Cross-correlation based Imaging of Defects in Plate using Ultrasonic Lamb Waves

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Abstract

Notch like defects may occur in thin metallic structures and components. These defects are often not detected due to their smaller size and if unattended may lead to leakages, causing industrial accidents and economic losses. Therefore, early detection of these defects is essential. The current work presents preliminary studies on an ultrasonic Lamb wave based imaging technique for detection of these defects. Experimental investigations are presented on a thin plate to demonstrate the developed technique. An A0 mode dominated Lamb wave field is generated using wedge technique which suppresses the higher Lamb wave modes. Notches of different sizes are created in the plate and are imaged in the plane of plate using the Planar Synthetic Aperture Focusing Technique (P-SAFT). The resulting images are improved by performing cross correlation between the received ultrasonic field and the incident A0 mode Lamb wave field. The SAFT images constructed from the cross-correlated waveforms show improvement over their counterparts generated through the conventional SAFT algorithm.

Keywords: Ultrasonic, Lamb wave, SAFT, Cross-correlation

1. Introduction

In the past few decades, plate structures are widely used in aerospace, civil or mechanical structures. Defects in these structures are very common in form of crack, delamination, corrosion and fatigue related damage etc. Therefore, regular inspection is essential to assure safety of these structures. Lamb wave based techniques, have been widely used for monitoring of plates [1-3] structures. The advantage of this wave is that, it propagates through the entire thickness of the thin plate and offer comprehensive information regarding the health of the structure [4, 8]. However, Lamb waves are dispersive and multiple modes are generated in the plate, which makes the propagating field highly complex. To eliminate the effect of dispersion and complexities in the received field, the applied excitation is limited to a frequency bandwidth, wherein the dispersion is minimal and only two fundamental modes (i.e., A0 and S0) are generated [5-7]. Application of a single mode Lamb wave for detection of defects in plates has been reported by several researchers [7-11]. The defects in a plate can be detected by studying the characteristics of the wave field reflected from the defect or the transmitted wave field that has propagated through the defect [8, 12-13]. Different signal processing and imaging based techniques viz. delay and sum [14], minimum variance [15], empirical mode decomposition [16], tomographic reconstruction [17] and several triangulation techniques [18] have been used to detect the defects in plates. The triangulation techniques require limited number of sensors, however, the resolution of the image is also limited. The tomographic reconstruction improves resolution of the images of the defects; however, a large number of sensors and a significant level of computational effort is necessary for its successful implementation.
This paper discusses on imaging of defects in an aluminium plates using a variant of Synthetic Aperture Focusing technique (SAFT) named as ‘Planar SAFT’ imaging. SAFT was initially used for radar based mapping using airborne vehicles. Although, SAFT has been widely explored in conjugation with compressional and shear waves in materials ranging from metals, composites and concrete [19-22], its application using the scattered Lamb wave field has rather been limited to a few examples and that too in the frequency domain [22]. The \( A_0 \) Lamb wave mode has been chosen in this study for inspection of plate. This particular mode is chosen over other modes because i) it can propagate longer distances with minimal dispersion, provided the bandwidth of the excitation is carefully chosen; ii) the out of plane amplitude of \( A_0 \) mode is significantly larger compared to other wave modes; iii) the shorter wavelength of the \( A_0 \) wave enhances the sensitivity of damage detection and also the image resolution and iv) this mode can be excited with a relatively simple setup [24]. The proposed technique requires only tied-together data collection strategy [25] instead of a more time consuming Full Matrix Capture (FMC) approach [20-21, 23]. The methodology also does not require information regarding the pristine condition of the medium and is, therefore, reference free.

2. Background Theory

2.1 Lamb waves in plates

Lamb waves are the guided waves that can travel long distances in thin plates. Symmetric (\( S_0, S_1, S_2 \ldots \)) and anti-symmetric (\( A_0, A_1, A_2 \ldots \)) modes are associated with the Lamb wave propagation. These modes are highly dispersive and their propagation velocities are dependent on the frequency and thickness product \((f \times d)\) of the plates [5]. Particle motion during the different mode of wave propagation (anti-symmetric and symmetric) is shown in Figure 1.

\[
\begin{align*}
\frac{\tan(\beta h)}{\tan(\alpha h)} &= -\frac{4\alpha \beta k^2}{(k^2 - \beta^2)^2} \\
\frac{\tan(\beta h)}{\tan(\alpha h)} &= -\frac{(k^2 - \beta^2)^2}{4\alpha \beta k^2}
\end{align*}
\]

Where, \(h = \frac{d}{2}, \alpha^2 = \frac{\omega^2}{V_p^2} - k^2\) and \(\beta^2 = \frac{\omega^2}{V_s^2} - k^2\). The other terms viz. \(d\) = thickness of the plate, wave number= \(k = \frac{\omega}{c_p} = \frac{2\pi}{\lambda}\); angular frequency= \(\omega = 2\pi f\); implies \(c_p; f\) = frequency of the propagating waveform, compressional wave velocity = \(V_p = \sqrt{\frac{E(1-\nu)}{\rho(1+\nu)(1-2\nu)}}\).
and shear wave velocity \( V_s = \sqrt{\frac{E}{2\rho(1+v)}} \), and phase velocity \( C_p \). Solution of Equation (1) denotes the symmetric \( (S_i) \) wave modes and Equation (2) provides the antisymmetric \( (A_i) \) modes of the Lamb waves. Aluminium plates are considered in the present study for the imaging of plates. Dispersion curves of Aluminium plates are generated using real solutions of these equations, which represent the un-damped propagating wave modes of the structures. The equations can be rewrite as following:

\[
\frac{\tan(\beta h)}{\beta} + \frac{4ak^2 \tan(\alpha h)}{(k^2-\beta^2)^2} = 0
\]

(3)

\[
\beta \tan(\beta h) + \frac{(k^2-\beta^2)^2 \tan(\alpha h)}{4ak^2} = 0
\]

(4)

The Equation (3) and Equation (4) is solved to obtain the dispersion curve for phase velocity of the plate. The group velocity \( C_g \) can be found out from the phase velocity using the following equation:

\[
C_g = \frac{d\omega}{dk} = C_p^2 \left[ C_p - \omega \frac{dC_p}{d\omega} \right]^{-1}
\]

(5)

Using \( \omega = 2\pi f \) the equation for group velocity can be written as

\[
C_g = C_p^2 \left[ C_p - (fd) \frac{dC_p}{d(fd)} \right]^{-1}
\]

(6)

Where \( fd \) denotes frequency times thickness.

The phase and group velocity dispersion curve for aluminium plate are shown in Figure 2a and 2b.

![Figure 2](image-url)

Figure 2 (a) Phase velocity (b) group velocity dispersion curve of aluminium plate

The product of frequency and thickness \( (fd) \) governs the phase velocity \( C_p \) and group velocity \( C_g \) of particular Lamb wave mode.

### 2.2 Generation of a pure A0 Lamb wave mode in plates

In the present study, pure antisymmetric Lamb wave \( (A_0) \) mode is generated using the wedge technique [4], which exploits the phenomenon of mode conversion of the waves incident at the wedge-plate interface (Figure 3). A compressional wave transducer is placed on the inclined face, at the critical angle \( (\theta_i) \) corresponding to the compressional wave velocity \( (C_{lw}) \) in the wedge material and the \( A_0 \) mode phase velocity in the plate \( (C_{A0}) \). The critical angle can be calculated using the Snell’s law [5], shown by the Equation 7.

\[
\theta_i = \sin^{-1} \left( \frac{C_{lw}}{C_{A0}} \right)
\]

(7)
In the present study, a teflon wedge is fabricated with the wedge angle tuned to generate a pure $A_0$ Lamb wave mode. The wedge angle equal to 32 degrees ($\theta_i=32^\circ$ from Equation 1) is calculated by considering the compressional wave velocity in Teflon ($C_{LV}$) to be 1520 m/s and the estimated $A_0$ Lamb wave phase velocity ($C_{A0}$) is to be 2870 m/s.

3. Experimental Investigation

An aluminium plates having thicknesses of 16 mm is considered for imaging of damage using Lamb waves. Vertical notches are created in the plates to simulate defects in the plate (Figure 3). The details of the experimental setup (dimensions, sensor locations, defect locations) are shown in Figure 4. The transmitters and the receivers are placed side by side and the ultrasonic signals are acquired in a tied together configuration at an interval of $10 \text{ mm}$ along the aperture of the specimens, as shown in Figure 4. The Lamb wave based inspection is carried out on the plate surface opposite to that of the notch opening to simulate an invisible damage (Refer to Figure 3). The notches are marked as I to III in Figure 4. The details of the notch dimensions are shown in Table 1 and Figure 4. Three different ratios of notch depth to thickness ($d/h$) are considered in the present study.

**Table 1: Detailed dimensions of the notches**

<table>
<thead>
<tr>
<th>Specimen $(h = 16\text{mm})$</th>
<th>Notch No.</th>
<th>Notch Length $(mm)$</th>
<th>Notch depth $d$ $(mm)$</th>
<th>Notch ratio $d/h$</th>
</tr>
</thead>
<tbody>
<tr>
<td>I</td>
<td>61</td>
<td>8.9</td>
<td>0.5563</td>
<td></td>
</tr>
<tr>
<td>II</td>
<td>95</td>
<td>12</td>
<td>0.75</td>
<td></td>
</tr>
<tr>
<td>III</td>
<td>45</td>
<td>5.45</td>
<td>0.3406</td>
<td></td>
</tr>
</tbody>
</table>
A representative photograph of the experimental setup is shown in Figure 5. A wide band excitation sources of 100 kHz center frequency is used. A 200 Volts square wave signal originating from a pulser-receiver circuit is used as the input and the received signals are digitized (by performing 128 averages) in an oscilloscope with sampling frequency of 70 MHz for a time window of 500 $\mu$s. Petroleum jelly is used as the coupling agent between the transducer and wedge and the wedge and plate interfaces.

**3.1 Planar Synthetic Aperture Focusing Technique (P-SAFT) algorithm**

A variant of SAFT algorithm, named as ‘Planar SAFT’ is used in this paper. The Planar SAFT generates an image in the plane of the plate, unlike conventional SAFT, which generates an image in a vertical plane passing through the aperture. The wave velocity considered in this algorithm is the Lamb wave group velocity and the imaging technique
shows the location of a vertical notch type defect in the plane of the plate. The information regarding the depth of the defect is not provided by the images. The plane of the plate (Figure 6) is discretized into a rectangular grid of pixels and each pixel is assumed to be a potential scatterer.

One transmitter (Tr) generates the A₀ Lamb wave mode and the scattered field reaches the receiver transducer (Re). The Time of Flight (TOF), corresponding to the travel path from Tr to Re via pixel \( P_{(m,n)} \) is calculated as:

\[
TOF = \frac{\left( |\vec{d}_{m,n}^s| + |\vec{d}_{m,n}^r| \right)}{C_{A₀}}
\]

where, \(|\vec{d}_{m,n}^s|\) is the distance from source to pixel location, \(|\vec{d}_{m,n}^r|\) is the distance from pixel to receiver location and \(C_{A₀}\) is velocity of the A₀ Lamb wave. The image value \( l_{(m,n)} \) at the pixel \( P_{(m,n)} \) corresponding to \( N \) tied-together positions is calculated according to Equation 9:

\[
l_{(m,n)} = \sum_{k=1}^{N} f_i(t = TOF)
\]

where, \( f_i(t = TOF) \) is the amplitude of the \( k^{th} \) A-scan acquired by the transmitter-receiver pair and TOF is given by Equation 8. The tied-together approach is adopted to enable faster data acquisition.

3.2 Cross-correlation based analysis

The images obtained from the Planar SAFT are contaminated with noises generated from the mode conversion of Lamb wave modes. A cross-correlation based approach is adopted in the present study to maximize the signal to noise ratio (SNR) of the received A-Scans thereby improving the SAFT images. Individual received A-scans are cross-correlated with the direct arrival of A₀ pure Lamb wave to maximize the SNR.

4. Experimental results and discussion

The captured ultrasonic field along the aperture of the plate, are used for the preparation of Planar SAFT images. The images, without any cross-correlation processing are the Raw SAFT image, which after carrying out the cross-correlations are termed as cross-correlated SAFT image. A magnified view of the images is also presented here for closer look at the defects. The defect no. I and II can be identified in the raw SAFT image (Figure 7), however,
defect no. III is not clearly visible and noise amplitudes are comparable with the defect no. III.

![Figure 7 Raw SAFT image; Colorbar represents the normalized amplitude.](image)

Now, the cross-correlated SAFT images are presented in Figure 8. All the notches (I, II and III) are mapped in the image.

![Figure 8 Cross-correlated SAFT image; Colorbar represents the normalized amplitude.](image)

The cross-correlation improves the resolution of the image by suppressing the noises. The capability of Lamb waves based reference free planar SAFT imaging technique is demonstrated in the present study. It is also observed from the study that cross-correlation improves the planar SAFT image quality. It suppresses the artifacts and noises, and increases the SNR of the image. The possibility of defect detection increases with the increase of notch depth to thickness ratio (d/h) and vice versa.

5. Conclusions
The study presented in this paper demonstrates that wedge based generation and reception of the $A_0$ Lamb wave field in an aluminium plate, can be utilized for detection and imaging of a notch like defects. A significant advantage of the proposed technique is that no prior information about the pristine condition of the medium is necessary. Therefore, the technique is reference free. Another advantage of this technique is, only one side access is necessary for the imaging. The developed imaging technique is faster compared to conventional SAFT technique which generate the images in the cross section of specimen. Whereas ‘Planar SAFT’ technique, shows the image in the plane of specimen. The cross-correlation SAFT is also shown here to increase the SNR of the image and suppress the artifacts. Notches of different depths and sizes in different plates are successfully detected using the proposed technique. The proposed technique also provides comprehensive information about the width of the invisible notches. The cross-correlation based planar SAFT imaging technique may be used for the imaging of defects in plate structures. The image is created by utilizing the tied-together ultrasonic data, which is more efficient than the FMC mode of data acquisition. Therefore, it enables faster scanning of structures compared to available techniques.

Future research will be carried out to investigate the capability of this technique to detect real cracks like due to fatigue, corrosion etc.

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