POWER DENSITY OPTIMIZATION OF AN ARRAY OF PIEZOELECTRIC HARVESTERS USING A GENETIC ALGORITHM

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ABSTRACT

In the last decade, several studies have shown an increasing interest in harvesting energy from vibrations. A common way to proceed consists of using integrated piezoelectric ceramics into a vibrating structure excited by a harmonic source. However, literature has shown that this kind of harvester is only effective under a narrow bandwidth of excitation frequency. If the excitation frequency shifts from this band, the power density of the harvester will significantly decrease. Different strategies have been used to enhance the harvesting performances when the vibration source has a larger frequency bandwidth. One of them is to use an array of harvesters which consists of multiple harvesters having different resonance frequencies in order to increase the harvested power on a wider frequency bandwidth. However, this leads to a harvester having a higher volume, which decreases its power density (mW.cm\(^{-2}\)). Design optimization can help to attenuate this decrease, but it has never been investigated to our knowledge. In this paper, we investigate the possibility to maximize the power density for a vibration source having a wider frequency bandwidth, using an array of two piezoelectric energy harvesters. It is shown that the optimization process can limit the decrease of the mean power density to 6% by adding a second harvester while increasing the mean power by 75%.

Keywords: Piezoelectric energy harvester, wideband vibrations, array of harvesters, genetic algorithm optimization, Rayleigh-Ritz method, power density.
INTRODUCTION

Mechanical energy harvesting from vibrating structures using piezoelectricity has been studied in several works of the last decade. It has been shown that a vibration energy harvester is only effective when the first resonance frequency of the structure matches the frequency of a harmonic vibration source. Different strategies have been proposed to increase the harvesting performances over a wider frequency bandwidth. For instance, Eichhorn et al. [1] have proposed to apply a mechanical stress to the converter in order to tune its resonance frequency. Moss et al. [2] have designed a harvester that up-converts a low-frequency excitation source to its resonance frequency by using impacts. Ferrari et al. [3] have proposed an approach based on a bistable nonlinear oscillator in order to use its instability state to widen the harvester’s effective bandwidth. Xue et al. [4] have proposed a multi-mode structure composed of ten piezoelectric bimorphs with different resonance frequencies. This structure increases the harvested power over a wide range of excitation frequencies, but decreases the power density of the harvester.

This paper investigates the design optimization of an array of two piezoelectric harvesters by using a genetic algorithm. An electromechanical model is first described. This model predicts the power density of an energy harvester using a semi-analytical formulation based on Rayleigh-Ritz method. The model is next integrated into a genetic algorithm in order to maximize this power density. Optimization problems are then formulated and solved for a single harvester (SH) and for an array of two harvesters (AH).

PIEZOELECTRIC ENERGY HARVESTING MODEL

Electromechanical model

The model detailed in this study is based on the work of Sodano et al. [5] who established the constitutive equations of a piezoelectric bimorph cantilever beam like the one shown in Figure 1.

A base acceleration $a(t)$ produces flexural vibrations to the bimorph while the piezoelectric ceramics generate a charge flow $q(t)$. The harvested power $P$ is modeled by the power dissipated across a resistive load $R$. One can show that the electromechanical model of this structure can be represented by the following set of differential equations:

$$M \ddot{h}(t) + C \dot{h}(t) + K h(t) + 2R \dot{q}(t) = Da(t)$$
$$\theta h(t) - C_v R \dot{q}(t) - q(t) = 0$$

(1)
where \( h(t) \) is the modal time response vector. In these equations, \( M \) is the modal mass matrix, \( K \) is the modal stiffness matrix, \( C \) is the modal damping matrix, \( \theta \) is the electromechanical coupling vector, \( D \) is the input mass vector and \( C_p \) is the capacitance of the piezoelectric volume. These parameters are defined by:

\[
M = \int_{V_b} \rho_b \phi(x) \phi(x)^T dV_b + \int_{V_p} \rho_p \phi(x) \phi(x)^T dV_p + \int_{V_b} \rho_b y^2 \phi'(x) \phi'(x)^T dV_b + \int_{V_p} \rho_p y^2 \phi'(x) \phi'(x)^T dV_p \\
K = \int_{V_b} E_b y^2 \phi''(x) \phi''(x)^T dV_b + \int_{V_p} E_p y^2 \phi''(x) \phi''(x)^T dV_p \\
C = \alpha M + \beta K \\
\theta = -d_{31} E_p \int_{V_p} y \phi'(x) \psi(y) dV_p \\
D = \int_{V_b} \rho_b \phi(x) dV_b + \int_{V_p} \rho_p \phi(x) dV_p \\
C_p = \varepsilon \int_{V_p} \psi^2(y) dV_p
\]

where \( \rho_b, \rho_p \) are material densities, \( E_b, E_p \) are modulus of elasticity, \( V_b, V_p \) are the volumes, \( d_{31} \) is the piezoelectric charge constant, \( \varepsilon \) is the permittivity constant, \( \psi(y) \) specifies the electrical field across the piezoelectric volume, \( \phi(x) \) is the transverse displacement distribution vector and \( (') \) denotes the first derivative with respect to \( x \). \( \alpha \) and \( \beta \) are the mechanical damping coefficients which represent the proportional damping in the structure. It should be noted that the subscripts \( b \) and \( p \) refer respectively to the beam and the piezoelectric ceramics. By assuming a harmonic base acceleration having an amplitude \( A \) and a circular frequency \( \omega \), one can solve the differential equations system (1) and obtain the following steady-state solution:

\[
\begin{bmatrix} h(t) \\ q(t) \end{bmatrix} = \begin{bmatrix} H \\ Q \end{bmatrix} \sin(\omega t - \phi)
\]

where \( H \) is the amplitude vector of each modal time response, \( Q \) is the amplitude of the charge function and \( \phi \) is the phase angle vector. One can then determine the harvested power with:

\[
P = \frac{R(\omega Q)^2}{2}
\]

In order to determine the harvested power, only the transverse displacement distribution vector \( \phi(x) \) still remains to be determined from the mechanical vibrations. A semi-analytical approach is presented in the following.
Semi-analytical mechanical model

Figure 2 shows the harvester under study. It is composed of a monolithic cantilever beam and two piezoelectric ceramics. $L$, $b$ and $t$ are respectively the length, the width and the thickness while the subscripts $1$, $2$ and $p$ refer respectively to the first section of the beam, the second section of the beam and one of the two piezoelectric ceramics.

![Fig. 2: Mechanical model of the piezoelectric energy harvester.](image)

Using Bernoulli’s assumptions, the harvester’s displacement field $\mathbf{u}$ can be defined as:

$$
\mathbf{u} = \begin{bmatrix}
u_x \\
u_y \\
u_z
\end{bmatrix} = \begin{bmatrix}
-y w'(x,t) \\
w(x,t) \\
0
\end{bmatrix}
$$

where $w(x,t)$ is the transverse displacement along the beam. Using Rayleigh-Ritz approach, one can approximate $w(x,t)$ as:

$$
w(x,t) = \mathbf{f}(x)^T \mathbf{g}(t) = \{f_1(x) f_2(x) ... f_N(x)\}^T \{g_1(t) g_2(t) ... g_N(t)\}^T
$$

where $\mathbf{f}(x)$ and $\mathbf{g}(t)$ are respectively the trial functions vector and the time functions vector. Using this formulation, the harvester is then approximated as a $N$ degree-of-freedom system. The trial functions set used in this study is a hierarchical trigonometric set defined as:

$$
f_i(x) = \sin \left( \frac{a_i x}{L} + b_i \right) \sin \left( \frac{c_i x}{L} + d_i \right) \\
i = 1, 2, ..., N
$$

This set has numerous advantages (e.g. numerically more stable) which are well discussed in [6]. Mode shapes can then be found using Lagrange’s equations with the generalized coordinates $g_i(t)$.
\[
\frac{d}{dt} \left( \frac{\partial T}{\partial g_i} \right) - \frac{\partial T}{\partial g_i} + \frac{\partial U}{\partial g_i} = 0 \quad i = 1, 2, \ldots, N
\]  

(8)

where \( T \) and \( U \) are respectively the kinetic energy and potential energy. \( T \) and \( U \) can be defined as:

\[
T = \frac{1}{2} \left( \int_0^L \rho_b A_b(x) \left( \frac{\partial w(x,t)}{\partial t} \right)^2 \, dx \right) + \int_0^L \rho_p A_p(x) \left( \frac{\partial w(x,t)}{\partial t} \right)^2 \, dx \\
+ \int_0^L \rho_p I_p(x) \left( \frac{\partial^2 w(x,t)}{\partial \chi \partial t} \right)^2 \, dx + \int_0^L \rho_p I_p(x) \left( \frac{\partial^2 w(x,t)}{\partial x^2} \right)^2 \, dx
\]

(9)

\[
U = \frac{1}{2} \left( \int_0^L E_b I_b(x) \left( \frac{\partial^2 w(x,t)}{\partial x^2} \right)^2 \, dx \right) + \int_0^L \rho_p I_p(x) \left( \frac{\partial^2 w(x,t)}{\partial x^2} \right)^2 \, dx
\]

where \( A_b(x), A_p(x) \) are the cross-section area functions and \( I_b(x), I_p(x) \) are the area moment of inertia functions. Next, (9) is inserted in (8) to determine a differential equations system. These equations can be represented using a matrix formulation, i.e.:

\[
\ddot{M}g(t) + \ddot{K}g(t) = 0
\]

(10)

where \( \ddot{M} \) and \( \ddot{K} \) are respectively the mass and stiffness matrices. The eigenvalue problem can be solved in order to find the mode shapes \( [p_1, p_2, \ldots, p_N] \) of the structure. Knowing the mode shapes of the harvester, it is possible to determine the transverse displacement distribution vector \( \phi(x) \) of the system which is defined by:

\[
\phi(x) = [p_1, p_2, \ldots, p_N]^T f(x)
\]

(11)

This model has been previously validated in [7] for a similar harvesting structure.

**GENETIC ALGORITHM OPTIMIZATION**

Genetic algorithm is an optimization method based on biological evolution which can optimize highly complex cost functions. The optimization problem solution is obtained by an iterative process by using the concepts of natural selection and mutation of individuals forming a population. The process stops according to a defined criterion of convergence. Figure 3 shows the flowchart of a typical genetic algorithm.
Fig. 3: Flowchart of a typical genetic algorithm.

The genetic algorithm process works according to the following procedure:

- Each harvester constitutes an individual of a population and is represented by a chromosome composed of genes which depend on the number of variables. Each gene is represented by a binary number composed of a specific number of bits.
- The initial population is randomly created by a specific number of individuals, their cost functions are evaluated and the individuals with the higher cost function are selected to compose the first generation.
- Population ranking is then executed in regard of the evaluation cost of each individual to apply a natural selection.
- The pairing of parents is determined in order to proceed to the mating which is the first process of the algorithm to explore the cost function. Offspring of the parents are introduced into the population.
- The second process to explore the cost function is the mutation which changes the genetic code of some individual. This process consists of randomly permute a percentage of bits of the population genetic code. At the end of this process, cost functions of the muted individuals are evaluated.
- This iterative process is repeated until the cost function reached a criterion of convergence.

For a more detailed analysis of this process, the reader is referred to Haupt and Haupt [8].

POWER DENSITY MAXIMIZATION

Optimization simulation parameters

Some parameters of the harvester are set to be constant:

- The harvester is composed of a brass beam and two PZT-5H ceramics.
- The mechanical damping coefficients $\alpha$ and $\beta$ are chosen to set the first damping ratio to 2%, which is a typical value.
The harvester is driven by a base acceleration amplitude $A$ of 1g, which is an arbitrary value frequently used in the literature.

Material properties and constant geometric dimensions are gathered in Table 1.

<table>
<thead>
<tr>
<th>Harvester properties</th>
<th>Symbol</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Brass density</td>
<td>$\rho_b$</td>
<td>8410 kg.m$^{-3}$</td>
</tr>
<tr>
<td>PZT-5H density</td>
<td>$\rho_p$</td>
<td>7800 kg.m$^{-3}$</td>
</tr>
<tr>
<td>Brass modulus of elasticity</td>
<td>$E_b$</td>
<td>103.4 GPa</td>
</tr>
<tr>
<td>PZT-5H modulus of elasticity</td>
<td>$E_p$</td>
<td>62 GPa</td>
</tr>
<tr>
<td>Permittivity of ceramics</td>
<td>$\varepsilon_S$</td>
<td>27.3 nF.m$^{-1}$</td>
</tr>
<tr>
<td>Piezoelectric charge constant</td>
<td>$d_{31}$</td>
<td>-320 pC.N$^{-1}$</td>
</tr>
<tr>
<td>Width of the piezoelectric ceramic</td>
<td>$b_p$</td>
<td>25 mm</td>
</tr>
<tr>
<td>Length of the piezoelectric ceramic</td>
<td>$L_p$</td>
<td>50 mm</td>
</tr>
<tr>
<td>Thickness of the piezoelectric ceramic</td>
<td>$t_p$</td>
<td>0.27 mm</td>
</tr>
<tr>
<td>Width of the first section of the beam</td>
<td>$b_1$</td>
<td>25 mm</td>
</tr>
<tr>
<td>Width of the second section of the beam</td>
<td>$b_2$</td>
<td>40 mm</td>
</tr>
<tr>
<td>Length of the first section of the beam</td>
<td>$L_1$</td>
<td>51 mm</td>
</tr>
<tr>
<td>Thickness of the second section of the beam</td>
<td>$t_2$</td>
<td>5 mm</td>
</tr>
</tbody>
</table>

In regards of the process of the genetic algorithm discussed previously, Table 2 shows the optimization parameters used for the simulations. In practice, a lot of simulations have been performed and these parameters have led to a good trade-off between optimization computing time and accuracy.

<table>
<thead>
<tr>
<th>Simulation parameters</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Number of individuals in a population</td>
<td>12</td>
</tr>
<tr>
<td>Number of bits by genes</td>
<td>8</td>
</tr>
<tr>
<td>Proportion of mutation</td>
<td>0.05</td>
</tr>
<tr>
<td>Number of generation having the same maximum</td>
<td>150</td>
</tr>
</tbody>
</table>

Two optimizations problems are formulated in the following.

**Optimization I – Mean power density optimization**

The cost function of this first optimization problem is to maximize the mean power density $\gamma_m$ over a certain frequency bandwidth. The power density $\gamma$ is defined as the ratio of the harvested power (see equation (4)) and the harvester volume ($V = b_1L_1t_1 + b_2L_2t_2 + 2b_pt_p$). The mean power density $\gamma_m$ over a specified bandwidth $[f_1, f_2]$ is then defined by:

$$\gamma_m = \frac{\int_{f_1}^{f_2} \gamma df}{\int_{f_1}^{f_2} \gamma^2 df} = \frac{\int_{f_1}^{f_2} \frac{R(\omega Q)^3}{2(f_2-f_1)(b_1L_1t_1 + b_2L_2t_2 + 2b_pt_p)} df}{\int_{f_1}^{f_2} \gamma^2 df}$$

(12)
In this work, $f_1$ and $f_2$ are arbitrary set to 90 and 110 Hz. For the optimization of the SH, the length $L_2$, the thickness $t_1$ and the resistive load $R$ have to be optimized by the genetic algorithm. The optimization problem can then be summarized as follows:

\[
\text{Find the maximum of: } \gamma_m(\mu_i) \quad (13)
\]

with $\mu_i = [L_2, t_1, R]$

Subject to:
\[
\begin{align*}
10 & \leq L_2 \leq 40 \quad \text{[mm]} \\
0.75 & \leq t_1 \leq 2.00 \quad \text{[mm]} \\
1 & \leq R \leq 10 \quad \text{[k}\Omega]\n\end{align*}
\]

In this paper, ranges for variables have been arbitrary set.

For an AH, preliminary simulations have shown that the optimization simply leads to two identical optimal SHs. This optimal AH allows harvesting of twice as much power than the optimal SH and it represents the same power density. However, the bandwidth of excitation frequency where the harvester is effective is still narrow and the power distribution is not wider than the optimal SH. So, another optimization approach is hereafter proposed to widen the power distribution.

Optimization II – Weighted mean power density optimization

In order to obtain a more uniform power density distribution in the frequency domain, we propose to modify the previous cost function by adding a weight factor corresponding to the ratio of the mean power density $\gamma_m$ and the maximum power density $\gamma_M$ of the distribution, i.e. $\gamma_m / \gamma_M$. For the optimization of the AH, six variables must be optimized and the optimization problem can be summarized as follow:

\[
\text{Find the maximum of: } \gamma_m(\mu_2) \sqcup \gamma_m(\mu_2) / \gamma_M(\mu_2) \quad (14)
\]

with $\mu_2 = [L_{2A}, t_{1A}, R_A, L_{2B}, t_{1B}, R_B]$

Subject to:
\[
\begin{align*}
10 & \leq L_{2A} \leq 40 \quad \text{[mm]} \\
0.75 & \leq t_{1A} \leq 2.00 \quad \text{[mm]} \\
1 & \leq R_A \leq 10 \quad \text{[k}\Omega]\n\end{align*}
\]
\[
\begin{align*}
10 & \leq L_{2B} \leq 40 \quad \text{[mm]} \\
0.75 & \leq t_{1B} \leq 2.00 \quad \text{[mm]} \\
1 & \leq R_B \leq 10 \quad \text{[k}\Omega]\n\end{align*}
\]

Optimizations results

Table 3 summarizes the results of both optimizations. The design obtained by optimization I is a SH having a volume of 5.7 cm$^3$ while the design obtained with optimization II is an AH having a volume of 10.7 cm$^3$. The volume of the optimal AH is thus 88% higher than the one of the optimal SH.
Table 3: Results of both optimization problems.

<table>
<thead>
<tr>
<th>Variable</th>
<th>Optimisation I</th>
<th>Optimisation II</th>
</tr>
</thead>
<tbody>
<tr>
<td>Thickness of the first section of the beam</td>
<td>$t_1$</td>
<td>1.21 mm</td>
</tr>
<tr>
<td>Length of the second section of the beam</td>
<td>$L_2$</td>
<td>17.56 mm</td>
</tr>
<tr>
<td>Resistive load</td>
<td>$R$</td>
<td>6.39 kΩ</td>
</tr>
</tbody>
</table>

Figure 4 (a) compares the harvested power for an optimal SH and an optimal AH as a function of the excitation frequency. The mean harvested power of the SH is 9.3 mW while the one of the AH is 16.3 mW. It means that the AH is able to harvest a power 75% higher than the one of the SH. Furthermore, the minimal power harvested by the AH is 364% higher than the SH (8.41 mW versus 2.31 mW), which is an advantage if a continuous minimal harvested power is required for an application.

Figure 4 (b) shows the power density of the optimal SH and the optimal AH as a function of the excitation frequency. The mean power density of the SH is 1.62 mW.cm$^{-3}$ while that of the AH is 1.52 mW.cm$^{-3}$. It represents a decrease of 6% of the mean power density. However, one can observe that the power density is higher for the AH when the excitation frequency is between 90 and 94.4 Hz and, between 102.4 and 110 Hz, which represents 60% of the frequency bandwidth under study.

Fig. 4: (a) Harvested power and (b) power density as a function of the excitation frequency of the optimal SH (solid line) and the optimal AH (dashed line).
CONCLUSION

The main objective of this paper was to maximize the power density of an array of two piezoelectric harvesters. An electromechanical model using a semi-analytical mechanical model was first described to predict the performances of a piezoelectric energy harvester. This model was then used to maximize the power density by using a genetic algorithm. This optimization method was next described and optimization problems for a single harvester and an array of harvesters were formulated and solved. By using an optimal array of two harvesters, we have shown that it is possible to increase the mean harvested power by 75% while limiting the decrease of the mean power density to 6%. Furthermore, the power density for the optimal array of two harvesters is higher than that of the optimal single harvester over 60% of the bandwidth of interest. In future works, we will experimentally validate the performances of these harvesters.

ACKNOWLEDGEMENTS

This work was supported by the Fond Québécois de la Recherche sur la Nature et les Technologies (FQRNT) and Natural Sciences and Engineering Research Council (NSERC).

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