Tomographic Algorithm for Industrial Plasmas

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Abstract

Tomographic algorithm has a great potential to understand the fundamental processes and phenomenon going on inside the industrial plasma. These algorithms provide the spatial profile of emission coefficient of plasma. Transform based algorithms give better reconstruction results for the cases where the large number of projections are available and these projections are distributed uniformly over 180 or 360 degree. Both of these requirements are essential for the transform based methods to produce reconstruction of desired accuracy. A typical problem encountered in plasma tomography is the limited number of views, because of the complexity of the plasma diagnostic setup. Iterative methods, popularly known as algebraic reconstruction methods, are an alternate choice (compared to transform methods) for limited number of rays/detector or views occurring due to certain engineering constraints. In the present work, multiplicative algebraic reconstruction technique (MART) has been employed to reconstruct emission profile of industrial plasma.

Keywords: Multiplicative algebraic reconstruction technique (MART), emission tomography, Convolution Back projection (CBP).

1. Introduction

The basic concepts of plasma emission tomography are the collecting projection data of the emitted radiation from plasma and process these projection data with tomographic algorithms to reconstruct cross-section of plasma. The emitted intensity of radiation in a particular direction is the line integral of the local emission coefficients. The line integral measured by single photodiode of a detector is given as,

\[ p(s, \theta) = \int e(r, \varphi) dl \]  \hspace{1cm} (1)

where \( e(r, \varphi) \) is the emissivity of plasma and \( p(s, \theta) \) is the projection data along a particular line.

Figure 1. Tomographic data collection geometry.
Plasma emission profile can be obtained by feeding projection data to reconstruction algorithms.

2. Mathematical formulation of MART

The algebraic reconstruction methods require the discretization of the object space by rectangular grids. The length of the \( t^{th} \) ray with the \( j^{th} \) pixel in a projection is called the weight function \( W_{ij} \), for \( i = 1,2,\ldots,M \) and \( j = 1,2,\ldots,N \). Accurate weight matrix can be formed by covering the actual geometrical parameter for better tomographic results. The emission assumed to be constant throughout the pixel. The total emission of \( t^{th} \) ray, denoted by \( P_t \), represent the line integral of the emission function along the path of rays. The line integral takes the form of finite sum and the model can be described by a system of linear equations,

\[
[W_{ij}] \{e_j\} = \{P_t\}
\]  

(2)

The problem now reduces to inversion of weight matrix. The weight matrix generally sparse and non-invertible, so iterative methods have been employed to get feasible solution of equation (2). Gordon et al. [7] introduced the concept of MART for the case of limited data tomography. Algorithm starts with initial guess of emissivity \( \tilde{e}_j \) for each pixel (grid). Emissivity \( \tilde{e}_j \) is modified in each iteration of MART algorithm until it converges to the true value of plasma emissivity.

3. MART algorithm steps

In this section, we summarized, the steps involves in the implementation of MART algorithm. Let \( P_t \) is the projection data for a selected ray \( j \) in a particular view \( \theta \). The approximated projection data \( \tilde{P}_t \) can be obtained as,

\[
\tilde{P}_{t\theta} = \sum_{j=1}^{N} W_{i\theta j} \ e_j
\]  

(3)

where \( M_\theta \) is the number of rays in a selected view. Start and close labels with statements are used in the description of algorithm to identify the iterative loop. The parameter to be varied in each loop is indicated in brackets.

Step 1 Start iteration \( (k) \).
Step 2 For each projection view \( (\theta) \).
Step 3 For each ray \( (i\theta) \),
   Compute approximate projection (Eq. 3),
   Calculate the correction as,
\[
\Delta P_{i\theta} = \frac{P_{i\theta}}{e_j}
\]
Step 4 For each pixel(grid)
   If \( W_{i\theta j} \) is nonzero then
\[
e_j^{\text{new}} = e_j^{\text{old}} \times \left( 1 - \lambda \times \frac{W_{i\theta j}}{(W_{i\theta})_{\text{max}}} (1 - \Delta P_{i\theta}) \right)
\]  

(4)

Check for stopping criteria,
\[
\left| \frac{e_j^{k+1} - e_j^{k+1}}{e_j} \right| \times 100 < \delta
\]

Where \( \delta \) is a suitable stopping criterion.

Step 5 Stop.
4. Modelling of tomography system for weight matrix formation

Tomography system for industrial compact plasma is designed and developed in the IIT-Kanpur. Compact plasma setup employed eight pinhole cameras at the periphery of the multicusp for data collection [2, 5].

We discretize the plasma cross-section with $J \times J$ square grids. The coordinate of one pinhole of camera is $(-R,0)$, where we assumed that the origin of Cartesian coordinate system coincide with centre of plasma cross-section and $2R/N$ is the length of each grid.

We can fetch the coordinates of other camera pin holes by applying rotation matrix on known coordinate,

$$
\begin{pmatrix}
    x_h \\
    y_h
\end{pmatrix} = \begin{pmatrix}
    \cos \theta & -n \sin \theta \\
    \sin \theta & \cos \theta
\end{pmatrix} \begin{pmatrix}
    -R \\
    0
\end{pmatrix}
$$

Let we have $N$ detectors for a particular view $\theta$ and $d$ is the total length of detector assembly, then the slope of each ray $i$ is $(\theta + \alpha)$, where $\alpha$ can be given as,

$$
\alpha = \tan^{-1} \left( \frac{d - d_i}{R} \right)
$$

Equations of rays are formed by availing the known coordinate of detector arrays and camera pinhole.

$$
y = y_h + \tan(\theta + \alpha) \times (x - x_h)
$$

Intersection points between rays and grids can be find by solving equation of rays and grids. We calculate length of each ray in each grid and divide this length by grid diagonal, which results weight of ray in that grid.
5. Results

Algebraic reconstruction algorithm, MART is employed to obtain 2D emission profile of plasma. This is the alternative solution for plasma imaging which already developed by convolution back projection (CBP) algorithm [2]. The reliability of MART code is checked with simulated Shepp-Logan phantom, before applying on the real experimental data of plasma. The original and reconstructed images of Shepp-Logan phantom by MART code are shown in Figure (3) & Figure (4). The reconstructed image in Figure 3(b) is similar to original image with L2 error of 0.1% while in Figure 4(b), L2 error is 13%.

The tomographic algorithms are applied on experimental projection data to obtain the real time plasma image. The experiment is performed for the plasma cross-section at the axial distance 200 mm with constant pressure 254 W and pressure 0.2 m Torr. Tomographic experimental data consists 8 views and 128 rays. We cannot apply directly transform based method for limited data scenario. We need to smooth the data by interpolation techniques to apply transform based method. The alternate way to deal with limited data situations is algebraic reconstruction method, which doesn’t require interpolation techniques for image reconstruction. Figure 5(a) shows plasma emission cross-section by MART code & Figure 5(b) shows same profile resulted by CBP. The difference of L2 error between CBP and MART is 13%. This difference is currently being investigated for better understanding for plasma properties.

![Figure 3](image1.png)  
(a) (b)

**Figure 3.** Tomographic reconstruction of phantom (a) Original phantom with (32×32) pixels (b) Reconstructed image by MART code.

![Figure 4](image2.png)

(a) (d)

**Figure 4.** Tomographic reconstruction of phantom (a) Original phantom with (64×64) pixels (b) Reconstructed image by MART code.
Figure 5. Tomographic reconstruction of Ar 750 nm line profile at 200 mm cross-section with constant 0.2m Torr pressure and 254 W power condition (a) MART result with (128×128) pixels (b) CBP result with (128×128) pixels.

6. Conclusions

Transform methods provide solutions only if interpolation is used, thus, sufficient smoothness must be assume in the projection data in limited data case. They have precise error estimate so they are very good for design purposes. Iterative methods, such as (MART), are found better choice (compared to transform methods) for limited number of rays/detector or views occurring due to certain engineering constraints.

The present effort focus on development of MART algorithm for plasma tomography geometry. This algorithm is used to reconstruct phantom data with realistic levels of noise from a number of different imaging geometries. The tomographic image developed by MART algorithm shows appropriate results for both smooth to noisy phantom within 1% error. The tomographic results for real plasma experimental data shows dark and bright concentric emission ring in plasma cross-section. MART algorithm shows inside view of object function in limited data scenario.

7. Reference
