A Novel Approach For Evaluation of Critical Surface Crack Sizes in Pressure Vessels Under Both Tension and Bending

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Abstract

Materials of high specific strength are commonly being used for fabrication of pressure vessels for launch vehicle and satellite applications. However brittle fracture of such materials is an important consideration in their structural design. Such members may often have some flaws that are either inherent or are introduced during fabrication processes like welding. These may have gone undetected during fabrication resulting in reduced load carrying capacity and hence operational life of the pressure vessel resulting in their failure. Among the various approaches suggested for premature control of fracture, the LEFM approach has been observed to be the best suited for design applications. This paper presents the details of fracture analysis carried out on a propellant tank used in an Indian satellite launch vehicle. LEFM based stress intensity factor solutions, for part through crack subjected to remote tensile and bending loads which are available from published literature have been used for the studies.

Key words: Titanium alloy, fracture, LEFM, stress intensity factor, surface crack

1 Introduction: A monocoque propellant tank used in an Indian launch vehicle comprises of two compartments separated by a thin common bulkhead. It is made of alpha-alpha beta alloy, Ti-6Al-4V, for its high specific strength. Though the tanks have been designed based on strength consideration, the structural integrity of the tanks is checked from fracture point of view too. Fracture mechanics study is warranted for this tank owing to:

- Quasi-brittle behavior of the material Ti-6Al-4V.
- The compartments are pressurised and the common bulkhead that separates the fuel and oxidizer compartments, is very thin and any crack present on this surface may grow and become a through fracture causing the fuel & oxidizer to mix, resulting in catastrophic failure.

Current study is limited to surface cracks on the tank walls with the stress field normal to plane of the crack, i.e. Mode 1 type of fracture with LEFM based approach.

2 Configuration of the propellant tank: The thickness of the tank varies from 1.5mm to a maximum of 2.7mm at the knuckle region near common bulkhead. The yield and ultimate strengths of the tank material are 825 N/mm² and 895 N/mm² respectively with percentage elongation of 10%. The fracture toughness of the material is 5MPa√m.

3 Theoretical basis for fracture studies: Two methodologies are discussed here. One is the Fracture Mechanics approach that is based on the stress intensity factor solutions and the other is the ligament snap through condition which is based on tensile instability of the un-cracked ligament.

3.1 LEFM formulation for a part through crack: Neglecting the small plastic zone at crack tip which is a singularity dominated zone, the stress state of the material is a function of the stress intensity factor, \( K_I \). The crack grows suddenly when
the stress intensity factor becomes equal to or greater than a critical value referred to as the fracture toughness, $K_C$, which is a material property viz. independent of the crack size or geometry and which can be evaluated only through experiments. This $K$ is set to $K_{IC}$ when certain conditions are met and the stress state is given by the relation

$$\sigma = \frac{K_{IC}}{\sqrt{a}} \gamma$$

where $K_{IC}$ is the plane strain fracture toughness of the material and ‘$\gamma$’ is flaw parameter that is function of crack length, specimen width, specimen thickness etc…

Rewriting the above equation, the stress intensity factor can be broadly rewritten as

$$K = \sigma \sqrt{a} \gamma$$

Equations for Stress-intensity factors (SIF) for a wide variety of three-dimensional crack configurations subjected to either uniform remote tension or both tension & bending loads as function of different parameters have been presented by J.C. Newman Jr & I.S Raju [1][2] as

$$K = \left(\sigma + H_j \sigma_b\right) \sqrt{\frac{a}{Q_1}} F_1(\beta, \alpha, \xi, \phi)$$

Substituting for all the variables in equation 3, viz the equation for stress-intensity factor, we observe that it is a function of two variables $\alpha$ and $\beta$.

Equation 3 can be re-written to express the fracture stress as a function of $K_{IC}$ and two variables $\alpha$ and $\beta$.

The solution is not unique since we have a single equation and two variables. We assume one variable ‘$\beta$’ as constant and vary the other variable ‘$\alpha$’ to obtain the fracture stress.

It may also be observed that as $\alpha$ tends towards 0, the theoretical value of fracture stress reaches infinity which is untrue. The limiting fracture stress value being the ultimate strength ‘$\sigma_u$’ of the material, Feddersen’ approach [3] is used for post yield correction and the final fracture strength curve is obtained by drawing a tangent from ‘$\sigma_u$’ to the curves obtained by Raju- Newman relation.

3.2 Ligament snap through condition: The fracture condition given by the Newman-Raju equation can sometimes be very large especially for thin gauge plates/shells. In such cases, the ligament failure stress (i.e. the average stress in the net un-cracked portion reaching the ultimate strength of the material) will be the deciding safe stress value. This can be written as

$$\sigma_f = f(\alpha, \sigma_u)$$

For a part-through semi elliptic crack, this equation takes the form,

$$\sigma_f = \sigma_u \left(1 - \frac{\pi}{4 \alpha}\right)$$

where, expression in parenthesis represents the average un-cracked ligament thickness over crack length $2c$. The above equation gives the nominal stress required to fracture the remaining ligament to result in a through-thickness crack of length $2c$.

4 Methodology: In the current study the following points are to be noted:

- Structure under consideration is subjected to both tensile as well as bending stresses.

- Described below is the step by step methodology used:

  Input data is tension ‘$\sigma_t$’ and bending ‘$\sigma_b$’ component of the stresses, plane strain fracture toughness of the material ‘$K_{IC}$’ and thickness ‘$t$’.

  $$\sigma_{\text{total}} = \sigma_t + \sigma_b$$

  Tensile component $\sigma_t$ is $\frac{\sigma_t}{\sigma_{\text{total}}} \times 100\%$ while that of bending is $\frac{\sigma_b}{\sigma_{\text{total}}} \times 100\%$.

  Step 1:

  1. In equation 3, replace stress intensity factor at crack tip with $K_{IC}$
Choose a particular location in the tank with wall thickness 't'. Allow crack depth 'a' to vary from 0.01 to t.

Evaluate the RHS of equation 3 for a given value of $\beta$.

Check if it is equal to $K_{IC}$.

If not, continue to increment 'a' and find that value at which equation 3 is satisfied. Evaluate the total stress viz. sum of tension and bending stresses ($\sigma_t + \sigma_b$) for this condition.

Repeat the steps iv to vi for different values of $\beta$.

Tabulate and plot $\alpha$ Vs $\beta$ for this total stress level.

Repeat step iii to viii for different % of $\sigma_t$ and $\sigma_b$ such that $\sigma_{total}$ remains the same.

Plot $\sigma_{total}$ Vs $\alpha$ for different $\beta$ that is a representation of the fracture strength curves.

Apply post yield criterion to the fracture strength curves thus drawn and evaluate the critical crack sizes for the applied stress level due to internal pressure loading at the chosen location.

Step 2:

Keeping $\sigma_{total}$ constant vary the percentage of $\sigma_t$ and $\sigma_b$.

Repeat steps i to x.

5 Case study: Fracture evaluation of the propellant tank: In order to carry out the fracture mechanics based analysis of the propellant tank, the knowledge of the stress field on the tank walls is a prerequisite.

5.1 Stress analysis of the propellant tank: Axi-symmetric finite element analysis of the propellant tank is carried out using ANSYS16.0 finite element package using multi-linear kinematic hardening material property of tank material. Figure 2 gives the finite element plot of the tank along with loading conditions.
The deformed shape as well as component stresses in the tank at ultimate pressure are given in Figure 27 and Figure 28 respectively.

The directional stresses at different thickness locations of the tank are tabulated in Table 3.

<table>
<thead>
<tr>
<th>Location</th>
<th>hoop</th>
<th>Stress, N/mm²</th>
<th>Stress employed for</th>
</tr>
</thead>
</table>

Figure 26: Finite element model of the propellant tank with boundary conditions & internal pressure

Figure 27: Deformed shape of the propellant tank

Figure 28: Stress distribution in the propellant tank at ultimate pressure
### Table 1: Stress Distribution

<table>
<thead>
<tr>
<th>Thickness</th>
<th>Stress, N/mm²</th>
<th>Tension</th>
<th>Bending</th>
<th>Total</th>
<th>Fracture Evaluation, N/mm²</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.5 mm thk</td>
<td>600</td>
<td>464</td>
<td>38</td>
<td>502</td>
<td>600</td>
</tr>
<tr>
<td>1.7 mm thk</td>
<td>830</td>
<td>486</td>
<td>120</td>
<td>606</td>
<td>830</td>
</tr>
<tr>
<td>2.3 mm thk</td>
<td>995</td>
<td>455</td>
<td>555</td>
<td>1010</td>
<td>1010</td>
</tr>
<tr>
<td>2.7 mm thk</td>
<td>634</td>
<td>311</td>
<td>720</td>
<td>1031</td>
<td>1031</td>
</tr>
</tbody>
</table>

The directional stresses at the knuckle region (2.7 mm thick) are given in Figure 29.

**Figure 29: Stress distribution in the knuckle region at ultimate pressure**

#### 6 Fracture analysis of the propellant tank

At locations other than at 2.7 mm thick region, the hoop stresses are greater than the meridional stresses. At these locations, Newman-Raju relations for tension alone [1] or equation 2 may be used with bending component zero and the fracture strength curves drawn.

At 2.7 mm thick region, the situation becomes trickier with the bending component dominating the tensile one. It is here that the methodology described in section 4.0 is applied.

Five cases have been studied:

- **Case 1.** $\sigma_t = 100\%$ of $\sigma_{\text{total}}$ and $\sigma_b = 0\%$ of $\sigma_{\text{total}}$
- **Case 2.** $\sigma_t = 70\%$ of $\sigma_{\text{total}}$ and $\sigma_b = 30\%$ of $\sigma_{\text{total}}$
- **Case 3.** $\sigma_t = 50\%$ of $\sigma_{\text{total}}$ and $\sigma_b = 50\%$ of $\sigma_{\text{total}}$
- **Case 4.** $\sigma_t = 30\%$ of $\sigma_{\text{total}}$ and $\sigma_b = 70\%$ of $\sigma_{\text{total}}$
- **Case 5.** $\sigma_t = 0\%$ of $\sigma_{\text{total}}$ and $\sigma_b = 100\%$ of $\sigma_{\text{total}}$

Case 1 is compared with fracture strength curves drawn using the relation for tension alone from reference [2] and found to have a good match.

**Figure 30: Fracture strength curves from [1] tension only**

**Figure 31: Fracture strength curves from [2] tension only**
7 Conclusions: The following conclusions are arrived at from the current study:

- A novel approach has been employed to draw the fracture strength curves when both tensile and bending stresses are present in the structure.
- Fracture mechanics based approach by using Raju- Newman relations [1][2] give identical results when used for tension alone (i.e. $\sigma_t = 100\%$; $\sigma_b = 0\%$).
- With progressive increase of bending stresses (with corresponding reduction in membrane stresses), the critical crack size increases.
- However, in all cases, ligament snap through occurs at $\alpha = 0.13$, though at different $\beta$ values.

8 References

Nomenclature

- $a$ - Depth of crack
- $c$ - Semi major axis of the part through crack
- $t$ - Thickness of structure under consideration
- $K$ - Stress-intensity factor
- $K_{IC}$ - Plane strain Fracture toughness
- $w$ - Width of the specimen
- $\alpha$ - Non-dimensional crack depth, $a/t$
<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\beta$</td>
<td>Non-dimensional crack length, c/t</td>
</tr>
<tr>
<td>$\zeta$</td>
<td>Non-dimensional width, a/w</td>
</tr>
<tr>
<td>$\gamma$</td>
<td>Flaw parameter</td>
</tr>
<tr>
<td>$\sigma_b$</td>
<td>Bending component of stress</td>
</tr>
<tr>
<td>$\sigma_f$</td>
<td>Failure stress (nominal stress at which a plastic hinge will be formed at the crack location)</td>
</tr>
<tr>
<td>$\sigma_t$</td>
<td>Tensile component of stress</td>
</tr>
<tr>
<td>$\sigma_{\text{total}}$</td>
<td>Sum of bending &amp; tensile stresses</td>
</tr>
<tr>
<td>$\sigma_y$</td>
<td>Yield strength of the material</td>
</tr>
<tr>
<td>$\sigma_u$</td>
<td>Ultimate strength of the material</td>
</tr>
</tbody>
</table>