THEORETICAL AND EXPERIMENTAL STUDIES OF GUIDED WAVES PROPAGATION IN DRY AND ONE-SIDE IMMERSED WAVEGUIDES WITH SURFACE INHOMOGENEITIES

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ABSTRACT

Guided waves can be efficiently used for non-destructive testing and structural health monitoring of elastic waveguides. In layered structures submerged into fluid guided waves have greater attenuation and leaky waves can be observed. Guided leaky waves still have great potential in non-destructive ultrasonic inspection of immersed thin-walled structures like ships or underwater tanks. Compared to bulk waves guided waves provide a longer inspection range and allow to identify internal and surface damages. Strong resonance effects have been predicted and experimentally observed in layered plates with a single strip-like cracks or notches. The present study focuses on numerical and experimental analysis of guided waves interaction with inhomogeneities at the surface of an elastic plate. An experimental setup includes Laser Doppler vibrometry, a water bath and isotropic plates with surface inhomogeneities. Mathematical models are based on the integral approach as well as the higher order finite element method also called the spectral finite element method. The semi-analytical nature of the mathematical models provides information on propagation of guided waves in structure and their interaction with inhomogeneities. Reflection and resonance effects observed numerically and experimentally are carefully analyzed.

Key words: guided wave, surface inhomogeneity, scattering, resonance.

1. Introduction

Guided wave propagation method is studied from the point of view of its application in Structural Health Monitoring (SHM) of various structures [1]. A propagating wave interacts with
damage. The changes in the wave field caused by the damage are tracked and assessed. Guided elastic waves can travel in the medium with relatively low attenuation allowing to monitor large structures. In the real application of the SHM approach it is important to consider the influences of external factors like changing temperature [2], load [3] or contact with other media like water [2] or sand [4]. This paper focuses on numerical and experimental analysis of guided waves interaction with internal and external elongated inhomogeneities attached to the surface of the half immersed plate. Problem of guided wave propagation in water immersed structures was investigated previously for wholly immersed plate [5] and half immersed plate [6]. In the experimental investigations Laser Doppler vibrometer was used, while mathematical models are based on the integral approach as well as the spectral finite element method.

2. Mathematical model

2.1 Formulation of the problem

Let us consider an isotropic rectangular block of height \( h \) and width \( w \) attached at the surface of an elastic layer. The Cartesian coordinates \( x = \{x_1, x_2, x_3\} \) are introduced in such a way that block occupies the domain \( \Omega_2 = \{0 \leq x_i \leq w_i \mid x_3 < \infty, 0 \leq x_2 \leq h\} \). Boundary of the block \( S \) is composed of four lines \( S = S_1 \cup S_2 \cup S_3 \cup S_4 \), see Figure 1. The lower boundary \( S_4 = \{0 \leq x_i \leq w_i \mid x_3 = 0\} \) coincides with the upper surface of the layered structure occupying the domain \( \Omega_1 = \{-\infty < x_1, x_2 < \infty, -H \leq x_3 \leq 0\} \) as shown in Figure 1. Hereinafter all the wavefields related to the domain \( \Omega_i \) are denoted by the upper index \( i \) \((i = 1 \text{ for layer and } i = 2 \text{ for block})\). Surface load is applied to the upper surface of the layer in the area \( S_{\text{PWAS}} \).

Material properties are given by the Lame constants \( \lambda^1, \mu^1 \) and mass density \( \rho^1 \) for the layer and \( \lambda^2, \mu^2 \) and mass density \( \rho^2 \) for the block. In the current work layer and block are made of aluminum.

The displacements \( u^1(x, t) \) and \( u^2(x, t) \) are governed by the Lame equations. Due to the linearity of the equations the Laplace transform is applied in the form [7]

\[
u^i(x, t) = \frac{1}{\pi} \text{Re} \left[ \mathbf{u}^i(x, \omega)P(\omega)^{-i\omega} \right]
\]

in order to simplify numerical solution scheme. Here \( P(\omega) \) is the Laplace transform of the input electric signal \( p(t) \). Thus, the problem is formulated and solved in the frequency domain for all frequencies \( \omega \) and Lame equations are reformulated for harmonic solution \( \mathbf{u}^i(x, \omega) \):

\[(\lambda + \mu) \nabla \mathbf{u}^i(x) + \mathbf{\mu} \Delta \mathbf{u}^i(x) + \rho \omega^2 \mathbf{u}^i(x) = 0,\]
The media considered are at the rest until the moment $t_0 = 0$:

$$u^i_j(x, t) = \frac{\partial u^i_j(x, t)}{\partial t} = 0, \quad t < t_0. \quad (3)$$

Displacement vector for in-plane problem studied here consists of two non-zero components $\{u_1, 0, u_2\}$. For convenience vector of normal and tangential stresses $\tau = \{\sigma_{12}, \sigma_{22}\}$ is introduced, so that stress-free boundary conditions are formulated at the surfaces of the block and the layer except the contact area $S_i$

$$\tau^2(x, t) = 0, \quad x \in S_2 \cup S_3 \cup S_4, \quad \tau^1(x, t) = 0, \quad \{x_2 = -H\} \cup \{x_2 = 0\} / (S_1 \cup S_{PWAS}). \quad (4)$$

At the contact area $S_i$ displacements and stresses are continuous:

$$\tau^1(x, t) = \tau^2(x, t), \quad x \in S_i \quad (5)$$

$$u^1(x, t) = u^2(x, t), \quad x \in S_i \quad (6)$$

In order to simulate coupling between the block and the layer unknown function of stresses

$$q^\infty(x_1) = \tau^1(x_1, 0) = \tau^2(x_1, 0), \quad x \in S_i \quad (7)$$

is be determined at the area $S_i$ common for layer and block.

2.3 Integral approach

Displacements in a stress-free layer with surface load $q_{PWAS}(x)$ applied to the surface of the layer can be constructed using the IA [7,8]. In accordance with the IA harmonic wave-fields $u_{PWAS}(x)$ induced by the load function $q_{PWAS}(x)$ are represented as the inverse Fourier transform with respect to horizontal coordinate $x_1$:

$$u_{PWAS}(x) = \frac{1}{2\pi} \int K(\alpha, x_1) Q_{PWAS}(\alpha)^{-i\alpha x_1} d\alpha. \quad (8)$$

Here $\alpha$ is Fourier transform parameter, $K(\alpha, x_1)$ is the Fourier transform of the Green’s matrix and $Q_{PWAS}(\alpha)$ is the Fourier transform of $q_{PWAS}(x)$ dependant on frequency $\omega$. Elements of the Green’s matrix can be derived as for isotropic layer analytically as for multi-layered and anisotropic structures [8]. The integration path $\Gamma$ goes along the real axis deviation from it in the Fourier transform of the Green Matrix $K$ singularity points in accordance with the principle of limiting absorption [7,8]. Induced by the load function $q_{PWAS}(x_1)$ wavefield is scattered by the block occupying domain $\Omega_2$ and therefore resulting wavefield in the layer can be defined as

$$u^i(x) = u_{PWAS}^i(x) + u^\infty(x), \quad (9)$$

$$u^\infty(x) = \frac{1}{2\pi} \int K(\alpha, x_1) Q^\infty(\alpha)^{-i\alpha x_1} d\alpha, \quad (10)$$

where $Q^\infty(\alpha)$ is Fourier transform of $q^\infty(x_1)$ defined by (7), which is to be determined from the solution of the coupled problem (1) – (7).

2.4 Hybrid scheme

In order to solve the coupled problem (1) – (7) a hybrid scheme based on the combination of two methods is applied. The SEM which has proved its efficiency for wave problems is used to simulate block [9,10], while wavefields in the layer are given by the representations (8)–(10). The SEM uses variational formulation of the governing equations (1) and decomposition of the domain $\Omega_2$ into finite elements. Test functions space is introduced as $W = \{v(x) | v_i(x) \in L_i^2(\Omega_2)\}$, while Gauss-Legendre-Lobatto polynomials [10] are used as test and basis functions. After SEM
is applied in order to discretise solution in the block the following SLAE for the coupled problem is constructed:

$$\mathbf{A} \cdot \mathbf{y} = \mathbf{f},$$

(12)

with respect to the unknown expansion coefficients of the surface load and displacements in the block $\mathbf{y}^T = \{u_1^T, u_2^T, q_1^T, q_2^T\}$. The schematic view of the SLAE (12) is shown in the Fig. 2.

![Fig. 2: Scheme of the SLAE matrix building-up.](image)

Left upper part of the matrix $\mathbf{A}$ is built up with the spectral element coefficients with respect to the boundary conditions (4). The right upper right part is built with respect to the circulation integral resulting from the implementation of the boundary conditions (5). The lower left part is filled with zeroes except for the lines corresponding to the nodal points on the contact area $S_1$, where diagonal elements are equal to one. The lower right part is built up with respect to the equation (10) and represents the realization of the IA. The upper part of the vector $\mathbf{f}$ is filled with zeroes and the lower part is built up with respect to the equation (8) and represents the load function $\mathbf{q}^{PWAS}(x_i)$ impact. The fulfillment of the lower rows and vector of the right part in such a way ensures the validity of the equations (8)–(10) and therefore guarantees satisfaction of the boundary conditions (6).

3. Experimental setup

In the research presented in this paper piezoelectric transducer was used to excite guided waves and PSV-400 Scanning Laser Doppler Vibrometer (SLDV) was utilized for sensing of the guided wave signals. Measurements were performed for aluminium plates with dimensions: 600 mm x 100 mm. First plate had bonded aluminium alloy block with dimensions $h=2$ mm, $w=20$ mm (Fig 3a), while the second plate was with the bonded aluminium alloy block with dimensions: $h=20$ mm, $w=5$ mm (Fig 3b).

![Fig. 3: Dimensions of blocks bonded on the plates: a) h=2 mm x w=20 mm, b) h=20 mm x w=5 mm.](image)
During the measurements each sample was installed in special water container. Measurements were performed for samples in dry condition and immersed in water up to half of their thickness. Complete experimental set-up is presented in Fig. 4. Guided waves were excited by the piezoelectric transducer. Its location can be seen in Fig. 4 (close to the left edge of the sample). Measurements were taken along a line located at the middle of the sample width (starting from the piezoelectric transducer and ending at the edge on the right). In this research, excitation signal in the form of tone burst (5 cycles of sine modulated by Hann window) was utilized.

![Fig. 4: Measurement stand including water container.](image)

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4. Analysis

4.1 Influence of fluid on guided waves propagation and scattering

Measurements of guided wave propagation for the plates with two different aluminum blocks were conducted for the plates in dry conditions as well as for the plates immersed in the water (half of the plate). As the results of measurements along the line located at the middle of the sample width the time-space matrix of signals was obtained $s(t,y)$. This matrix can be presented in the form of waterfall plots in time-space domain $(t,y)$.

![Fig. 5: Waterfall plot for plate with the block 2 mm x 20 mm: a) dry plate, b) immersed plate; excitation frequency 15 kHz.](image)

In the first case, measurement were taken for the aluminum plate with the block with dimensions 2 mm x 20 mm. Measurements were taken for the dry plate and plate immersed in the water. In Fig 5 results of measurements in the form of waterfall plots for dry and immersed plate are presented. In this case excitation carrier frequency was equal to 15 kHz. In the results for case of dry plate (Fig. 5a) it can be noticed that elastic waves partially propagate through the block and
are partially reflected by it. Based on the time-of-flight and known distance of propagation it was determined that it is the A0 wave mode. This mode arrived to the edge of the plate at about 0.5 µs after excitation. Guided wave S0 mode that propagates much faster can be also noticed in the waterfall plot. It arrived to the edge of the plate at about 0.2 µs. Moreover, reflection of S0 mode from the block can be also noticed around the time 0.1 µs. However amplitude of S0 mode is much lower than A0 mode. In the case of immersed plate (Fig. 5b) the same waves can be noticed. Moreover in this case large reduction of amplitude of A0 wave mode can be noticed. In the case of A0 mode, displacements dominate in the out-of-plane directions and generate normal pressure loading onto the water which can propagate further into the water. For the case of S0 mode the amplitude reduction is negligible because in this case in-plane displacements dominate and they are source of shear loading onto the water. However, shear waves cannot propagate in the water therefore amplitude reduction will be negligible [11]. It needs to be underlined that such effects for S0 mode cannot be investigated here due to its very low amplitude.

![Waterfall plot for plate with the block 20 mm x 5 mm: a) dry plate, b) immersed plate; excitation frequency 15 kHz.](image)

In the second case measurements were taken for the aluminum plate with the block with dimensions 20 mm x 5 mm. In this case measurements were also taken for the dry plate and plate immersed in water. In the Fig 6 waterfall plots for dry and immersed plate for excitation carrier frequency 15 kHz were presented. In the case of dry sample (Fig. 6a) it can be observed that A0 mode partially propagates through the block and partially reflect from it. Here the reflection of A0 wave mode is much stronger that in the previous case where block with different dimensions was bonded to the plate. In this case time of arrival of A0 at the plate edge is equal to ~0.6 µs (in previous case it was about 0.5 µs) because the transducer was located farther than in the previous experiment. It is impossible to analyze propagation of S0 mode in this case. In the case of water immersed plate (Fig. 6b) reduction of A0 amplitude can be observed as influence of water contact. Moreover, in this case propagation of S0 mode and its reflection from the block (~0.2 µs) as well as the plate edge (~0.4 µs) can be noticed.

![Waterfall plot for plate with the block 20 mm x 5 mm: a) dry plate, b) immersed plate; excitation frequency 280 kHz.](image)
In the Fig. 7 waterfall plots for dry and immersed plate were presented for excitation frequency 280 kHz. In the case of dry sample (Fig. 7a) it can be observed that A0 mode partially propagates through the block and partially reflects from it (~0.09 µs). Than A0 mode which propagate through the block, reflects from the plate edge at time ~1.8 µs. Propagation of other mode with much stronger amplitude can be observed. It reflects from the block at time 1.1 µs and from the edge of plate at time ~2 µs. This second mode is neither A0 nor S0 mode but mode created by interference of A0 mode reflections from the plate edges located near the excitation point (see Fig. 4). In the case of water immersed plate (Fig. 7b) reduction of amplitude of A0 mode due to the contact of plate with water can be observed. Moreover less reflections of A0 mode from the block can be observed. The S0 mode is not visible.

4.2 Resonances and eigenfrequencies
Applying Fourier and wavelet transforms to experimental data strong resonances cannot be observed. Mathematical model provides eigenfrequencies of aluminum layer with block, which are roots of characteristic equation $\det A(\omega) = 0$ obtained from system (12). Corresponding dimensional frequencies $\omega_n$ are given in Table 1 for both blocks used in the experimental setup. It can be seen that absolute values of imaginary parts of $\omega_n$ are large enough for both blocks so that attenuation of trapped modes is great and it cannot be observed in the experiment.

<table>
<thead>
<tr>
<th>No</th>
<th>Block h=2 mm x w=20 mm</th>
<th>Block h=20 mm x w=5 mm</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>46 377 – 37.561i</td>
<td>38 773 – 38 836i</td>
</tr>
<tr>
<td>2</td>
<td>116 320 – 60 185i</td>
<td>60 929 – 46904i</td>
</tr>
<tr>
<td>3</td>
<td>191 171 – 72 113i</td>
<td>120 035 – 32 869i</td>
</tr>
<tr>
<td>4</td>
<td>267 623 – 34 142i</td>
<td>184 759 – 4 155i</td>
</tr>
<tr>
<td>5</td>
<td>360 648 – 37 580i</td>
<td>239 332 – 35 500i</td>
</tr>
<tr>
<td>6</td>
<td>399 758 – 20 363i</td>
<td>322 482 – 126 575i</td>
</tr>
</tbody>
</table>

Fig. 8: Resonance frequencies $\omega_n$ dependence on the width $w$ of the block (height $h = 2$ mm).

The developed model allows also performing parametric analysis of the resonance frequencies with dependence of the block sizes. Fig. 8 shows eigenfrequencies $\omega_n$ of aluminum layer with block of the height $h = 2$ mm in dependence on the width $w$. It is natural that resonance
frequencies tend to decrease with the block width extension. A similar situation is observed for
delaminations and notches, where strong localization is possible due to much lower $|\text{Im} \omega_n|$ values [12]. At the same time $\omega_n$ do not approach real axis, that may lead to wave localization
and it is much close to the case of piezoelectric transducer.

5. Conclusions

A hybrid mathematical model has been developed in order to simulate wave motion of layered
structure with rectangular block on the surface. It was revealed that complex-valued resonance
frequencies are denser with block width increase, but they do not come close to real axis so that
strong resonances cannot be observed in the experiment. An experimental study was performed
to analyse influence of fluid on guided waves propagation and scattering by block. Analysis of
conversion of A0 and S0 waves in dry and immersed plates due to block is provided.

6. Acknowledgments

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