DAMPING CONTROL OF A PZT MULTILAYER VIBRATION USING NEGATIVE IMPEDANCE CIRCUIT

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Abstract

The control of damping was demonstrated by means of a negative impedance circuit. The generator excited vibration in a system consisting of a metal block and a jointed multilayer PZT rod. A small accelerometer attached to the metal block detected the vibration of the system. The free damped vibration of the system before and after the cutoff of the generator was recorded by an oscilloscope. It was found that the adjustment of a variable resistance in the negative impedance circuit either increased or decreased the decay time of the damped vibration. The resonant frequency, the decay factor and the initial acceleration were recorded as functions of the dividing ratio of the variable resistance. The theoretically calculated curves fitted well with experimental results.

Keywords: Elasticity, Piezoelectric Materials, Negative Impedance Circuit, Damping
INTRODUCTION

Previous papers have reported that the elasticity of piezoelectric materials can be controlled by incorporating a negative capacitance circuit [1-6]. On receiving the sound or vibration, piezoelectric materials generate piezoelectric voltage. This voltage is amplified by an electric circuit working as a negative capacitor and the amplified voltage is fed back to the materials. The piezoelectric strain induced by the feedback voltage may increase or decrease the original strain induced by sound or vibration, depending on the design of the circuit. The effective acoustic impedance of piezoelectric materials is hardened or softened by connecting the circuit. The hardening phenomenon has been utilized in sound shielding with a piezoelectric PVDF film: once excited by sound, the polymer film is hardened and reflects the sound [7-10]. The softening phenomenon has been utilized for vibration shielding by piezoceramic PZT plate: once excited by vibration, the PZT plate is softened and the resonant frequency of the vibration system is decreased [11-14].

In these studies, the main interest has been focused on controlling the magnitude of the elastic constant of piezoelectric materials. However, the dynamic phenomena like sound and vibration always involve the viscous properties of materials. An example is the free damping vibration of a bar. The decay time of vibration is governed by the viscosity of the material.

The complex elastic modulus \( Y \) of piezoelectric material is expressed as \( Y = Y' + iY'' \), where \( Y' \) is the real part and \( Y'' \) the imaginary part of the elastic modulus. The control of the viscosity or the imaginary part \( Y'' \) of the piezoelectric ceramic PZT is discussed in this paper. We expect that the decay of a vibration system involving the PZT rod may be controlled by combining the negative impedance circuit.

EXPERIMENTS

A simple vibration system is constructed as shown in Fig. 1. The vibration system consists of a brass disk with a mass of 84 gram and a multilayer PZT rod with a length of 4 cm and a diameter of 1 cm (NEC Tokin). The vibration is excited by a function generator and a driver at the bottom of the PZT rod. The vibration is detected by a small accelerometer put on the disk. The temporal change of the amplitude of vibration is recorded in the oscilloscope.

![Fig. 1: Experimental setup for a system of a brass disk and a multilayer PZT rod, which shows the free damping vibration.](image)

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The schematic diagram of a negative impedance circuit connected to the PZT rod is shown in Fig. 2.

**Fig. 2**: The negative impedance circuit connected to PZT rod

Piezoelectric materials (PZT) are insulators and can be modeled as capacitors with a capacitance $C_s$. One electrode of the PZT is connected to the ground and the other electrode is connected to the input of a negative impedance circuit, whose apparent capacitance is denoted as $C_c$. The circuit consists of a high gain operational amplifier (Burr-Brown OPA551) with a 20 Volt DC supply, capacitance $C_o$ and resistance $R_o$ connected in parallel and a variable resistance $R_1+R_2$. The input of the circuit is connected to the negative input terminal of the operational amplifier (op-amp), so that the output of the op-amp is returned 180° out of phase to the input. This negative feedback is the origin of the name “negative impedance circuit”, or the negative capacitance circuit.

Piezoelectricity is the interaction between mechanical properties and electrical properties, the transduction between elastic strain and electric polarization. The idea to control the mechanical strain by the electric circuit originates from the fact that the complex elastic modulus of piezoelectric materials is equivalent to the electric impedance of the circuit.

In Fig. 2, the impedance combining a capacitance $C_o$ and a resistance $R_o$ corresponds to the mechanical impedance of PZT, which consists of elasticity and viscosity. In other words, $C_o$ corresponds to $Y'$ and $R_o$ to $Y''$. The role of the variable resistance $R_1+R_2$ is to control the degree of feedback between them. As is seen in the figure, $C_s$, $C_o$, $R_1$ and $R_2$ form the four arms of a bridge surrounding the op-amp, whose two inputs are in equal potential. Defining $C_c$ as the capacitance of the whole circuit, we have the following relation

$$C_c = -(C_0 + \frac{1}{i\omega R_0}) \frac{R_2}{R_1}$$

(1)

The relation of the complex elastic modulus $Y = Y' + iY''$ of the piezoelectric material of the capacitance $C_c$ of the circuit is given in Date’s original paper [1] as follows,
\[
\frac{Y' + iY''}{Ye} = \left(1 - \frac{k^2}{1 + \frac{Cc}{Cs}}\right)^{-1}
\]

where \(Y_e\) is the elastic modulus when the circuit is shorted, and \(k\) is the electromechanical coupling coefficient, which determines the efficiency of the transduction between mechanical and electrical energies.

Combining Eqs. 1 and 2, one can see that \(Y\) is a function of \(R_1/R_2\) if other parameters are fixed. Since the capacitance of \(Cs\) is 15 \(\mu\)F, \(Co=17 \mu\)F and \(Ro = 15.8\) k\(\Omega\) are chosen and a variable resistance \(R_1+R_2=100\) k\(\Omega\) is used. The experiments have been undertaken to observe how the mechanical vibration of a system in Fig. 1 is influenced by adjusting a variable resistance.

**PRELIMINARY EXPERIMENTS**

Fig. 3: Free damping vibration of a system of a disk and a multilayer PZT rod connected to a negative impedance circuit. 1: The output of generator, 2: The amplitude of vibration, and 3: The input to the negative impedance circuit. The decay factor is \(\gamma = 274\) s\(^{-1}\) for A: the circuit disconnected, \(\gamma = 259\) s\(^{-1}\) for B at \(R_1/R_2 = 1.01\), and \(\gamma = 484\) s\(^{-1}\) for C at \(R_1/R_2 = 1.05\).

Fig. 3 shows the results of measurement when the circuit is disconnected to the PZT rod. (A-1) indicates the time change of the output voltage of the function generator. The sinusoidal vibration at the resonant frequency of 420 Hz is excited until the cutoff. (A-2) shows the time change of the amplitude of vibration detected by the accelerometer. The amplitude builds up and decays before and after the cutoff time. The smooth curve indicates the exponential decay in the form of \(\exp(-\gamma t)\)
and the decay factor $\gamma = 274 \text{ s}^{-1}$ is obtained. The voltage proportional to the initial amplitude of vibration is 10 mV. (A-3) shows that the input voltage of the circuit is zero.

Fig. 3B shows the damping vibration when the circuit is connected to PZT. (B-1) indicates the time change of the output voltage of the function generator to be the same as (A-1). (B-2) shows that the amplitude of vibration builds up and decays with time. The black line indicates that the decay is slower than that in (A-2) when the circuit is disconnected. The decay factor is $\gamma = 259 \text{ s}^{-1}$. (B-3) shows the effect of time change of the input voltage to the circuit, which is similar to the amplitude of vibration in (B-2). The measured values of $R_1 = 48.9 \Omega$ and $R_2 = 48.2 \Omega$ give $R_1/R_2 = 1.01$.

Fig. 3C shows the similar result to Fig. 3B when $R_1 = 49.5 \Omega$, $R_2 = 47.3 \Omega$ and $R_1/R_2 = 1.05$. The decay of vibration is quicker and the decay factor is $\gamma = 484 \text{ s}^{-1}$. The observations shown in Fig. 3 prove that the adjustment of the variable resistance in the circuit alters the decay behaviour of the free vibration of the system. The decay factor $\gamma$ can be either increased or decreased by adjusting the value of $R_1/R_2$.

**COMPARISON OF THEORY AND EXPERIMENT**

Based on the preliminary results, the dependence of the frequency, the decay factor and the initial acceleration in the damping vibration on the value of $R_1/R_2$ have been investigated in more detail. The experimental results are presented in Figs. 4, 5 and 6. The sensitive dependence on $R_1/R_2$ is clearly observed.

![Graph](image.png)

**Fig. 4:** Dependence of resonant frequency on $R_1/R_2$

Fig. 4 shows that the resonant frequency $f$ decreases from 439 Hz to 372 Hz with increasing $R_1/R_2$. This indicates also that the spring constant $K$, and thus the elastic constant of PZT decreases with $R_1/R_2$. 

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Fig. 5: Dependence of decay factor on $R_1/R_2$

Fig. 5 shows that the decay factor increases with increasing $R_1/R_2$. It suggests the increase of the viscosity $\eta$ of PZT.

Fig. 6: Dependence of initial acceleration on $R_1/R_2$

Fig. 6 shows the initial amplitude of acceleration versus $R_1/R_2$. The initial acceleration is proportional to the mechanical energy input to the vibration system. Its decrease suggests the loss of the mechanical energy introduced to the system owing to viscosity. Close correspondence is seen between the increase of $\gamma$ and the decrease of the input mechanical energy.

The simple equation for the forced vibration of the system of a mass and a viscoelastic body is given by

$$m \frac{d^2x}{dt^2} + \eta \frac{dx}{dt} + Kx = F$$

(3)
where $F$ is force, $x$ displacement, $m$ mass, $\eta$ viscosity, and $K$ elastic constant. The general solution of $x$ for the free vibration at $F = 0$ is given by

$$x = x_0 \exp(-\gamma t) \cos(\omega t + \delta)$$

(4)

where $\gamma$ is the decay factor and is given by

$$\gamma = \frac{\eta}{2m}$$

(5)

and $\omega$ is the resonant angular frequency. The frequency $f$ is given by

$$f = \frac{1}{2\pi} \left( \frac{K}{m} \right)^{1/2}$$

(6)

For the forced vibration of $x = x_0 \exp(-\omega t)$, we have

$$x (-\omega^2 m + K + i \omega \eta) = F$$

(7)

It is seen that $K + i \omega \eta$ is proportional to $Y = Y' + i Y''$. If the proportionality coefficient is assumed to be 1, which is determined only by dimensions, we have

$$Y' = K$$

(8)

$$Y'' = \omega \eta$$

(9)

If the proportionality coefficients, $L_1$, $L_2$, and $L_3$ are used, the frequency $f$ is given from Eq. (6) as

$$f = (L_1 Y')^{1/2}$$

(10)

The decay factor $\gamma$ is given from Eq. (5) as

$$\gamma = L_2 Y''$$

(11)

The initial acceleration at resonance $A = \omega^2 x_0$ is given from Eq. (7) as

$$A = L_3 / Y''$$

(12)

The calculation of $f$, $\gamma$, and $A$ are performed by means of Mathcad program, combining Eqs. (1), (2), (10), (11), and (12). The actual values assumed for fixed parameters are as follows: $k = 0.2$, $C_s = (15 - 0.5) \mu F$, $C_0 = 17 \mu F$, $R_0 = 15.8 k \Omega$, $f = 440Hz$, $Y_e = (1 + 0.1i) \cdot 1000$. The variable is $x = R_1 / R_2$. The values of $L_1$, $L_2$, and $L_3$ are determined by the data at $x = 0$.

The calculated curves fit qualitatively well to the observed data as shown in Figs. 4, 5 and 6. The agreement may suggest the validity of Eqs. (10), (11) and (12).
SUMMARY

The mechanical damping characteristics of a system consisting of a metal disk and a jointed multilayer PZT rod connected to a negative impedance circuit were studied. When the excitation was cut off, the system underwent the free damped vibration. The wave forms were recorded by an oscilloscope. The frequency, the initial acceleration and the damping factor of vibration were measured, while the ratio of resistances $R_1/R_2$ in the circuit was varied over a wide range.

In a preliminary experiment, the damping factor $\gamma$ was 274 s$^{-1}$ without circuit control, but 258 s$^{-1}$ and 484 s$^{-1}$ with circuit control, depending on the value of $R_1/R_2$. The value of $\gamma$ can be either increased or decreased by the control of $R_1/R_2$ in the circuit.

The theoretical calculation represented well the experimental data for the frequency $f$, the decay factor $\gamma$, and the initial acceleration $A$ measured at various values of $R_1/R_2$. It concludes that $f$ is proportional to the root of $Y'$, $\gamma$ to $Y''$, and $A$ to $1/Y''$.

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