal space, remaining \( L - k \) eigenvectors span the noise space.

7. Fourier transforms of eigenvectors give arrival time spectrum.

8. Repeat on different frequency bands to get a spectrogram.
Arrival time estimation

- Solving for the exact time(s) of the $k$ impulses reduces to a root finding problem.

$$W[f] = \text{sum of autocorrelations of noise eigenvectors}$$

$$\sum_f W[f] e^{(j2\pi tf_s/N)f} = 0$$

$$\sum_f W[f] z^f = 0$$

$$t_i = \frac{N}{2\pi f_s} \text{imag}(\ln z_i).$$
Experiment

Frequency selective arrival time estimation for guided waves in plates.

Identify separate arrival times for frequency bands with “Rayleigh” mode and $A_0$ mode.
Arrival time estimation for Rayleigh and A0 mode

Root–MUSIC estimation of arrival times, D=70h

- Rayleigh autocorrelation width L
- Rayleigh estimation range M
- A0 autocorrelation width L
- A0 estimation range M
- Measured Rayleigh arrival time
- Measured A0 arrival time

Time (µs) vs Frequency (MHz)
Experimental results from Lamb wave propagation

Estimation of propagation distances

\[ d = (v_{A0} - v_{Rayleigh}) \ast (t_{Rayleigh} - t_{A0}) \]
Comparison with Hilbert transform method

Time-estimation MUSIC

Peak of Hilbert transform

(note: MUSIC failed at $d=5h$)
Conclusions

- MUSIC has potential for frequency selective arrival time estimation for nonlinear ultrasound.
- Can provide improved arrival time estimates from very narrow frequency bands.
- Does indeed work on real experimental (Lamb wave) ultrasound waveforms.
- Requires good signal-to-noise ratios ($\sim 3.0$).
Experimental results from Lamb wave propagation

Normalized source-receiver separation (thicknesses)
Percent error
Opposite side source, $d > 20h$
Same side source, $d > 5h$
On eigenvectors of Toeplitz matrices

“The proofs... do not completely capture the intrinsic mathematical beauty... and shed little light on the basic structure of Toeplitz matrices that leads to these remarkable properties”