Extension and Inflation of an Auxetic Cylindrical Stent
S.K. Bhullar, Martin B.G. Jun
Mechanical Engineering, University of Victoria, Canada
E-mail: sbhullar@uvic.ca; mbjun@uvic.ca

ABSTRACT

Fatty deposits can block blood flow through arteries and cause pain. A piece may break off, form a clot, and cause a heart attack or stroke. A tiny, expandable mesh tubes called stents help to open the blockage and keeps it open, which improve blood flow, allows blood to flow smoothly and reduces pain and risks of clots forming. These kinds of stents can be made of either metal or polymers. Ideally, the elastic response of a biomaterial should be matched with the biological function and mechanical properties of native tissue. Poisson’s ratio, which essentially describes the deformations in the transverse direction, is an important parameter, determining the complete elastic response of any biomaterial. Further it is generally believed that synthetic biomaterials have better controlled physical and mechanical properties and can be used to tailor for both soft and hard tissues. Poisson’s ratio of man-made materials also can be tuned by patterning polymers with an artificial lattice of rib-containing unit-cells. The materials tailored through negative Poisson’s ratio are called auxetic materials. Studies and experiments demonstrated that these materials offer a huge potential in biomedical industry. Therefore, the main purpose of this work is to develop a mathematical model to discuss mechanical properties such twisting moment, extension and inflation of an polymer auxetic stent. To summarize auxetic stents can help minimise the negative effects of current stent designs through their geometry, deformation mechanism and enhanced mechanical properties. Therefore, auxetic stent will be beneficial to improve early and delayed complications of stenting hence quality of life of patients.

Keywords – Auxetic material, Tailored geometry, Deformation mechanism, Cylindrical stent, Enhanced mechanical properties.
1. INTRODUCTION

1.1. Problem Area

Generally the build-up of fatty material and other substances on the internal surface of the blood vessels reduce supply of blood and oxygen which may cause sudden death or limit normal daily activity. One strategy used to control symptoms or restore blood supply for example is Percutaneous Transluminal Coronary Angioplasty (PTCA) in which a small elongated balloon is inflated at the site of the plaque, compacting the deposited material against the vessel wall. However, nearly 70% of the lumen size gained during PTCA is often lost due to arterial elastic recoil and other early and delayed complications. The concept of remodeling the artery was in 1964 and entered into mainstream medicine 1977 [1]. Its advantages include ease of operation and speed of recovery. Thus, the concept of the stent arose as a means to mitigate elastic recoil of the artery and to strengthen the artery wall [2-4]. Therefore a stent is placed in an artery to restore blood flow through narrow or blocked arteries which helps support the inner wall of the artery in the months or years. Stents usually are made of metal and polymers but sometimes they are made of fabric called stent grafts and are used in larger arteries. Also some stents are coated with medicine that is slowly and continuously released into the artery. These stents are called drug-eluting stents. The medicine helps prevent the artery from becoming blocked again. To date a variety of stents with different materials are fabricated for example [5-10]. Further, despite the development and progression of metallic stents, they continue to have limitations such as stent thrombosis, which requires prolonged antiplatelet therapy, and mismatch of the stent to the vessel size, which often results in a smaller lumen after stent implantation. [11]. In order to overcome some of these limitations polymeric stents emerged as a promising alternative to existing stents. Polymeric stents have good biocompatibility, biodegradable and can be readily loaded with drugs in the polymer bulk [12]. In addition the tailored mechanical properties plays potential role to make them compatible with human body. Therefore, the challenging desire of being able to tailor materials with any specified mechanical and elastic properties has been expressed by several researchers in recent years. From last three decades researchers have been working on the designing materials with negative Poisson’s ratio. In 1980’s it was studied that materials with negative Poisson’s ratio such Polyurathylene as foam [13] and PTFE (polytetrafluoroethylene) [14] have better indentation resistance, shock absorption and fracture toughness. In the next subsection a brief introduction to materials having negative Poisson’s ratio tailored with enhanced mechanical properties are discussed.

1.2. Auxetic Materials

Over the last three decades study of materials with tailored mechanical properties called smart materials is important area of research. The of materials having negative Poisson’s ratio called auxetic materials become wider when stretched [1] as illustrated in Fig. 1.
This unusual behavior has been an accepted consequence of classical elasticity theory for over 150 years [15] and they are thermodynamically stable materials. In fact Poisson's ratios of isotropic materials can not only take negative values, but can have a range of negative values twice that of positive ones [16]. The concept of negative Poisson’s ratio was matured by Roderick Lakes and many others in 1980’s [13-14] and since then a variety of auxetic materials and structures have been discovered, fabricated or synthesized ranging from the microscopic down to the molecular levels. However, some very important biological materials are also auxetic such as several types of skin including catskin, cow teat skin, salamander skin and the load-bearing cancellous bone from human shins [17-19] as well. Also, the arterial endothelium tissue demonstrates negative Poisson’s ratio [20]. Thus under the catagories of smart materials auxetic materials with tailored enhanced mechanical properties such as indentation resistance, fracture toughness, resilience, vibration control and shear resistance [21-23] have received much attention and offer a huge potentials to biomedical industry (but not limited to) along with the natural sciences, the aerospace and defense. The applications of auxetic materials in biomedical for example auxetic scaffolds, stents and implants with enhanced mechanical properties through negative Poisson’s ratio tailored by geometrical structure hence unique deformation has been discussed in [24-26].

With the above background in this paper we studied effect of axial forces (normal forces), twisting moment and inflation (pressure) of a cylindrical polymer auxetic stent. The material Polytetrafluoroethylene (PTFE) thin cylindrical tube of inner radius 1.44mm and outer radius 2mm is selected for the auxetic stent as illustrated in Fig. 2. Then, the results are discussed followed by some conclusions. To our knowledge extension and inflation in an auxetic stent of PTFE have never been reported.

Fig. 1: The auxetic (left) versus non-auxetic (right) behavior of materials.

Fig. 2: PTFE cylindrical tube for stent application
2. MATERIAL AND METHODOLOGY

2.1. Material Selection

The polymer Polytetrafluoroethylene (PTFE), a highly anisotropic non-linear elastic material having certain similarities between the microstructure, mechanical and flow parameters and better matched to those of the native tissues in the application of artificial implants, is selected for the auxetic stent. In addition it has outstanding physical, chemical, mechanical, and thermal properties, flexibility with a high tensile strength and resistance to fatigue [14]. Further, consisting of a carbon backbone covalently bonded to a uniform sheath of fluorine atoms Polytetrafluoroethylene (PTFE) can be manipulated and engineered into a variety of forms. This expanded form is referred to as expanded PTFE or ePTFE and takes many shapes including tapes, membranes, films, tubes, fibers, sheets, and rods [27]. The molecular structure of PTFE produces properties that are beneficial when considering its use in medical applications. In addition, it resist wetting by biological liquids and is not chemically changed or degraded by medical fluids (extremely hydrophobic) and the material performance is maintained over time. The vascular graft is a classic example of where PTFE/ePTFE (expanded PTFE) is used in medical devices [28]. It is found that polymer PTFE has negative Poisson’s ratio and the auxetic behavior is exhibited through the microstructure of which is comprised an interconnected network of nodules and fibrils as illustrated in Fig. 3 [14-29].

![Fig. 3: Microstructure of PTFE](image)

2.2. Inflation and Extension of the stent

As a model for stent is considered as an auxetic hollow cylinder with inner radius $a$ and outer radius $b$, height $l$ and thickness $h$. The ends of cylinder are subjected to tractions that supply a net twisting moment $M$ and normal force, $N$ while the curved surface is traction free and cylindrical polar co-ordinates $X_1, X_2, X_3 = R, \theta, Z$ and $x_1, x_2, x_3 = r, \theta, z$ before and after deformation respectively are described as:

$$r = r(R), \quad \theta = \Theta + \psi Z, \quad z = \lambda Z$$

where, $\lambda$ is the axial stretch ratio, $\psi$ is the angle of twist and in further calculations dot is used to denote differentiation with respect to $R$. Further, the stress components for a cylinder composed of compressible material are:
As we assumed that compressible cylinder is composed of Blatz-ko-compressible material, therefore
\[
W = C \left[ (I_1 - 3) + \frac{1-2\nu}{\nu} \left( \frac{I_3^{3-2\nu}}{I_3} - 1 \right) + C_4^{\lambda-1} (I_4 - 1)^2 \right] \tag{7}
\]
\[
W_1 = C_4, \ W_2 = 0, W_3 = -C_1 I_3^{3-2\nu}, W_4 = -2C_4 (I_4 - 1), W_5 = 0 \tag{8}
\]
and the corresponding stress from equation (1) – (6) are
\[
J\sigma_{11} = 2C_1 \lambda^2 - 2C_1 I_3^{3-2\nu} \tag{9}
\]
\[
J\sigma_{22} = 2C_1 \lambda^2 + 2C_1 \lambda^2 - 2C_1 I_3^{3-2\nu} + 4C_4 (I_4 - 1) \lambda^2 \tag{10}
\]
\[
J\sigma_{33} = 2C_1 \lambda^2 - 2C_1 I_3^{3-2\nu} + 4C_4 (I_4 - 1) \lambda^2 \tag{11}
\]
\[
J\sigma_{23} = 2C_1 \lambda^2 + 4C_4 (I_4 - 1) \lambda^2 \tag{12}
\]
where, \( I_3 = \frac{\lambda^2}{R^2}, I_4 = \lambda^2 + \psi^2 r^2 \tag{13}\)

The equilibrium equation in terms of Cauchy stress is
\[
r \frac{d\sigma_{11}}{dr} + \sigma_{11} - \sigma_{22} = 0 \tag{14}
\]
\[
J(\sigma_{11} - \sigma_{22}) = 2C_1 \left( \frac{\lambda^2}{R^2} - \frac{\lambda^2}{R^2} - \psi^2 r^2 \right) - 4C_4 (\lambda^2 + \psi^2 r^2) \tag{15}
\]
\[
J = \frac{r^2 \lambda}{R} \tag{16}
\]
Since for the purposes of illustration we consider an isotropic case to study auxetic and non-auxetic behaviour of materials and as cylinder is composed of Blatz-ko-compressible material, therefore, in this case we have
\[
W = \frac{\mu}{2} \left[ \left( \frac{I_2}{I_3} - 2 \sqrt{I_3^3 - 5} \right) \right] \tag{17}
\]
where, \( I_2 = \frac{\lambda^2}{R^2} + \lambda^2 + \psi^2 r^2 \), \( I_3 = \left( \frac{r^2 \lambda}{R} \right)^2 \), \( \mu = \frac{E}{2(1+\nu)} \), \( \nu \) and \( E \) are Poisson's ratio and Young's modulus respectively. The corresponding stress components are
\[
\sigma_{11} = \mu \left( 1 - \frac{R \lambda}{\psi^2 r^2} \right) \tag{18}
\]
\[
\sigma_{22} = \mu \left( 1 - \frac{R \lambda}{\psi^2 r^2} \right) \tag{19}
\]
\[
\sigma_{33} = \left( 1 - \frac{R(1+\psi^2 r^2)}{r^2 \lambda^2} \right) \tag{20}
\]
\[
\sigma_{23} = \mu \frac{\psi^2 r^2}{r^2 \lambda^2} \tag{21}
\]
The equilibrium equation (14) corresponds to equilibrium in the radial direction and upon using equations (18)-(21) along with the boundary condition $\sigma_{11}(b) = 0$, the obtained stress components are

$$\sigma_{33} = \left(1 - \frac{r(1+\psi^2+r^2)}{rr^2 \lambda^2}\right)$$

$$\sigma_{23} = \mu \frac{\psi r}{\lambda}$$

$$\sigma_{11} = \sigma_{22} = \sigma_{12} = \sigma_{13} = 0$$

The expressions for resulting normal forces, twisting moment $M$ and pressure (inflation) for the stent which is considered as a hollow cylinder with internal radius $a$ and external radius $b$

$$N = \pi \mu \int_a^b \sigma_{33} r \, dr$$

$$M = \pi \mu \int_a^b \sigma_{32} \lambda^3 \, dr$$

$$P = \frac{\mu}{a^2} \int_a^b \sigma_{33} r \, dr$$

3. RESULTS AND DISCUSSION

The deformation of the auxetic stent of PTFE is determined completely by the deformed inner radius, the twist angle $\psi$ and the axial stretch $\lambda$. To compare the auxetic versus non-auxetic behaviour of stent the results of a polyurathylene (PU) stent are also illustrated. The values of Poisson’s ratio are taken as $\nu = -0.7$ for PTFE stent [14] and $\nu = 0.46$ for PU stent. Auxetic versus non-auxetic pressure, axial and twisting moment versus radius curves are illustrated in Fig. 4 to Fig. 9.

Fig. 4: Pressure-radius curves of auxetic PTFE stent (left) and non-auxetic stent (right).
Fig. 5: Comparison of auxetic versus non-auxetic behaviour  
Pressure- radius curves of stents.

Fig. 6: Axial force – radius curves of an auxetic stent (left) and non-auxetic stent(right).

Fig. 7: Comparison of axial force – radius curves for an auxetic versus non-auxetic stents.
It is observed in Fig. 5 that the stent stiffness increases with axial stretch $\lambda$ and the upward movement of pressure – radius curves show the pressure required to maintain increased radius for an auxetic and non-auxetic stent. It is very clear from Fig. 6 that almost more than twice pressure is needed in the case of non auxetic stent compared to auxetic one. Further, increase or decrease in axial force $N$ depend on the value of axial stretch $\lambda$ as illustrated in Fig. 7 for auxetic and non-auxetic stents whereas their comparison is shown in Fig.8. This complex behavior is observed because of coupling between axial and circumferential strain (extension and inflation). For example, tension is induced in both axial and circumferential directions whereas circumferential stretch alone is accompanied by axial shortening. Also, twisting moment $M$ increases with radius as illustrated in Fig. 9 for a fixed axial stretch $\lambda = 1.5$ which is almost half in the case of auxetic stent as shown in Fig. 10. Further, these outcomes are improved through the effect of negative Poisson’s ratio tailored in polymer PTFE due to its deformation mechanism of microstructure which is comprised an interconnected network of nodules and fibrils as illustrated above in Fig.4. It can be seen from Fig. 11 (right top) [14] movement of the nodules caused by fibrils stretched or pulled tight, producing a lateral expansion and Fig. 11 (right bottom) describes the rotation of the nodules producing further lateral expansion and were confirmed by SEM examination[14].

Fig. 8: Twisting moment – radius curves for an auxetic(left) and non-auxetic stent.

Fig. 9: Comparison of twisting moment – radius curves of a non-auxetic stent.
4. CONCLUSION

4.1. Study Highlights

4.1.1. What is current knowledge.
During the last three decades considerable research effort has gone into measuring the mechanical properties of stents, scaffolds, implants and prostheses and their possible or actual synthetic replacement. With the requirement for future economic growth, auxetic materials are increasingly recognized as integral components of the smart and advanced materials and auxetic polymers offer a huge potential to biomedical industry (but not limited to). For example auxetic polytetrafluoroethylene (PTFE), ultra-high molecular-weight polyethylene (UHMWPE) and polyurethylene (PU) are used as the expansion member in a dilator device for coronary angioplasty, liner component for an intraluminal catheter, stents, artificial blood vessels, annuloplasty prosthesis, compression bandages, auxetic artificial intervertebral discs and scaffolds. To our knowledge elastic behavior such as extension and inflation of blood vessel stent in the context of elasticity modeled as an auxetic PTFE cylinder is not reported yet.

4.1.2. What is new in our research study.
Since stents play an important role improving the quality of life. Our research study represents real life configuration where blood vessel stent are considered as hollow cylinder which perform important and necessary work, allowing the blood to flow treating blocked or diseased arteries. This noval polymer auxetic stent has specifically tailored mechanical properties which are achieved through aucticity (influence of negative Poisson’s ratio) shows enhanced functional characterization over non-auxetic stent. This noval stent based on auxetic microstructure could be promising in the treatment of blocked arteries and other stenting applications. In addition initially having a small diameter could be beneficial in deployment of the stent.

REFERENCES