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NATURAL CONVECTION HEAT TRANSFER MODELLING OF SHAPE MEMORY ALLOY WIRE

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ABSTRACT

Shape memory alloy (SMA) wires are becoming increasingly popular for automotive applications. One of the difficulties in control of electrically-heated SMAs is monitoring the temperature and hence, actuation to avoid damage due to overheating. First-order convective heating models are typically used, but there is often significant uncertainty in the heat transfer coefficient due to lack of existing correlations for thin cylinders, where curvature effects are significant. In this paper, a natural convection heat transfer correlation is developed for NiTi SMA wire. The correlation may be used in thermal models in order to predict the temperature of a current carrying SMA wire without using direct temperature measurement methods. The results reported in this paper are based on experiments for a 0.5mm diameter SMA wire with a Rayleigh number range: $2.6\times 10^{-8} \le Ra_D \le 6\times 10^{-1}$. A pressure variation method was used to obtain a range of Rayleigh numbers for a heated SMA wire. The ambient pressure was controlled within a vacuum chamber, from 1 atm to 2×10^{-4} atm (0.1 MPa to 2×10^{-5} MPa). Data were collected for the wire at various angles between horizontal to vertical at each set pressure. The new correlation can be used to determine the convective heat transfer coefficient of an SMA wire of known diameter and inclination angle. The convection coefficient (h) is determined using the correlation along with the Prandtl number (Pr), air dynamic viscosity (μ), air compressibility factor (Z), air thermal conductivity (k), and gas constant (R).The wire temperature can then be determined by substituting this coefficient into the convective heat transfer equation.

Keywords: Shape Memory Alloy, Convection Coefficient, Natural Convection Heat Transfer, Heat Transfer Correlation, SMA Thermal Modelling

NOMENCLATURE

A,C,n,	equation constants ;
D,	wire diameter [m] ;
g,	acceleration due to gravity $[ms^{-2}]$;
Gr_D	Grashof number ;
h,	convective heat transfer coefficient $[Wm^{-2}K^{-1}]$;
I_{SMA},	SMA wire current [A] ;
k,	air thermal conductivity $[Wm^{-1}K^{-1}]$;
L, l,	wire length [m] ;
Nu_D,	Nusselt number = hDk^{-1};
P,	pressure [Pa] ;

Pr,	Prandtl number ;
Qconv,	convection heat transfer rate [W];
Qrad,	radiation heat transfer rate [W];
R,	gas constant $[kJ\,kg^{-1}K^{-1}]$;
R_{SMA},	SMA wire resistance [Ω] ;
Ra_D,	Rayleigh number = $[g\beta(T_{SMA} - T_\infty)D^3v^{-2}.Pr]$;
T_{SMA},	wire surface temperature [K] ;
T_∞,	ambient temperature [K] ;
V_{SMA},	SMA wire voltage [V];
Z,	compressibility factor;

Greek Symbols

ε, emissivity ;
μ, dynamic viscosity $[kg\,m^{-1}s^{-1}]$;
σ Stefan-Boltzmann constant = $5.67 \times 10^{-8}[W\,m^{-2}K^{-4}]$;
φ, wire inclination angle [rad.] ;

INTRODUCTION

Shape memory alloys (SMAs) have attracted attention in recent years because of their ability to undergo a phase transformation near room temperature. A shape memory alloy that is deformed at a low temperature (martensite phase) can return to its original shape when heated to a higher temperature (austenite phase). The change in phase between martensite and austenite occurs without diffusion with only the crystal structure of the material changing during this process. Additionally, the phase transformation occurs with strain recovery associated with a large force and the ability to perform mechanical work that gives the alloy the potential to be used as an actuator. In order to use the SMA wires as an actuator a reliable heat transfer model is important to protect the wire from overheating and control the time required to generate a shape recovery motion. The SMAs are usually heated via a joule effect by passing an electric current through the wire, while cooling to ambient temperature is achieved by natural or forced convection. There are no specific types of heat transfer correlations for SMA wires readily available in the literature and therefore, heat transfer correlations for cylinders or wires are used for the SMA wires heat transfer modeling.

A natural convection heat transfer correlation for long horizontal cylinders was developed by Churchill and Chu [1] in 1974 for $10^{-5} < Ra < 10^{12}$ as

$$Nu = \left(0.6 + \frac{0.387\,Ra^{1/6}}{[1+(0.599/Pr)^{9/16}]^{8/27}}\right)^2 \tag{1}$$

Morgan [2] introduced a power form of natural convection heat transfer correlations for long horizontal cylinders as

$$Nu = A(Ra)^m \tag{2}$$

where

$A = 0.675$ $m = 0.058$ for $10^{-10} < Ra < 10^{-2}$

$A = 1.02$ $m = 0.148$ for $10^{-2} < Ra < 10^{2}$

$A = 0.85$ $m = 0.188$ for $10^{2} < Ra < 10^{4}$

Fujii et al. [3] introduced a natural convection heat transfer correlation for long horizontal thin platinum wire for $10^{-8} < Ra_D < 10^{6}$ as

$$\frac{2}{Nu} = \ln\left(1 + \frac{3.3}{C(Pr)Ra_D^n}\right) \tag{3}$$

where

$$C(Pr) = 0.671 \Big/ \left\{1 + \left(0.492/_{Pr}\right)^{9/16}\right\}^{4/9}$$

$$n = 0.25 + 1/(10 + 5Ra_D^{0.175})$$

For vertical cylinders Fujii et al. [4] introduce a correlation as

$$Nu_D = C\,(Pr)(Gr_D \Pr D/1)^{1/4} + 0.763C(Pr)^{1/6}(Gr_D \Pr\ D)/1)^{1/24} \tag{4}$$

for $C\,(Pr)(Gr_D \Pr D/1)^{1/4} > 2 \times 10^{-3}$

Nagendra et al. [5] introduced the following equation for vertical wires

$$Nu = 0.87(Ra\frac{D}{L})^{0.05} \tag{5}$$

where D/L is the ratio of wire diameter to its length.

In this paper a natural convection heat transfer correlation in the range of $2.6 \times 10^{-8} \leq Ra_D \leq 6 \times 10^{-1}$ is introduced for SMA wires at $Pr = 0.7$ and wire inclination angles from horizontal to vertical.

EXPERIMENTS

Apparatus

All the experiments were carried out in an NRC 3117 vacuum chamber composed of an 18” dia × 30” tall Pyrex bell jar. The system allows control of the ambient pressure from 1 atm (760 torr) down to 10^{-7}torr with using mechanical and diffusion pumps. A Keithley Model 2700 data acquisition system was used to collect the data, and then the data were recorded using a computer running an ExceLINX program. The SMA wire was heated with a BK PRECISION 1760A power supplier, its current was determined using a 0.05 Ω shunt resistor. The SMA wire orientation angle was controlled by connecting one end of the wire test fixture to another plate with a hinge and a rod is connected to the both plates. The top plate can be inclined and fixed at any angle from horizontal to vertical. The rod is marked at intervals so that the devices fixed angles can be measured to show its orientation angle (Fig.1).

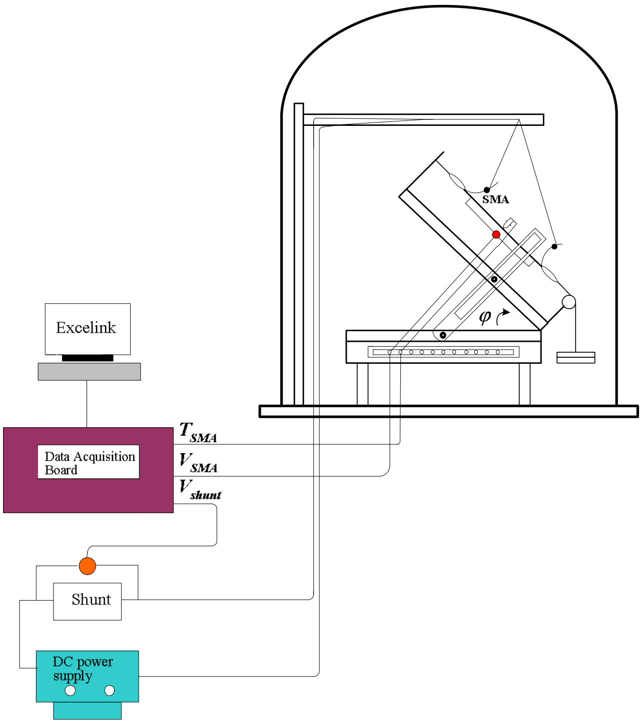

Fig.1: Experiment setup

Experimental Procedure

A Dynalloy nickel-titanium (Ni-Ti) SMA wire of 500 mm total length and 0.5 mm diameter with A_s = 90 ° C was sanded with a fine sandpaper sheet to achieve a better electrical contact. An E type, 40AWG thermocouple was spot welded to the center of the SMA wire for temperature readings. Two constantan 40 AWG thermocouple wires were spot welded 2.5 cm in from both ends of the screw clamps for the wire's voltage measurements. The wire was connected to the power supply circuit. The power supply circuit, the shunt resistor and the SMA wire were connected as shown in (Fig.1). The wire was heated by using the DC power supply via the direct current or Joule effect method. The Q_{rad} and Q_{conv} data were collected for the wire under a 100 MPa load and with a stress-free condition.

In order to develop a heat transfer correlation for the wire, the natural convective heat transfer coefficient was measured over a wide range of Rayleigh numbers by changing the air pressure using the vacuum station. At different pressures (700 torr, 600 torr, 500torr, 400 torr, 300 torr, 200 torr, 100 torr, 60 torr, 30 torr, 5torr and 0.1 torr) the SMA wire at horizontal orientation was heated from room temperature by applying current in 200 mA increments, starting from 60 mA, until the temperature of the wire reached 100$^\circ C$ as determined by the thermocouple. The current was then discontinued and the wire cooled by natural convection by coming into equilibrium with the ambient medium. A pause after each incremental change allowed the temperature to reach a stable condition (i.e. when the temperature readout on ExceLINX did not increase more than 1% in 10 successive temperature readings). The voltage drop across the wire was measured using the Keithley

2700 Data Acquisition system when the wire reached a steady state at 100°C. At each set pressure, the data for Q_{conv} were collected for the wire at 0°, 15°, 30°, 45°, 60°, 75° and 90 ° inclination angles.

Q_{conv} for the SMA wire can be calculated when the wire is heated by passing current through it. Heat transfer occurs between the heated wire and its surrounding, by convection, conduction and radiation. When the wire reaches a steady state condition with its surrounding, the power input to the wire (VI) is equal to the convection, conduction and radiation heat transfer rate. The conduction heat transfer rate can be neglected for the wire; therefore, for the SMA wire at steady conditions Q_{conv} at each set pressure will be equal to the power input to the wire minus Q_{rad}

$$Q_{conv} = V_{SMA} I_{SMA} - Q_{rad} \tag{6}$$

For measuring Q_{rad} the air pressure inside the vacuum jar was reduced to 10^{-7} torr. The SMA wire was heated up at horizontal orientation by applying the current step by step (60 mA, 100 mA, 200 mA and 250 mA) until the temperature of the SMA wire reached 100°C. The current was then shut down, and the wire was cooled. A pause after each incremental change allowed the temperature to reach a stable condition (when the temperature readout on ExceLINX does not increase more than 1% in 10 temperature readings). The voltage of the wire was measured using the Keithley 2700 Data Acquisition system when the wire had reached a steady state at 100°C. At 10^{-7} torr, it can be assumed that the power input to the wire will be only equal to the radiation heat transfer rate.

RESULTS AND DISCUSSION

Development of Convective Heat Transfer Correlation for the SMA Wire

An empirical correlation is developed for the SMA wires in the power form as:

$$Nu_D = A + C(Ra_D)^n \tag{7}$$

The values for the constants $A, C, and\ n$ for various angles are shown in Tables 1 and 2. The empirical correlation for the SMA wire under a 100 MPa stress in the range of $2.6 \times 10^{-8} \leq Ra_D \leq 6 \times 10^{-1}$ is

$$Nu_D = (-0.03\varphi + 0.16) + (-0.1\varphi^2 + 0.03\varphi + 0.81)Ra_D^{(-0.03\varphi + 0.16)} \tag{8}$$

For a wire under stress-free conditions in the range of $2.6 \times 10^{-8} \leq Ra_D \leq 6 \times 10^{-1}$, the Nusselt number can be shown as

$$Nu_D = (-0.04\varphi + 0.19) + (-0.09\varphi^2 + 0.003\varphi + 0.82)Ra_D^{(-0.03\varphi + 0.17)} \tag{9}$$

where φ is the inclination angle of the SMA wire in radians from 0 to 1.57 radians (0 to 90°).

For a horizontal SMA wire under a 100 MPa applied stress the correlation has the form

$$Nu_D = 0.81(Ra_D)^{0.16} + 0.16 \tag{10}$$

and for a vertical SMA wire under 100 MPa applied load

$$Nu_D = 0.61(Ra_D)^{0.11} + 0.11 \tag{11}$$

Table 1: Constants for the wire under 100 MPa applied load in range of $2.6 \times 10^{-8} \leq Ra_D \leq 6 \times 10^{-1}$

φ *Rad. (deg.)*	**C**	***A = n***
0	**0.810**	**0.160**
0.26 (15°)	**0.811**	**0.152**
0.52 (30°)	**0.798**	**0.144**
0.79 (45°)	**0.772**	**0.136**
1.05 (60°)	**0.732**	**0.129**
1.31 (75°)	**0.678**	**0.120**
1.57 (90°)	**0.610**	**0.113**

Table 2: Constants for the wire under stress-free conditions in the range of $2.6 \times 10^{-8} \leq Ra_D \leq 6 \times 10^{-1}$

φ *Rad. (deg.)*	***A***	***C***	***n***
0	**0.190**	**0.820**	**0.170**
0.26 (15°)	**0.180**	**0.815**	**0.162**
0.52 (30°)	**0.169**	**0.797**	**0.154**
0.79 (45°)	**0.159**	**0.767**	**0.146**
1.05 (60°)	**0.148**	**0.724**	**0.139**
1.31 (75°)	**0.138**	**0.670**	**0.131**
1.57(90°)	**0.127**	**0.603**	**0.123**

As shown in Tables 1 and 2 the constant values of A, C, and n decreases with increasing the wire inclination angle. The relation between Nu_D and Ra_D is shown in Figures (2) and (3) for the wire in different angles based on constant values calculated in Table 1 and 2.

The applied load to the SMA wire does not have a significant impact on the natural convection heat transfer correlation for the wire. As shown in Figures 2 and 3 there is less than a 4% difference between the Nu values for the wire under 100MPa stress and the stress-free condition.

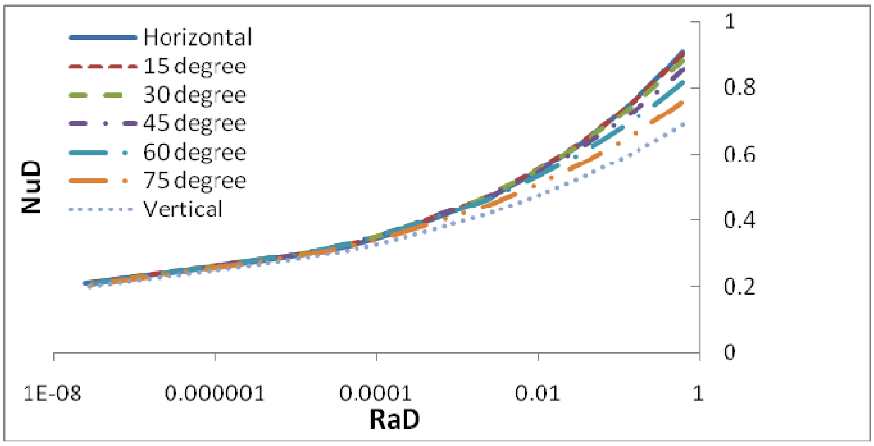

Fig.2: Relation between Nu_D and Ra_D for the wire at various angles under 100MPa applied load.

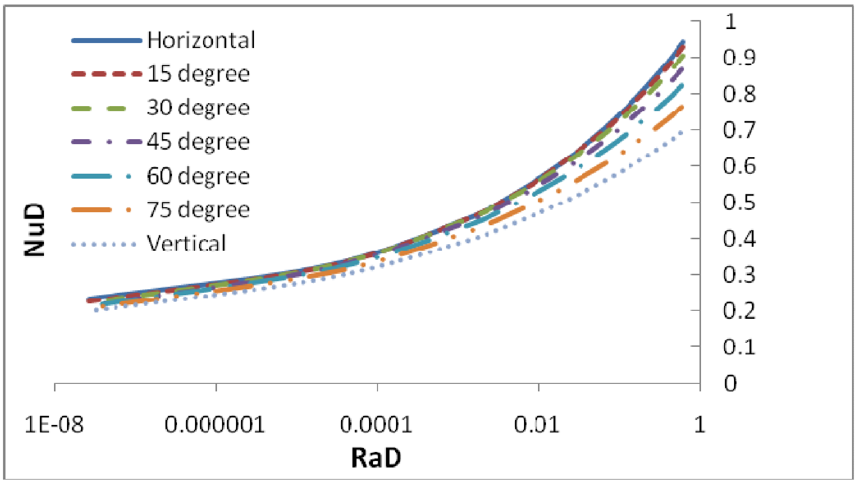

Fig.3: Relation between Nu_D and Ra_D for the wire at various angles under stress-free conditions.

At higher Rayleigh values the Nu_D decreases around 25 % by changing the wire position from horizontal to vertical. At lower Rayleigh values the Nu_D values decrease around 5% by changing the wire position from vertical to horizontal.

The experimental values for Nu_D are plotted against wire inclination angle in Figures 4 and 5. The Nu_D values decrease by increasing the wire inclination angle but as the Rayleigh number decreases and reaches to 2.5×10^{-8} the Nu_D values approach a straight line and does not change significantly by changing the wire inclination angle.

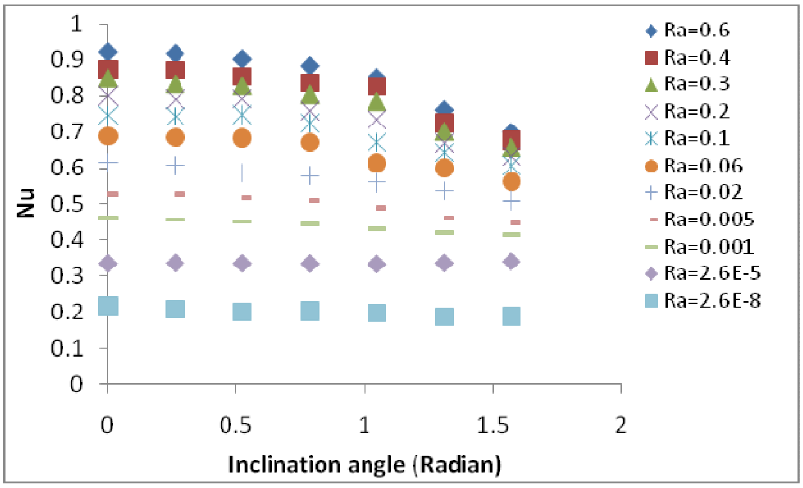

Fig.4: Effect of inclination angle on Nu_D values for the SMA wire under 100 MPa applied load.

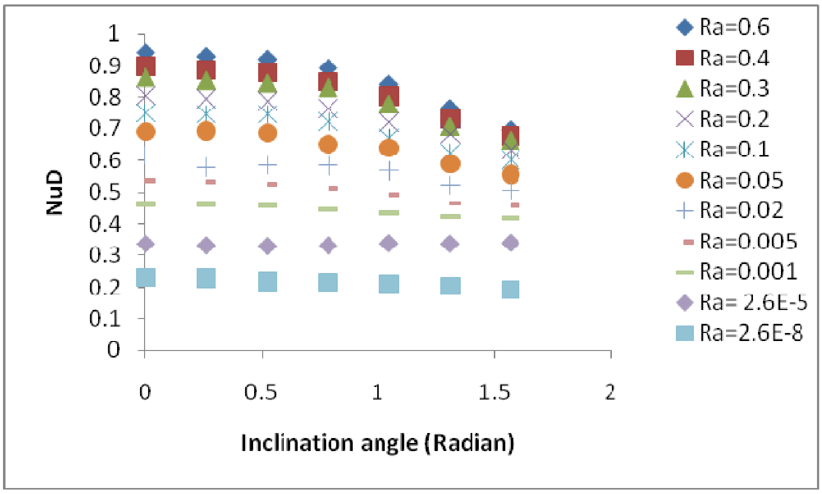

Fig.5: Effect of inclination angle on Nu_D values for the SMA wire under stress free conditions.

As shown in Figures 6 and 7, the new correlation has the same trend as other correlations and is positioned in the middle among the existing correlations: lower than Fujii and Morgan and higher than Churchill and Chu. For the horizontal wire, the new correlation agrees with Fujii et al. correlation for platinum wire and Morgan for long horizontal cylinders this is especially true in the region of $10^{-3} \leq Ra_D \leq 6 \times 10^{-1}$. At higher Ra_D range the correlation is less than 5% lower than Fujii and Morgan and at lower Ra_D range the difference is less than 17%. For the wire in the vertical position, the correlation is lower than Muller and higher than Fujii et al. and Nagendra. The correlation is less than 20% lower than Muller and less than 15% higher than Nagendra.

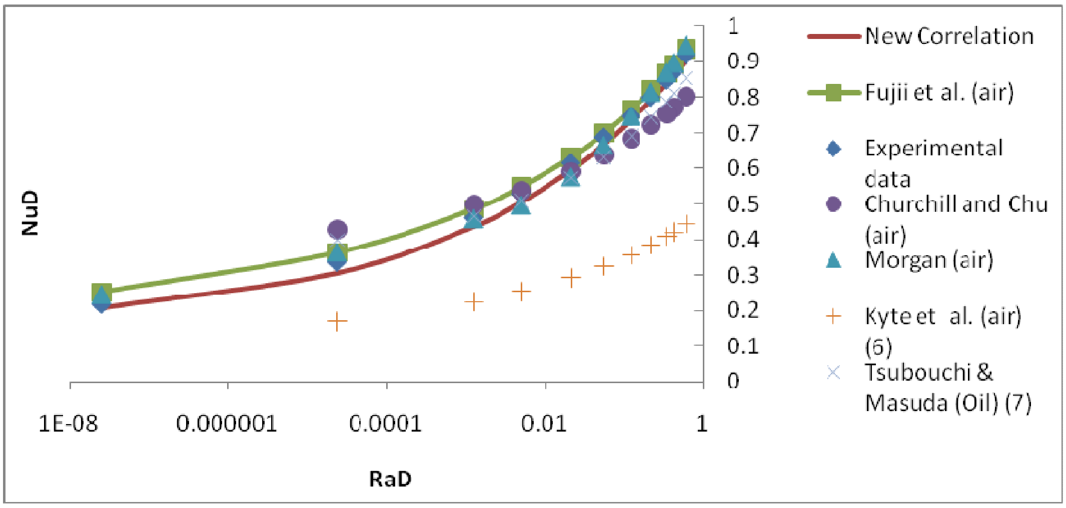

Fig.6: Comparison of present data for the SMA wire at horizontal position under 100MPa with existing natural convection heat transfer correlations.

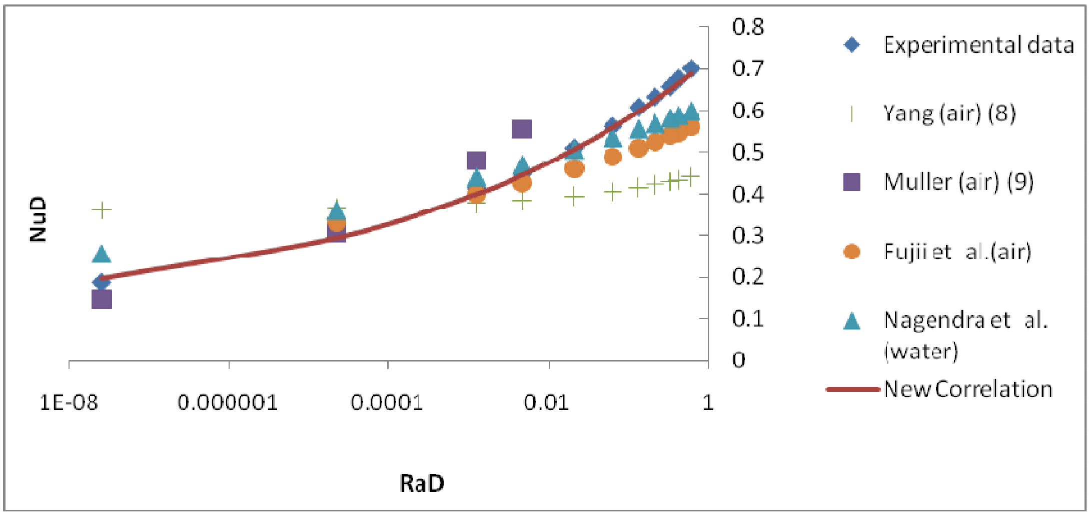

Fig.7: Comparison of present data for the SMA wire at vertical position under 100 MPa with existing natural convection heat transfer correlations.

Temperature Prediction of SMA Wire Using New Correlation

The standard convection and radiation heat transfer equation can be applied to a heated SMA wire when it reaches a steady state condition as

$$I^2R_{SMA} = hA_s(T - T_\infty) + \varepsilon\sigma A_s(T^4 - T_\infty^4) \tag{12}$$

The heat transfer coefficient based on equation (7) for a SMA wire can be calculated as

$$h = \frac{k}{D}\,(A + C\,(Ra_D)^n) \tag{13}$$

For example when a 0.5 mm diameter current carrying SMA wire under a 100 MPa stress in 1 atm air pressure (760 torr) is inclined from horizontal at 15°, then equation (7) becomes

$$Nu_D = 0.152 + 0.811(\mathrm{Ra_D})^{0.152} \tag{14}$$

and equation (13) for this wire becomes

$$h = \frac{k}{D}\,(\,0.811(\frac{g(T_{SMA} - T_\infty)D^3P^2}{\mu^2T_\infty R^2Z^2(\frac{T_{SMA}+T_\infty}{2})^2}.\mathrm{Pr}\,)^{0.152} + 0.152) \tag{15}$$

Substituting equation (15) into (12) the heat transfer equation of the SMA wire becomes:

$$I^2R_{SMA} = \frac{k}{D}\,(0.811\,(\frac{g(T_{SMA} - T_\infty)D^3P^2}{\mu^2T_\infty R^2Z^2\left(\frac{T_{SMA}+T_\infty}{2}\right)^2}\,.Pr)^{0.152} + 0.152) \times \\ (\pi Dl(T_{SMA} - T_\infty) + \varepsilon\sigma\pi Dl(T_{SMA}^4 - T_\infty^4)) \tag{16}$$

by substituting the constant values of $g = 9.8\ m/s^2, R = 0.287\ kJ/(kg.K)$, $Z = 1$, $\varepsilon = 0.63$ and $\sigma = 5.67 \times 10^{-8}\ W/m^2K^4$ in to the equation (16) the temperature of the current carrying SMA wire can be predicted.

CONCLUSIONS

Experiments were carried out to investigate natural convection heat transfer from a SMA wire of 0.5 mm in diameter and 500 mm in length to air. A decrease in Nu values was observed by increasing the wire inclination angle at higher Rayleigh ranges but as the Rayleigh values approach to 2.6 $\times 10^{-8}$ no significant change in the Nu values was observed which means at lower Rayleigh ranges Nu values are independent of wire inclination angle.

A new heat transfer correlation for the SMA wires based on experimental results is proposed which agrees with existing correlations for the range of $2.6 \times 10^{-8} \leq Ra_D \leq 6 \times 10^{-1}$ and $Pr = 0.7$. The new correlation is based on the inclination angle of the SMA wire and can be used for wires with inclination angle range of $0^{\circ} \leq \varphi \leq 90^{\circ}$.

The correlation was developed for the wire under both stress-free and 100MPa conditions and no significant difference was observed. The new correlation can be used to predict the temperature of current carrying SMA wires for a given wire diameter and inclination angle.

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