The 8th International Conference of the Slovenian Society for Non-Destructive Testing
»Application of Contemporary Non-Destructive Testing in Engineering«
***September 1-3, 2005, Portorož, Slovenia*, pp. 217-221**

MAGNETIC FIELD AROUND TWO SEPARATED MAGNETIZING COILS

B. Brudar

Breg 146, 4274 Žirovnica, Slovenia, *E-mail:* darko.brudar@prompt.si

ABSTRACT

If the arrangement of coils with the eddy currents testing is not a standard one it is very important to understand the physics of the new arrangement to be able to interpret the results properly. A very useful tool is the mathematical treatment of field equations. The numerical methods are often applied, since Maxwell equations are rather complex partial differential equations and there are usually real and imaginary components of the field to be taken into account. In the article there are basic algorithms given that are used with the method of finite differences. There are only main ideas given how to solve more complicated problems. The actual concrete results are part of a broader project and will be published later.

Keywords: Numerical solution of partial differential equations

1. Introduction

In order to be able to detect longitudinal as well as perpendicular surface cracks in ferromagnetic bars of circular cross-section it is necessary to make a special construction of magnetizing and secondary coils [1].

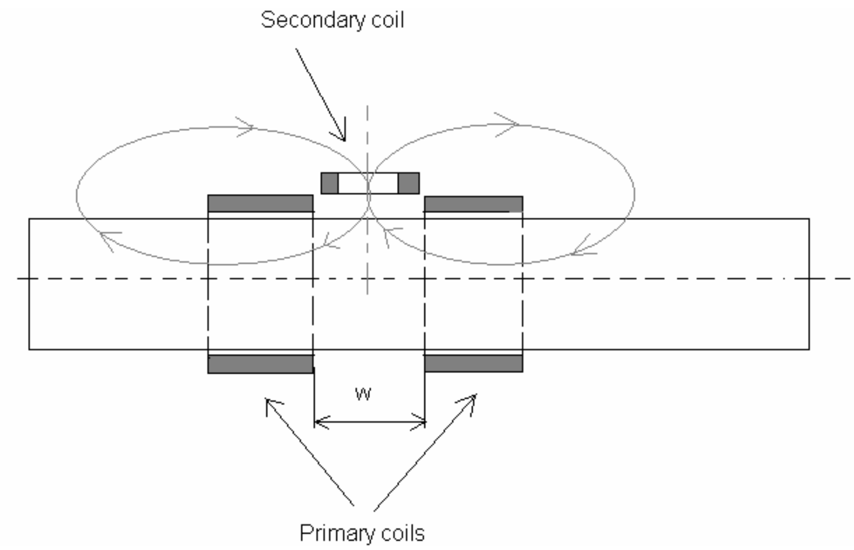

Fig. 1: Primary and secondary coils.

The magnetizing coil is made of two equal parts being separated longitudinally. The secondary coil is detecting radial magnetic flux and is placed in the middle of the primary coil according to Fig. 1.
It is not possible to speak about homogenous magnetic field. It is very important to define the proper optimal distance (W) between both parts in order to make the whole system sensitive to both types of cracks. The magnetic field in the neighborhood of all three coils can be calculated assuming some idealization of coils. Computer simulation of different positions and dimensions helps a lot with construction of actual arrangement. It is shown in due text how it is possible to assess the necessary separation in the primary coil and how it is possible to assess the induced voltage in the secondary coil influenced by moving of a defective bar through the whole arrangement.

2. Maxwell equations for the magnetic field

For the case of a ferromagnetic bar with the circular cross-section it is very convenient to start with the calculation of vector potential in cylindrical coordinate system [2].

$$\nabla^2\vec{V} - \sigma.\ \mu.\mu_0.\frac{\partial\vec{V}}{\partial t} - \mu.\mu_0.\varepsilon.\varepsilon_0.\frac{\partial^2\vec{V}}{\partial t^2} = 0 \quad (1)$$

By introducing new variable: $\vec{V} = \vec{W}.e^{i\omega t}$, we can have the real and the imaginary component of the vector potential : $\vec{W} = \vec{A} + i.\vec{A}^*$. It is possible to write two separated equations for the real and for the imaginary component.

$$\nabla^2\vec{A}\ + \omega.\mu.\mu_0.\sigma.\vec{A}^* = 0 \quad (2)$$

$$\nabla^2\vec{A}^* - \omega.\mu.\mu_0.\ \sigma.\vec{A} = 0 \quad (3)$$

The vectors A and A^* have generally three components in space and so we have a system of 6 partial differential equations to solve, for each space component and for the real and imaginary part.
To illustrate the procedure let us limit to the system of two dimensions. Namely the magnetizing coil has the form of a cylinder and if we choose the source of the coordinate system in the axis of the coil, the problem can be much simplified. If the problem is rotationally symmetrical, only one component $A = A_\varphi$ is different from zero.
The following pair of equations is to be solved in cylindrical coordinate system assuming that the problem is rotational symmetrical:

$$\frac{\partial^2 A}{\partial r^2} + \frac{1}{r}.\frac{\partial A}{\partial r} - \frac{A}{r^2} + \frac{\partial^2 A}{\partial z^2} + F.A^* = 0 \quad (4)$$

$$\frac{\partial^2 A^*}{\partial r^2} + \frac{1}{r}.\frac{\partial A^*}{\partial r} - \frac{A^*}{r^2} + \frac{\partial^2 A^*}{\partial z^2} - F.A = 0 \quad (5)$$

where $F = \omega.\mu.\mu_0.\sigma.a^2$.

On the other hand it is possible to write the corresponding expression for the two components of the magnetic field density B :

$$B_r = -\frac{\partial A}{\partial z} \qquad B^*{}_r = -\frac{\partial A^*}{\partial z}$$

$$B_z = \frac{1}{r}.\frac{\partial}{\partial r}(r.A) \qquad B_z^* = \frac{1}{r}\,\frac{\partial}{\partial r}.(r.A^*)$$

2.1 Boundary conditions

For the mesh points that are lying on the boundary between the air and the material, the basic boundary condition must be fulfilled. One must keep in mind that when crossing the boundary the normal component of the magnetic field density (B) must be preserved. On the other hand the tangential component of the magnetic field strength (H) must be preserved as well.

It is practically impossible to solve the system of equations generally. For some special simplified cases it is possible to find maybe even analytic solution, but much more often it is necessary to use some numerical methods.
There are several algorithms available but it depends on the experience of the research worker which method should be used. It is not necessary to calculate the vector potential to some great precision. A more or less rough assessment is usually good enough.

2.2 Method of finite differences

We have solved this problem by the method of finite differences. The coordinate system was chosen as shown on Fig. 2. Instead of looking for the general solution for the unknown vector potential we wish to find the solution in discrete mesh points as shown in Fig. 2.

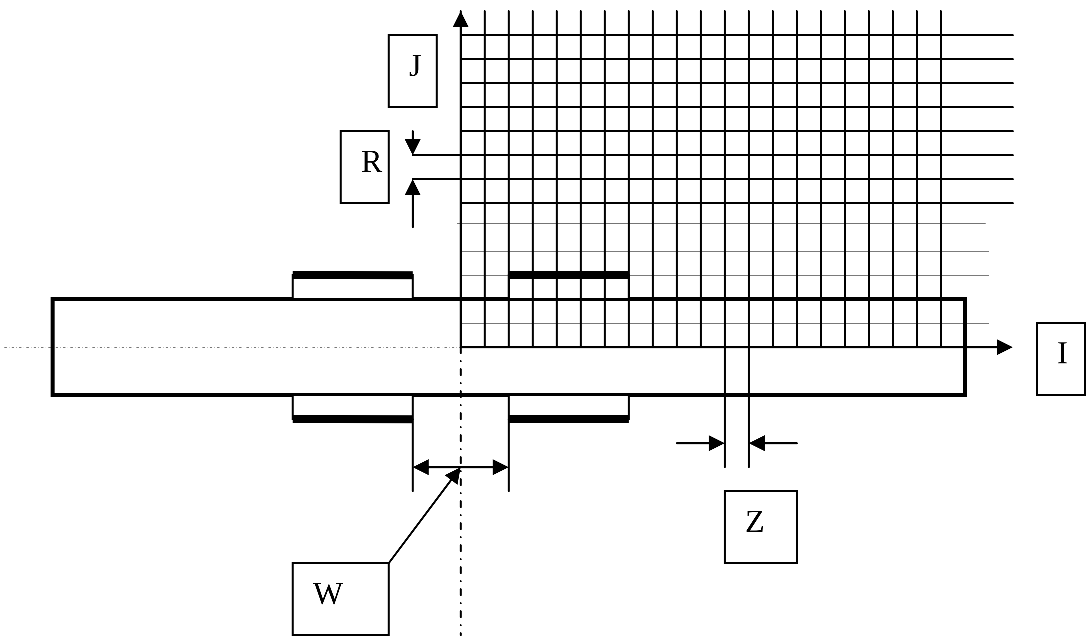

Fig. 2: The mesh points where the vector potential will be calculated.

For each mesh point (I, J) a linear numerical expression corresponding to the partial derivatives from Equation 4 or Equation 5 can be written.
For example: Instead of Equation 1 and Equation 2 the following linear expression can be written for the point (I, J):

$$\frac{A(I,J-1)-2*A(I,J)+A(I,J+1)}{R^2}+\frac{1}{R*(J-1)}*\frac{A(I,J+1)-A(I,J-1)}{2*R}-$$
$$-\frac{A(I,J)}{(R*(J-1))^2}+\frac{A(I-1,J)-2*A(I,J)+A(I+1,J)}{Z^2}+F*A^*=0 \quad (6)$$

$$\frac{A^*(I,J-1)-2*A^*(I,J)+A^*(I,J+1)}{R^2}+\frac{1}{R*(J-1)}*\frac{A^*(I,J+1)-A^*(I,J-1)}{2*R}-$$
$$-\frac{A^*(I,J)}{(R*(J-1))^2}+\frac{A^*(I-1,J)-2*A^*(I,J)+A^*(I+1,J)}{Z^2}-F*A=0 \quad (7)$$

In Equations 6 and 7 R means the mesh distance in radial direction and Z in longitudinal direction.
If there are IKON mesh points chosen in the direction I and JKON mesh points in the direction J, it is necessary to find the solution to IKON*JKON*2 linear equations with the same number of unknowns. From the solutions in discrete mesh points it is also possible to calculate the values of the real and the imaginary component of magnetic field density.
The problems how to write the corresponding numerical difference equation in the corners and on the lines of symmetry can be avoided by application of commonly used algorithms in methods of finite differences [3].
It is also convenient to use unequal spaced mesh. Far from the coils where nothing is being changed any more, the logarithmic mesh is very often applied. Also the mesh points inside the material are sometimes denser close to interesting spots. All these modifications of the mesh represent some minor additional difficulty and some mathematical experience is needed.

2.3 Explanations of symbols used in Equations 1-7

A, A^*	real and imaginary components of the amplitude of vector potential
a	radius of the bar
B_r, B_z	real components of the magnetic field density
B_r^*, B_z^*	imaginary components of the magnetic field density
F	dimensionless frequency
r	radius
t	time
$\vec{V}$	vector potential
$\vec{W}$	amplitude of vector potential
z	coordinate z
σ	electric conductivity
μ_0	permeability of empty space
μ	relative permeability
$\omega = 2\pi * f$	frequency

3. Radial field between the two parts of the primary coil

The arrangement of coils was simulated according to Fig 2 and the distance W between two parts was varied. There were two equal parts of magnetizing coil simulated with a ferromagnetic rod in the middle. The radial component of the magnetic field strongly depends on this distance and on the frequency and on the gap between the secondary coil and the surface of the bar. The relative permeability of the ferromagnetic bar in the middle is also of decisive importance. All

this data together are giving the necessary information to calculate the distribution of the magnetic field in the vicinity of defective spot. From the computer calculations also the radial magnetic flux could be evaluated. On basis of these simulations we could construct a very sensitive apparatus for detection of surface cracks of both kinds on a ferromagnetic bar.
The most important issue is that the separation of the primary coils must not be too small. It must be big enough to “bring” the field from inside of the bar across the surface to the outside where the radial secondary coil can “catch” the flow lines emerging from the interior. Since both parts of the primary coil are as equal as possible the secondary coil acts as the differential arrangement of a pair of secondary coils. The only difference is that in this case there are not two induced voltages subtracted but the two parts of magnetic field are flowing in opposite direction.
It is interesting to simulate different geometry and different physical properties on the distribution of the magnetic field at different frequencies. The results are part of a project where so called BRUDAR region around coils mentioned above will be investigated thoroughly.

3.1 Radial magnetic field at the points J=JTUL between two parts of magnetizing coil

We solved a simple case and here only main final conclusions are given.
Only one fourth of the whole cross-section is taken into account due to the symmetry according to Fig. 2. The length of part of magnetizing coil is chosen is 6*Z. The radius of the rod a= 6*R, the radius of the coil is 9*R, where R and Z can be chosen deliberately.
The number of points in radial direction JKON= 30, the number of points in longitudinal direction IKON= 30, the points on the surface of the rod in radial direction JK= 7, the points on the surface of the coil in radial direction JTUL=10.
Practically we chose the diameter of the bar 27 mm, the length of one part ot the primary coil was 50 mm long and the secondary coil had 20 mm diameter. At the frequency of 5-7 kHz the arrangement of coils was extremely sensitive for the longitudinal and perpendicular surface cracks of the bar. The calculations of the radial component of the magnetic field were done also for the moving rod through the coils bearing an uniform radial longitudinal surface crack with a good defined start and good defined end. All these calculations are part of a more complex project that is not yet complete.

4. Conclusions

The mathematical methods for solving Maxwell equations are an excellent tool to verify new ideas and when looking for new possibilities. The emerging field from the inside of the non-ferromagnetic bar can help detecting perpendicular cracks since the eddy currents cannot flow so easy if the field turns to radial direction. In this case namely a perpendicular crack represents some hindrance to eddy currents flowing in axial direction. .
Using a personal computer it is possible to simulate various cases of circular symmetry also tubes, combinations of ferromagnetic and non-ferromagnetic materials. All this gives much better understanding of NDT method itself and represents a new tool for further investigations.

5. References

[1] Brudar B.: Magnetic Leakage Fields as Indicators of Eddy Current Testing, The 8th International Conference of the Slovenian Society for Non-Destructive Testing, September 1-3, 2005, Portorož, Slovenia.
[2] Brudar B.: The Electromagnetic Field in the Neighborhood of a Defect in a Material, Research Techniques in NDT, Vol. V, Academic Press, London, 1982, 169-238.
[3] Smith G. D.: Numerical Solution of Partial Differential Equations, Oxford University Press, Oxford, 1971.