REFLECTION AND TRANSMISSION OF $A_0$ MODE IN METALLIC SUB-BEAMS

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ABSTRACT

When the fundamental anti-symmetric Lamb mode ($A_0$) propagating in a top or bottom metallic sub-beam is incident at the edge of a semi-infinite horizontal crack, it undergoes reflection, gains transmission into the main beam and also propagates from the top to the bottom sub-beam and vice-versa. The Lamb mode propagating from one sub-beam to the other is termed as ‘Turning Lamb Mode’ (TLM). Reflection and transmission factors, defined based on wavelet transform, of $A_0$ mode propagating in the sub-laminates were computed at an assortment of locations of horizontal crack across the beam thickness and frequencies of excitation. Moreover, the power reflection and transmission coefficients were also estimated. It was found that the reflection and transmission factors and power coefficients vary with the thickness ratio, but, the variation with respect to frequency of excitation from 150 kHz to 225 kHz is found to be negligible.

Keywords: Turning Lamb mode, Crack, Sub-beam, Transmission and reflection, Wavelet – transform

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The frequency of excitation was swept from 150 kHz to 225 kHz in steps of 25 kHz and mode of investigation was Ao. The type of crack considered in this work is ‘semi-infinite’ as shown in Figure 1(a). The wave groups captured were propagating towards and/or from the edge of horizontal crack. At the location of crack, the beam was divided into two sub-beams, top sub-beam and bottom sub-beam. The location of semi-infinite crack can be anywhere across the thickness. This location determines the thicknesses of sub-beams.

Table 1: Material properties

<table>
<thead>
<tr>
<th>Material</th>
<th>E in (GPa)</th>
<th>υ</th>
<th>ρ in kg/m³</th>
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<tr>
<td>Aluminium</td>
<td>70</td>
<td>0.30</td>
<td>2750</td>
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Transmission from the Top Sub-beam to Main Beam

Figure 2(d) shows A-scan, captured by receiver R₃, of Lamb mode transmitted from top sub-beam to the main beam. Using WT technique, wavelet coefficients were computed and plotted as a function of time. ‘Transmission factor’ is defined as the ratio of wavelet coefficient (peak value) of the transmitted wave group to the wavelet coefficient (peak value) of the incident wave group, which is shown in Figure 2(d).

This completes one set of analysis for thickness ratio and excitation frequency of 1/6 and 150 kHz respectively. In the next analysis, crack location across the thickness was moved to 2 mm, measured from the top surface. This corresponds to thickness ratio of 2/6. Again similar numerical analysis, illustrated above, was carried out for an excitation frequency of 150 kHz. This was continued for other thickness ratios 3/6, 4/6 and 5/6 as well. For each thickness ratio, reflection factor, transmission factor of TLM and transmission factor of Lamb mode propagated into the main beam were estimated. Procedure akin to the above was followed for the frequencies from 150 kHz to 225 kHz in steps of 25 kHz.

DATA ANALYSIS

Figure 3(a) shows the variation of WT based reflection factor, transmission factor of TLM and transmission factor of Lamb mode into the main beam with respect to thickness ratio for various frequencies from 150 kHz to 225 kHz. With increase in thickness ratio, the reflection factor decreases (Figure 3(a)) and transmission factors increase as shown in Figures 3(b) and 3(c).

Power Reflection and Transmission Coefficients

Power reflection and transmission coefficients of the reflected and transmitted Lamb wave groups in the sub and main laminates were estimated through numerical simulations. The following expression [8] gives the time averaged power flow, $<P>$, across any cross-section.

$$<P> = \frac{1}{t_a} \int_0^{t_a} \int_0^h \left( \sigma_{xx} \dot{u} + \tau_{xx} \dot{w} \right) dz dt$$  (1)

Fig. 2: (a) Interaction of Lamb mode with crack edge and propagation of ‘Turning Lamb mode’. A-scans obtained at receivers (b) R₁, (c) R₂ and (d) R₃ at an excitation frequency of 150 kHz
Fig. 3: Variation of (a) reflection factor, (b) transmission factor of TLM and (c) transmission factor of Lamb mode into the main beam, with respect to thickness ratio for various frequencies.

Fig. 4: Variation of (a) power reflection coefficient, (b) transmission coefficient of TLM and (c) transmission coefficient of Lamb mode into the main beam, with respect to thickness ratio for various excitation frequencies.

Results and Discussion

Numerical simulations were carried out on aluminum beam containing a semi-infinite horizontal crack showed that reflection and transmission factors were found to decrease and increase respectively, with respect to thickness ratio, in the frequency range 150 kHz to 225 kHz, as shown in Figure 3(a).
Reflection factors decreased from 0.85 (approx) to 0.175 (approx) when thickness ratio varied from 1/6 to 5/6, whereas transmission coefficients increased continuously from 0.120 (approx) to 0.980 (approx) with increase in thickness ratio as shown in Figures 3(a) and 3(b). The variations of reflection and transmission factors with respect to frequency (in the range 150 kHz to 225 kHz) were found to be almost same at a given thickness ratio. In other words, the influence of frequency on reflection and transmission factors is negligible.

Figure 4 shows the variations of power reflection and transmission coefficients of Lamb mode. Power reflection coefficient decreases with increase in thickness ratio. When thickness ratio was 1/6, the power reflection coefficient was around 0.72 (approx). With subsequent increase in thickness ratio from 1/6 to 5/6, power reflection coefficient decreased from 0.72 (approx) to 0.03 (approx). The reflected power, when thickness ratio was 5/6, was almost 1/24th (approx) of that at 1/6 thickness ratio in the frequency range of 150 kHz to 225 kHz. An interesting phenomenon was noticed in case of TLM. Power associated with the TLM was found to increase when the crack location across beam thickness was moved away from the top surface. When crack was at the mid-plane (3 mm from the top surface) the power transmission coefficient of TLM was high in the frequency range from 150 kHz to 225 kHz as shown in Figure 4(b).

Power transmission coefficient was found to decrease when crack was moved further down from the mid-plane. Initially, power transmission coefficient of TLM increased from 0.10 (approx) to 0.33 (approx) when thickness ratio increased from 1/6 to 3/6, respectively. When thickness ratio was further increased from 3/6 to 5/6, power transmission coefficients varied from 0.32 (approx) to 0.16 (approx), respectively as shown in Figure 3(c).

When Ao was incident at the edge of crack, it transmitted into the main laminate through the crack edge. Figure 4(c) shows power associated with the Ao Lamb mode transmitted into the main laminate. Power transmission coefficient of Ao Lamb mode transmitted into the main laminate was found to increase with increase in thickness ratio. Power transmission coefficients vary from 0.07 to 0.77 (approx) for a thickness ratio variation of 1/6 to 5/6, respectively in the frequency range from 150 kHz to 225 kHz as shown in Figure 3(c).

Reflection factor and power reflection coefficient defined based on WT and power respectively, of Ao Lamb mode exhibit a decreasing trend with increases in thickness ratio as shown in Figures 3(a) and 4(a), respectively. Transmission factor and power transmission coefficient defined based on WT and power respectively, of TLM were found exhibit a dissimilar behavior with increase in thickness ratio as shown in Figures 3(b), and 4(b) respectively.

Transmission factor increases with increase in thickness ratio as shown in Figure 3(b), whereas power transmission coefficient reaches a maximum value at thickness ratio 3/6 (crack at the mid-plane), then starts decreasing with increase in thickness ratio as depicted in Figure 4(b) in the frequency range 150 kHz to 225 kHz.

This study has revealed the fact that in the frequency range from 150 kHz to 220 kHz, albeit the trend in variation of reflection and transmission factor based on WT is similar to variations in power reflection and transmission coefficient, the variation of power transmission coefficients of Ao TLM is completely different from the variation of transmission factors based on WT.

CONCLUSIONS
Numerical simulations carried out on transmission and reflection characteristics of Ao Lamb modes in sub-beams, in the frequency range 150 kHz to 225 kHz, revealed the fact that the variation of transmission coefficients of Ao TLM follow an increasing and decreasing trend with increase in thickness ratio, while the transmission factors of Ao TLM keep increasing with increase in thickness ratio.

It was observed that power reflection coefficients and reflection factors both keep decreasing with increase in thickness ratio. Transmission factors and power transmission coefficients of transmitted Ao Lamb mode into the main laminate also followed an increasing trend with increase in thickness ratio.

However, the power transmission coefficients of Ao TLM exhibited an anomalous behavior when compared to the transmission factors of Ao TLM.

REFERENCES