Numerical and experimental study of the influence of multiple scattering on surface waves dispersion curves

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Abstract
Seismic surface wave dispersion analysis allows the non-invasive diagnosis and evaluation of residual life of structures and to retrieve their mechanical intrinsic properties, which are all important issues for civil engineering. For heterogeneous medium, like concrete, multiple scattering theories allows to retrieve the effective dispersion curve of the media but under the assumption of a infinite space. Consequently, these theories do not take into account the various wall and boundary effects. Laboratory experiments on analog small-scale models can be efficient but are limited by the production of large number of random heterogeneous models. Here, we demonstrate that a two-dimensional spectral element method (SEM) is able to reproduce laboratory experiment results and can be used to produce efficient dispersion curves, from 50 random heterogeneous models, in a good agreement with analytic homogenization results.

Keywords: heterogeneous media, surface waves, multiple scattering, forward problem, physical modeling

1. Introduction
The diagnosis of structures and the evaluation of their residual life is an important issue for civil engineering, [1, 2]. Surface waves offer an effective mean for retrieving in a non invasive manner mechanical intrinsic properties of material as a function of depth, [3, 4]. In most cases which deal with cover-concrete non destructive evaluation, the assumption of homogeneous layers can be debatable when the aggregates represent approximately 50% in weight and have sizes comparable to the expected depth resolution [5, 6]. Indeed, multiple scattering has an impact on the surface wave effective phase velocity dispersion curve when the incident wavelengths are similar to the radius of inclusions [7]. Multiple scattering theories, such as ISA model, [8], describes the propagation of coherent bulk waves propagating in infinite heterogeneous medium. ISA is known to be theoretically available for a dilute medium and weak mechanical contrast between the inclusions and the matrix. In a semi-infinite medium, two phenomena affect the dispersion curve : the wall and the boundaries effects, [9, 10].
In order to validate the assumption that Rayleigh wave velocities can be computed with the effective compression and shear waves velocities, an experimental and numerical approach have been developed. We study the elastic wave propagation in heterogeneous small-scale models made of epoxy-resin containing a fix amount of randomly distributed circular inclusions having a constant radius. First, a laboratory experiment result on a unique small-scale model is compared with a numerical equivalent generated by a code based on the Spectral Element Method [11]. Then, given the difficulty to produce dozens of random physical small-scale models to reach an effective field, the SEM method is used for this purpose and the resulting effective phase velocity is compared with the predictions obtained with the ISA model.

2. Small-scale modeling

The laboratory experiment is realized on the MUSC laboratory (Non-contact Ultrasonic Measurement, Mesures Utrasonores Sans Contact in French) at IFSTTAR in Nantes, France [12, 13]. The model used in this experiment is designed to be a simplified analog of concrete: an homogeneous matrix with a random distribution circular heterogeneities. The model of size 300×150×80 mm, invariant along the $y$-axis, is made of a charged epoxy-resin, named F50240% for the matrix and 12 %, in terms of surface concentration, of aluminum cylinders of 5.0 mm diameter for inclusions. Contrary to some other small-scale experiments [14, 15], each face of the model is in contact with air and acts, consequently, as a free surface. The estimated properties of epoxy-resin are: $V_P = 2968 \text{ m.s}^{-1}$, $V_S = 1494 \text{ m.s}^{-1}$ and $\rho = 1822 \text{ kg.m}^{-3}$, and properties of inclusions are: $V_P = 5630 \text{ m.s}^{-1}$, $V_S = 3225 \text{ m.s}^{-1}$ and $\rho = 2700 \text{ kg.m}^{-3}$.

![Random heterogeneous model and the piezoelectric transducer ACSYS®.](image)

Here, we consider an acquisition geometry for which the source is on the top-left corner of the model (0 mm distance) and 600 receiver positions with spacing of 0.5 mm from left to right corner. Source and receivers are on the same vertical plan parallel with the $x$-axis and centered on the $y$-axis. The source is a Ricker with a central frequency $f_0$ of 120 kHz and is injected in the model using a piezoelectric transducer, ACSYS®, in the normal direction of
the top free surface. The receiver is a laser interferometer, BOSSA NOVA®, which record
the vertical particular displacement on the surface at each receiver position with a precision
of few nanometers. The source and the laser interferometer are controlled by two automated
arms which allow a positioning with a precision of 10 μm for both source and receivers po-

tions.

3. Numerical modeling

Modeling the wave propagation needs the use of a numerical method fixed by the studied
medium, the attempted accuracy of the results and the numerical cost [16]. In the scope of
this study, the numerical method must be able to generate surface waves data in a media
with circular interfaces with a great accuracy and a low numerical dispersion.

The most widely used is the Finite-Differences (FD) method [17, 18, 19] which estimates
each derivative on a regular Cartesian grid using a Taylor development [20] of order n. FD is
simple to implement but the Cartesian grid defined by the minimum propagated wavelength
(λmin) in the full media is unable to reproduce properly complex interfaces such as circular
inclusions. Moreover, 60 points by λmin are needed to model Rayleigh wave in order n = 2
[21], where only 15 points are required for body waves [22], which increases drastically the
computational cost in case of near-surface modeling experiment.

The Finite-Elements Method (FEM) is another popular method used for wave propagation
modeling [23, 24]. FEM is based on a variational formulation of the equation of motion and
gives a continuous approximate solution in space using polynomial basis functions defined
on each node of each cell of the mesh. The natural boundary conditions of FEM is the
free surface and the triangular unstructured meshes (in two dimension) are well adapted
to complex media. However, low polynomial basis are inadequate to obtain accurate and
non-dispersive solution: cell sizes must be at least 1/20 of the dominant wavelength for het-
erogeneous soils to reduce numerical dispersion [24], which represents, again, an important
computational cost even in the two-dimensional approximation.

The Spectral Element Method [25, 11, 26, 27] is based on a high-order polynomial approx-
imation of the weak formulation of the wave equation and inherits the flexibility and the
natural free surface condition of the FEM [28]. The high-order formulation allows to obtain
an accurate and non-dispersive solution while its explicit scheme in time facilitate parallel-
ization and reduce the computational cost. The mesh is composed of quadrangles which
can be strongly distorted [29] and allows accurate spatial representations of complex medias
[30]. In polynomial order n = 4, the cell size should be between λmin/2 and λmin which is a
coarse mesh compared to FEM.

Given the invariance of the model in the y – axis and the need to generate accurate surface
waves propagation results, we choose a two-dimensional elastic SEM. Figure 2 shows a zoom
on a small-scale model meshed using the GMSH open-source software [31].
4. Comparison between experimental and numerical data

Resulting seismograms for laboratory and numerical experiments are shown in figures 3-a and 3-b, respectively. For visualization purpose, each trace of seismograms is normalized by its maximal amplitude. Both seismograms show clearly P-wave and first surface wave arrivals. Following seismic noises, composed of reflections and diffractions on inclusions and free surfaces appear different. This can be explained by the absence of the quality factor in numerical modeling and the echo of the experimental source [13]. However, this should not have a critical impact on the following results.

Experimental and numerical seismograms are multiplied, around the apparent velocity of the surface waves, by a window which is the best compromise between the resolution in frequency (that requires a large window) and the localization in time (that calls for a sharp
window), defined as fellow:

\[
f(t) = \begin{cases} 
    t_0 \frac{1}{2} \sin \left( \frac{\pi}{2} \cdot \frac{\pi + \omega_0 + t}{2\pi - \omega_0 t} \right), & \text{if } \frac{-\pi}{\omega_0} < t < \frac{\pi}{\omega_0} - t_0 \\
    1, & \text{if } \frac{\pi}{\omega_0} - t_0 < t < \frac{-\pi}{\omega_0} + t_0 \\
    t_0 \frac{1}{2} \sin \left( \frac{\pi}{2} \cdot \frac{\pi + \omega_0 - t}{2\pi - \omega_0 t} \right), & \text{if } \frac{\pi}{\omega_0} + t_0 < t < \frac{\pi}{\omega_0} \\
    0, & \text{otherwise.}
\end{cases}
\]

(1)

in which \( \omega_0 \) and \( t_0 \) must be chosen so that \( \pi < \omega_0 t_0 < 2\pi \). Then, the phase velocity dispersion curve \( V_v(f) \) is computed using the Slant-Stack procedure [32]. The maximum of the transform in the \((V - f)\) plan yields to the phase velocity of the surface waves and give the so called dispersion curve. Resulting dispersion curves, for laboratory (red line) and numerical (blue line) data, in terms of frequency and wavelength are presented in figure 4-a and 4-b respectively. The error of the dispersion curve (red dots) is calculated for the laboratory experiment result according to [33]. The variations along dispersion curves are essentially related to wave reflections and diffractions on inclusions which will add the two-dimensional modeling theoretical approximations and the fact that the quality factor is neglected. However, the numerical dispersion curves follow the same trend than the experimental ones and stay mainly in the experimental dispersion curve error area.

Thus, we can consider the use SEM to produce data for a large number of random heterogeneous models in order to estimate the dispersion curves for an effective field.

Figure 4: Comparison between dispersion curves from numerical (blue curve) and experimental modeling (red curve) in terms of (a) frequency and (b) wavelength.

5. Effective medium

The effective field, obtained after averaging the field in all the disordered configurations, corresponds to that of waves propagating in an effective homogeneous medium. For numerical
modeling, we limit the number of simulation, $N_{\text{simu}}$, to 50 which is enough for the surface concentration used [7].

The time domain numerical simulations are performed in a bounded computational domain $[x_1, x_2] \times [z_1, z_2] = [0,300] \times [0,80]$ mm. With a surface concentration at 12%, $N = 147$ scatterers, with a constant radius $a = 2.5$ mm, are uniformly distributed in the subdomain $D = [x_1 + \frac{1}{4}a, x_2 - \frac{1}{4}a] \times [z_1 + \frac{1}{4}a, z_2 - \frac{1}{4}a]$. The distribution of the centers of circles $S_k = \{C_i, \ i = 1, \ldots, N\}$ required minimum exclusion distance $2a + \xi$. $\xi = \frac{1}{4} \times a$ increases with the mesh size. Several algorithms to simulate the $C_i$ may be found in the literature [34]. Here we propose one algorithm.

**Algorithm :**

- for $k = 1$ to $N_{\text{simu}}$
  - choose randomly $C_1$ in $D$.
  - for $i = 2$ to $N$
    * choose $C_i$ in $D$.
    * if $C_i C_j \geq 2a + \xi$ ($j = 1, \ldots, i - 1$) then $C_j$ is kept.
    * otherwise choose another $C_i$.
  - if $k = 0$, $S_k$ is kept.
  - if the correlation $\text{coor}(S_k, S_l) \leq 0.15$, ($l = 1, \ldots, k - 1$) then the coordinates vector, $S_k = \{C_i\}$ with $i = 1, \ldots, N$, is kept.
  - otherwise choose another coordinates vector $S_k$.

Figure 5 represents the mean of the 50 random distributions of inclusions and the profile of concentration with the depth. The variation of concentration from 0% to 13% with a pic at 14.5% reveals the presence of a wall effect around the boundaries of the domain. This wall effect has been studied by [10] and depends on the concentration and the maximum size of inclusions. Nevertheless, the variations of the concentration over the depth after 7 mm is less than 1%. The mean distribution of inclusions gives a good homogenization.

The effective surface waves velocity is obtained after averaging the numerical seismograms computed from the 50 random distributions of inclusions (figure 6). It is interesting to compare the results of the effective surface waves velocity computed numerically with an analytical homogenization such as the Independent Scattering Approximation (ISA), [8] (figure 7). Even though the validation limits of multiple scattering theories models are generally not well defined, ISA is known to be theoretically available for a infinite dilute medium and weak mechanical contrast between the inclusions and the matrix. The study [7] demonstrates that ISA remains valid up to a 12% inclusion concentration for concrete properties. ISA homogenization is computed with the estimated mechanical properties of epoxy-resin and the mechanical properties of aluminum used for the numerical calculations. Figures 7-a and 7-b shows the comparisons of the resulting dispersion curves for numerical and ISA homogenization in terms of frequency and wavelength, respectively. These
results show good agreements, at first order, between numerical and analytic results for the wavelengths greater than 15 mm. For the wavelengths lesser than 15 mm, the numerical phase velocity are 20 m/s weaker. The differences can be explained by the influence of the wall effect, with a thickness of h=7 mm, on the dispersion curve for the wavelength which approximately are lesser than 2 thickness of the wall effect. As described in the previous section, two-dimensional approximation and a neglected quality factor, is a complementary explanation.

6. Conclusions

Produce an effective field from laboratory experiments is very difficult given the required number of random small-scale model. In this scope, it is necessary to use numerical methods, as SEM, which allow to generate precise data with a low numerical dispersion and a reasonable computational cost.

Thus, we have demonstrate that 2D SEM, even without quality factor, is able to produce
dispersion curve comparable to laboratory experiments. More, 2D SEM for 50 random heterogeneous models gives an effective dispersion curve in agreement with an analytical solution designed for infinite medium.

However, it is necessary to go further by considering, for examples: (1) surface concentration of inclusions up to 60%, (2) a matrix with a gradient of physical properties, and (3) an analytic forward modeling code able to take into account a finite medium.

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References