A model for the ultrasonic field radiated by an Electro-Magnetic Acoustic Transducer in a ferromagnetic solid

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Abstract
An Electro-Magnetic Acoustic Transducer (EMAT) is made of two main components: a permanent magnet and an electric coil driven with an alternating current at ultrasonic frequencies. The two main mechanisms to generate elastic waves through magnetic field interactions are the Lorentz force, only requiring that the material is conductive, and the magnetostriction force, occurring for ferromagnetic materials. Models of magnetostriction and Lorentz forces are derived whatever the relative values of the static and dynamic fields. At first, attention is focused on the fact that, for a given driving frequency, in addition to waves generated at this frequency, both forces also generate ultrasonic waves at harmonic frequencies of the driving one. Then, a transformation of the two body forces into equivalent surface stresses is proposed allowing the EMAT model to be readily connected to existing models for radiation and propagation of waves (bulk and guided waves).

Keywords: Electro-Magnetic Acoustic Transducer (EMAT), magnetostriction, ferromagnetic materials, frequency spectra

1. Introduction

In the context of ultrasonic nondestructive testing, Electro-Magnetic Acoustic Transducers (EMATs) are used in many industries such as petrochemical or in nuclear power plants, for material characterization, thickness measurement and flaw detection. The CETIM (Centre Technique des Industries Mécaniques) also uses EMATs with magnetostrictive strips to generate ultrasounds in both plates and pipes: the elastodynamic excitation of the material is due to the transmitted strip vibrations. In this case, the problem of ultrasonic wave generation is similar to that of an EMAT used in a ferromagnetic medium.

In this context, EMATs are often used to induce guided waves, specially shear and torsional waves which are difficult to generate with more conventional transducers as piezoelectric transducers. EMATs are also efficient in hostile environments, for example in high temperature or high pressure fields. However, the piece under test needs to be conductive. In fact, EMATs generate ultrasounds through the Lorentz force mechanism, so that eddy currents and magnetization must exist in the medium under test, eddy currents being only created in conductive materials. EMATs generally have a low signal to noise ratio which can be increased when EMATs are used together with magnetostrictive patches or in ferromagnetic materials [1]. In this case, EMATs generate ultrasounds not only through the Lorentz force but also through the magnetostriction process [2, 3]. While the well known Lorentz force can be easily modelled, the magnetostriction phenomenon requires more attention [3]. Ferromagnetic materials present highly non linear magnetic behaviour resulting in the generation of excitation frequency harmonics. The harmonic generation is limited to the second harmonic in the case of the Lorentz force and can be calculated analytically, as recall hereafter. For the magnetostriction, the generation of harmonics is more complex and cannot be analytically derived.

The aim of the present paper is to compute the contribution of the various harmonics of both forces to the radiated ultrasonic field. The calculation of the force frequency spectra is made...
in three dimensions without considering a particular configuration. It allows dealing with bias fields of arbitrary direction and intensity and with arbitrary distributions of dynamic field. Once these force spectra are known, both forces are transformed into surface stresses in order to use existing semi-analytical propagation models ([4] for bulk waves and [5] for guided waves) used in the simulation platform CIVA. This transformation only assumes that the body forces are created in a shallow depth of the material (the ultrasonic wavelengths are assumed to be larger than the spatial variations of the forces). This transformation was already explained in previous papers [6, 7]. The order of magnitude of the equivalent surface stresses was also compared, in [8], to Ribichini’s results [9] with a good estimation. Finally, as an example, the modal amplitude of the SH0 mode for the different components of the frequency spectrum is computed for two current intensities. It can then be concluded that the harmonic contributions cannot always be neglected.

2. Frequency spectra of the Lorentz and magnetostriction forces

Both components of an EMAT generate a magnetic field in a ferromagnetic material. The permanent magnet creates a static field $B_s$ whereas the coil creates a dynamic magnetic field $B_d e^{j\omega t}$. The angular frequency $\omega_0$ [rad.s$^{-1}$] corresponds to the current excitation frequency, $j = \sqrt{-1}$ and $t$ represents the time [s].

2.1 The Lorentz force

The interaction between the eddy currents $J$ and the magnetic field $B$ creates a force called the Lorentz force, expressed as follows

$$F_L = J \times B.$$  \hspace{1cm} (1)

If the magnetic induction field $B$ is decomposed into its static $B_s$ and dynamic $B_d e^{j\omega t}$ parts, the Lorentz force can be re-written as

$$F_L = J_d \times B_s e^{j\omega t} + J_d \times B_d e^{2j\omega t},$$  \hspace{1cm} (2)

where eddy currents are dynamic quantities with $J = J_d e^{j\omega t}$. The double frequency effect appears in the second right-hand term through the $e^{2j\omega t}$ dependence. So, if the dynamic magnetic field amplitude is not negligible compared to the static one, the double frequency effect of the Lorentz force must be taken into account in the wave generation and component at twice the excitation frequency appear in the received signal.

2.2 The magnetostriction force

The calculation of the magnetostrictive force was formulated by Thompson [1] and Hirao and Ogi [2]. It relies on the evaluation of the magnetostrictive strain tensor $\varepsilon_{MS}$. This tensor is expressed in the time domain by extending the method proposed by Hirao and Ogi [2] for any bias field value. The usual assumption of low dynamic field compared to the static field is no more required. The total calculation steps, in three dimensions and in the time domain, were previously explained by the authors [8]. The only data needed is the macroscopic magnetostriction curve $\varepsilon (\|H\|)$. This curve represents the elastic strain $\varepsilon$ of a ferromagnetic material subjected to magnetization $H$ where the strain $\varepsilon$ must be measured in the direction parallel to the applied magnetic field $H$. It is assumed that the magnetostriction curve can be used for any bias field direction and is the same at any point of the material. The calculation
of the magnetostrictive strain tensor also requires that the magnetostriction effect keeps the volume constant. These two assumptions are fulfilled since the static fields used in EMAT applications are low and since the tested materials are magnetically disordered.

If the static field direction is along the \( x \) axis of the local reference system (calculation point reference system), the magnetostriction strain tensor \( \varepsilon_{MS} \) can be expressed as

\[
\varepsilon_{MS} = \begin{pmatrix}
\varepsilon & e^{xy} \cos^2(\theta_{xy}) - \frac{1}{2} e^{xy} \sin^2(\theta_{xy}) & e^{xz} \cos^2(\theta_{xz}) - \frac{1}{2} e^{xz} \sin^2(\theta_{xz}) \\
-\frac{1}{2} e^{xy} & -\frac{1}{2} e^{xy} \cos^2(\theta_{xy}) + e^{xy} \sin^2(\theta_{xy}) & -\frac{1}{2} e^{xz} \cos^2(\theta_{xz}) + e^{xz} \sin^2(\theta_{xz}) \\
0 & 0 & 0 \\
0 & 0 & \frac{3}{2} e^{xz} \sin(2\theta_{xz}) \\
0 & \frac{3}{2} e^{xy} \sin(2\theta_{xy}) & 0
\end{pmatrix},
\]

where the first column represents the magnetostrictive strain along the \( x \) axis with a strain in this direction denoted by \( \varepsilon \); the second represents the magnetostrictive strain in the \((x,y)\) plane with a strain \( e^{xy} \) in the direction of the total (static and dynamic) magnetic field in this plane and an angle \( \theta_{xy} \) between the \( x \) axis and the direction of the magnetic field. Accordingly, the third represents the magnetostrictive strain in the \((x,z)\) plane with a strain \( e^{xz} \) and an angle \( \theta_{xz} \). The various quantities appearing in equation (3) are time-dependent.

The magnetostrictive force can be calculated, following the next steps. The piezomagnetic strain tensor \( d \) is calculated with the previous magnetostriction strain tensor by

\[
d_{ij} = \frac{\partial \varepsilon_{MS}}{\partial H_j}.
\]

Then, the piezomagnetic stress tensor \( e \) is determined with the help of the elastic stiffness tensor \( C \):

\[
e = C d.
\]

The magnetostrictive stress tensor \( \sigma_{MS} \) is expressed as

\[
\sigma_{MS} = -e H.
\]

The stress free boundary conditions at the surface of the material are then integrated in the magnetostrictive stress tensor calculation by imposing the condition

\[
\sigma_{MS} \cdot n = 0
\]

at the part surface. Finally, the magnetostriction force \( F_{MS} \) is expressed by

\[
F_{MS} = \nabla \cdot \sigma_{MS}.
\]

Since the previous calculation is made in the time domain, the expression of the frequency spectrum of the magnetostriction force does not appear explicitly. By means of a Fourier series, they can be evaluated. Examples of frequency spectra are shown in figure 1 for a 3-mm-thick plate made of iron. The magnetostriction curve used is presented figure 2 [2]. The static field considered is weak \((0.5 \times 10^4 \text{ A.m}^{-1})\) while the dynamic field intensity has
respectively moderate (0.1 $10^4$ A.m$^{-1}$), high ($0.5$ $10^4$ A.m$^{-1}$) and very high ($1$ $10^4$ A.m$^{-1}$) values. The calculation point is placed at the plate surface below the middle of a wire. For each spectrum, the normalization is made relatively to the highest component. The results correspond to the tendency described in the literature [10]. The nonlinear frequency behaviour is highlighted at high dynamic fields and low bias fields.

![Figure 1](image1.png)

**Figure 1.** Normalized spectra of the magnetostriction force for different amplitudes of dynamic magnetic fields. Magnetic field amplitudes in $10^4$ A.m$^{-1}$

![Figure 2](image2.png)

**Figure 2.** Steel magnetostriction curve, used for the magnetostriction frequency spectra displayed figure 1; the static field amplitude is $0.5$ $10^4$ A.m$^{-1}$

### 3. Body force transformation into surface stresses

The Lorentz and magnetostriction forces are generated in a volume near the surface, and then act as source terms in the equation of motion governing the propagation of elastic waves. However, this equation is more easily solved by considering surface source terms as done in the simulation platform CIVA for bulk and guided waves [4, 5]. The body forces are thus transformed into surface stresses as previously described [7, 8]. While, for the Lorentz force, an integral over the depth of the material is appropriate to transform it into equivalent surface stresses [3, 11]; it was demonstrated by Thompson [1] that such an integration leads to a non-physical result in the case of the magnetostriction force (the integral vanishes). Thus an alternative way to transform these forces has been proposed by Thompson [6]. The main steps of this derivation are given next with the quantities used in the expressions of the equivalent surface stresses. Two important facts have to be considered. First, the transformation only assumes that the wavelengths of the elastodynamic Green’s functions are larger than the spatial variations of the forces. Second, two formulations are given whereas Thompson had considered only one of them. These two formulations are relevant due to their respective applications, as it is described below.

The transformation starts by considering the elastic displacement $u$ written as a volume convolution integral of the Green’s tensor $G$ with the body forces $f$: 
The desired form for the displacement is that of a surface convolution integral, written as

\[ u_i(r) = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} G_{ij}(r, r_0) f_j(r_0) \, dx_0 \, dy_0 \, dz_0. \]  

where the integration over the depth disappeared and the source points are only at the surface of the material. The transformation of equation (9) into equation (10) starts with the following assumption. The spatial variations along the material depth of the body forces are weak compared to the spatial variations of the Green’s functions. The former are related to the penetration depth while the latter are related to the ultrasonic wavelengths. This hypothesis is not restrictive for EMAT NDT applications because, for the frequencies and materials used in this context, the penetration depth not exceeds a few hundredths of a millimetre while the wavelengths are of the order of a millimeter. This assumption allows the application of a Taylor series on the Green’s functions along the material depth. This Taylor series yields

\[ u_i(r) = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} G_{ij}(r, R_0) \sigma_j(R_0) \, dx_0 \, dy_0, \]

where the force moment quantities appear, defined by

\[ \{M_x^k \} = \int_{0}^{\infty} f_j(R_0) \, dz_0 : j \in [1,2,3] \quad k \in [0,1,2]. \]  

The other transformation steps are detailed by Thompson [6]. They were also revised, detailed and extended by Rouge et al. in [7]. These steps consist in writing the local equilibrium relationship as a function of the displacement and in considering the boundary conditions of free surface in order to eliminate the derivatives on the Green’s functions in equation (11). At the end of the transformation, two equivalent formulations of the displacement field are found.

In the first formulation, the displacement field is written

\[ u_i(r) = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \sigma_j^1 \left[ G_{ij}(r, R_0) \right] dx_0 \, dy_0, \]

Figure 3. Reference system with point source \( r_0 \) and calculation point \( r \) notations
depth anymore. This formulation may seem uncompleted, comparing that with the sought form of equation (10). It is however well adapted to guided waves: the surface stresses calculation is conducted in a half-space but the assumptions made are still valid for a wave guide so the results can be used for guided wave generation. In this case, the spatial variations of the Green’s functions are characterized by the wave numbers, which are known analytically in the case of SH (Shear Horizontal) waves and can be known numerically for the Lamb waves from for example a SAFE (Semi Analytical Finite Element) code. From a formula given by Ditri and Rose [5], both the SH waves [12] and the Lamb waves [13] modal amplitudes can be derived by a combination of derivative operators and of Green’s functions. The second formulation leads to

\[
u_i(r) = \int \int G_{ij}(r, R_0) \sigma_{ij}^\parallel(R_0) \, dx_0 \, dy_0,
\]

where the equivalent surface stresses \( \sigma_{ij}^\parallel \) are directly related to the force moments. Thus, properties on the Green’s functions no more need to be known. The equivalent surface stresses are then directly evaluated and can be directly substituted into existing generation and propagation models based on surface source terms for bulk waves [4]. The detailed expressions of the two equivalent stresses in equations (13-14) are given in [7].

4. Guided wave radiation by an EMAT in a ferromagnetic plate

Combining the forces calculation, their transformation into surface stresses and previous models based on surface source terms, the modal amplitude of the guided waves generated by an EMAT in a ferromagnetic plate can be computed (SH [12] and Lamb waves [13]). Here, an example of SH modal amplitude calculation is given. The configuration used is shown in figure 4 and consists in a meander-coil associated with a tangential static magnetization. The 3-mm-thick plate is made of iron characterized by the previously shown magnetostriction curve given by Hirao and Ogi [2] and shown in figure 2. The study consists, for two current intensities, in calculating the modal amplitude of the first SH mode, SH0, as a function of the frequency. Only the magnetostriction force is considered. The excitation frequency varies from 0.1 to 0.2 MHz. The SH0 modal amplitudes are represented in figure 5 for a low current intensity, 40 A (left), and for a high current intensity, 400 A (right). The modal amplitudes of the excitation frequency, also called the fundamental frequency, are marked by blue circles. The modal amplitudes of the first four harmonics of the magnetostriction force are also calculated and are marked by green squares, black plus, clear blue triangles and pink cross respectively. For all the frequencies considered (that is to say, the excitation frequency spectrum plus that of its four harmonics), the amplitude of the fundamental component is also calculated and represented by the red curve.
5. Conclusions

A model for the ultrasonic field radiated by an Electro-Magnetic Acoustic Transducer in a ferromagnetic solid has been proposed. Both the Lorentz and magnetostriction forces have been considered. The calculation of the magnetostriction force is not restricted to the usual case of a low dynamic field; any static and dynamic magnetic field intensities and directions can actually be considered. This fact results from the method used to calculate the frequency spectra of the magnetostriction force. Combining the force calculation with the mathematical transformation of body forces into surface stresses, the contributions of the fundamental excitation frequency and of its harmonics for both the Lorentz and magnetostriction forces to any propagative mode (guided, bulk and surface waves) can be determined. Besides, the use of the platform CIVA also allows considering any EMAT design so that the overall model can help designing EMAT for specific and new applications. Future works will be concentrated on modelling the field radiated by a magnetostrictive patch and to adapt the modelling approach to a multilayered material with a cylindrical geometry, as required to deal with magnetostrictive strips applied on pipes.
References