Characterization and Modeling of Ultrasonic Scattering in the Diffusive Regime using Phased Arrays

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Abstract
Structural noise due to multiple scattering is important in steel samples when grain sizes are of the same order as the ultrasonic wavelength or larger. It can be a limiting factor for non destructive evaluation. In this communication, an approach to characterize multiple scattering in metal samples by performing a single experiment is presented. Beamforming post-processing techniques are used to obtain three parameters: the elastic mean free path, the correlation distance of the speckle field, and the diffusion constant. These parameters can be used as input for a simulation of the diffuse multiple scattered fields. Outputs of the simulation method are compared to experimental data.

Keywords: Ultrasound, structural noise, multiple scattering

1. Introduction

During the ultrasonic non destructive evaluation of metallic samples, waves are scattered by the microstructure, leading to the recording of structural. This noise limits the efficiency of nondestructive testing by reducing the signal-to-noise ratio. Moreover, in thick samples, or when the grain size equals or exceeds the wavelength, multiple scattering becomes important. Characterizing this noise can provide information on the microstructure. It can also serve as a basis to simulate structural noise and help predict the detection capabilities of an inspection procedure.

This communication is organized in two sections. First, a characterization approach is presented. It consists in performing a single experiment with a phased array to measure three characteristic parameters of multiple scattering. Secondly, a simulation of the structural noise in the diffusive regime is presented. It uses the three parameters as inputs.

2. Experimental characterization of multiple scattering

2.1 Measurement setup

In the experiment, a linear phased array is placed parallel to a steel sample in a water tank (figure 1).
The array is moved and for each position the response matrix $K$ is recorded. This matrix contains the $N^2$ signals measured for each of the $N$ elements used as an emitter and as a receiver. The recorded signals contain the structural noise and several geometry echoes. The measured signals are post-processed using beamforming techniques, in order to form incident and observed plane waves of interest. Three techniques are used for the calculation of three parameters: the elastic mean-free path, the correlation distance of the speckle field, and the diffusion constant.

### 2.2 Elastic mean free path

The elastic mean free path $l_e$ is the characteristic decay length of a coherent wave. This decay can be observed using the successive reflections of a plane wave at normal incidence. The plane wave is obtained by using all emitting elements with no delay law and a Hann window as an amplitude law (the Hann window limits the divergence of the emitted wave). The measured signals are summed over all receiving elements and averaged over all probe positions, which increases the amplitude of the coherent wave relative to the structural noise. The elastic mean free path is then obtained by fitting an exponentially decaying function to the amplitudes of the successive reflection echoes, corrected by reflection coefficients.

### 2.3 Noise correlation distance

Even though the measured structural noise varies as a function of position, there is a correlation for positions close to each others. To observe that correlation, we consider a configuration where all elements of the array are emitters and calculate the correlation between the structural noise signals measured by different elements. First, to obtain only structural noise signals without coherent echoes, we subtract the average of signals over positions from the signals. Then, we calculate the correlation between the signals measured at different elements. It appears that, as could be expected, this correlation decreases as the distance $\delta$ between elements increases. We fit that decrease to an exponential function in order to define a typical noise correlation distance $\delta_C$:

$$
\text{NoiseCorrelation}(\delta) = \exp\left(-\frac{\delta}{\delta_C}\right).
$$

### 2.3 Diffusion constant

The diffusion approximation can be applied to ultrasonic scattering when multiple scattering is largely dominant [1]. In this approximation, the propagation of ultrasonic energy is analogue to the propagation of heat and entirely determined by the diffusion constant.
The diffusion constant can be determined experimentally by using the coherent backscattering effect [2]. This effect is an enhancement of the scattered intensity around the backscattering direction, and its evolution as a function of time can be related to the diffusion constant. Beamforming was used to observe coherent backscattering and calculate the diffusion constant, following the approach presented by Aubry [3]. Beamforming is used to obtain the signals corresponding to emission and reception of plane waves with different angles. Then, the measured intensity is considered as a function of time and as a function of the angle between the emitted and received plane waves. It forms a figure known as the backscattering cone (Figure 2).

For early times, the angular width of the backscattering cone $\Delta \theta$ decreases as a function of time $t$. Theoretical considerations concerning the spreading of ultrasonic energy in the diffusion approximation lead to the following relation [3]:

$$\Delta \theta^{-2} = \frac{k^2 D}{\log 2} t.$$  \hspace{1cm} (2)

where $k$ is the wave number. $D$ can therefore be obtained by calculating the slope of the function $\Delta \theta^2(t)$ for early times.

It should be noted that this approach is only valid in the diffusive regime, where the diffusion approximation is valid and $D$ is a relevant parameter to characterize the scattering. The validity of the diffusion approximation can be tested by checking if $\Delta \theta^2$ is proportional to $t$, because this proportionality relationship stems from the diffusion approximation.

### 2.4 Example of results

Table 1 is an example of characterization results obtained on a sample of stainless steel using two measurement systems with bandwidths centered on two different frequencies. All the parameters are lower at the higher frequencies. Lower values of $l_e$ and $D$ indicate an increasing of scattering at high frequency, which was expected.
Table 1. Characterization of a sample at two frequencies.

<table>
<thead>
<tr>
<th></th>
<th>$l_e$</th>
<th>$d_e$</th>
<th>$D$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1 MHz</td>
<td>27 mm</td>
<td>1.4 mm</td>
<td>20 mm²/µs</td>
</tr>
<tr>
<td>5 MHz</td>
<td>5 mm</td>
<td>0.5 mm</td>
<td>4.4 mm²/µs</td>
</tr>
</tbody>
</table>

3. Simulation of structural noise

The three parameters obtained with the method described above can be used as input to simulate structural noise. The simulation method that uses them can be decomposed in four steps: the emission by the probe, the diffusion of the average ultrasonic energy, the conversion from average energy to an ultrasonic field, and the reception by the probe. It relies on the diffusion approximation, which means that it is expected to be valid only when multiple scattering is dominant over single scattering.

3.1 Incident energy

The first step is the computation of the emission of the ultrasonic wave by the probe before any scattering occurs. In reality, some scattering starts occurring as soon as the wave enters the scattering material. But parts of the incident energy propagate over a certain distance in the part before being deviated by scattering. The elastic mean free path $l_e$, that can be measured using the method detailed above, is the characteristic distance travelled by the wave energy before it is scattered. In the simulation method, we make the approximation that no scattering occurs before the wave has traveled the distance $l_e$ in the medium and that the energy scatters according to the diffusion equation afterwards. It is a simplified view of the problem, because in reality there is not an abrupt transition between regimes.

We therefore perform a computation of the ultrasonic field emitted by the probe at depth $l_e$ assuming no scattering. We use the ultrasonic field computation module of CIVA, which is based on a paraxial beam method [4], for this computation. The ultrasonic energy corresponding to this field serves as a source term for the next step of the computation.

3.2 Diffusion of the energy

In this step, the average of the energy over realizations of the microstructure, noted $<E>$, is considered. In the diffusion approximation, it follows a simple equation governed by the diffusion constant $D$:

$$\frac{\partial <E>}{\partial t} = D \frac{\partial^2 <E>}{\partial x^2} + \text{source}.$$ (3)

This approximation is used here to model scattering. The sample is assumed to have a parallelepiped shape and perfectly reflecting boundaries, which allows an analytical solution for the equation [5]. The assumption of perfectly reflecting boundaries leads to neglecting the loss of energy that occurs at the interface between the sample and the water.

Using the incident energy of the previous step as input, the solution of the diffusion equation allows it to be used to obtain the average energy at the surface facing the receiving probe.

3.3 Scattered energy
The preceding step yields an energy averaged over realizations of the microstructure, which corresponds experimentally to an average over probe positions. In order to model specific noise signals instead of averaged quantities, realizations of the ultrasonic field are randomly generated.

The signal measured by the receiving probe is proportional to displacements and stresses in specific directions. In the current version of the method, these fields are not determined precisely. A generic quantity \( \Phi \), defined as the weighted sum of displacements and stress that is measured by the probe, is considered. A proportionality relationship with \( <E> \) is assumed. The proportionality factor is not determined. As a consequence, the outputs of the simulation will not be calibrated.

\[
\langle \Phi(x,t) \rangle \propto \langle E(x,t) \rangle.
\]  

(4)

The statistical properties of the simulated structural noise will stem from the statistical properties of \( \Phi \). Structural noise is often assumed to follow a zero-centered normal distribution. In order to ensure that this is the case for the structural noise simulated here, \( \Phi \) follows a zero-centered normal distribution. Additionally, as a measured structural noise has correlations for time and measurement position, \( \Phi \) is computed in a way that ensures it has such correlations. First, a white noise function of time and space is computed. Then it is convolved in time and space by functions that ensure correlations. The convolution in time is done based on the frequency contents of the incident pulse. The convolution in space is done with an exponentially decaying function whose decay length is related to the parameter \( \delta_c \), the noise correlation distance obtained with the characterization method.

### 3.4 Structural noise signal

A structural noise signal is used by summing over the surface of the sample the product of the probe sensitivity with the randomly generated \( \Phi \) obtained in the previous step. The probe sensitivity is obtained using a CIVA computation. Several noise signals can be obtained by using several randomly generating realizations of \( \Phi \).

### 3.5 Example of results

Structural noise measurements were performed along with the characterization measurements presented in the first section, in the two measurement configurations that correspond to the results of table 1. The three parameters of this table were used as inputs to simulate the noise. The envelopes of measured and simulated noise as a function of time were computed for several realization of the noise and averaged to obtain figure 3. As mentioned before, the results of the simulation are not calibrated. In this figure, the amplitudes are adjusted so that the simulated and measured noises are at the same level at the end of the plot. Only the evolution of noise as a function of time is meaningful.

At 1 MHz, simulated and measured noises are different. There are peaks at 23 and 39 \( \mu \)s: they can be due to geometry echoes or reflected forward-scattered noise and are not taken into account by our simulation method. The slopes of the curves are also different, which could be due to the fact that the diffusive regime is not established in the experiment. The agreement is
better at 5 MHz, which was expected: as the frequency increases, scattering is more important and the diffusion approximation becomes more appropriate.

Figure 4. Averaged noise as a function of time. The front face echo corresponds to 0 µs (the signal from 0 to 5 µs is masked by this echo and is not shown here).

4. Conclusion

A single experiment with a phased array allows obtaining several characteristic parameters of multiple scattering. These parameters could be used to gain information on the microstructure of the material or the scattering regime. They also served as input for a simulation of noise that is able to correctly predict the evolution of noise in a case where multiple scattering is dominant.

References