Characterizing the Complex Modulus and Poisson’s Ratio of Asphalt Concrete Specimens through Modal Testing

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Abstract

The viscoelastic complex modulus and complex Poisson’s ratio of asphalt concrete is fundamental to modern pavement design due to the time and temperature dependency of bituminous materials. Conventional cyclic loading to measure these material properties at strain levels of about 50 micro strains are expensive and complicated to perform. Low strain (~0.1 micro strains) stress wave measurements are faster and simpler to perform and open up the possibility for future non-destructive field quality control of pavements. This non-destructive modal testing is performed by measuring frequency response functions of specimens using an impact hammer and an accelerometer. A comparison between the two methods show that the modal test systematically give a slightly higher absolute value of the complex moduli compared to the conventional cyclic testing. However, the modal and cyclic testing resulted in similar values of the complex Poisson’s ratio despite the different applied strain levels.

Keywords: Complex modulus, complex Poisson’s ratio, modal testing, frequency response functions (FRFs), finite element method (FEM)

1. Introduction

The time and temperature dependent stiffness of asphalt concrete is directly related to pavement quality and is fundamental to modern pavement design. For example, a too high stiffness in cold climates can lead to cracking while a too low stiffness at warm temperatures can lead to permanent deformation. Therefore, it is important with accurate characterization of the viscoelastic behavior of asphalt concrete over a wide range of temperatures and loading frequencies. Conventional complex modulus measurements of asphalt concrete are performed at different temperatures and frequencies (~0.01 to 25 Hz) through cyclic loading at strain levels of about 50 micro strains. These tests are expensive, time consuming and complicated to perform [1]. Test methods based on low strain nondestructive modal testing are faster and simpler to perform, and have a better repeatability [2-4]. In addition, laboratory resonance testing can provide the necessary link to surface wave measurements in the field to enable nondestructive quality control of pavements [5].

The natural frequencies of a solid with free boundary conditions are a function of the mass, the dimensions and the elastic constants. This relation has been widely used to derive elastic constants of many different materials through resonance frequency measurements [6]. It has also been applied to asphalt concrete specimens where the complex moduli have been determined from resonance testing by using both simplified analytical formulations and numerical computations [7-11]. The used analytical approximation is only valid for the fundamental resonance frequency and for certain geometries with a length (L) to diameter (d) ratio (L/d) larger than two. Hence, it is possible to determine only one modulus at each measurement temperature, which is not enough to characterize the frequency dependency of asphalt concrete [7, 8]. In contrast to this, modal testing in combination with numerical
computations enables a characterization of the modulus over a sufficiently wide frequency range to determine master curves that describe the material properties over a wide range of frequencies and temperatures [10]. In this approach, frequency response functions (FRFs) are computed through the finite element method from assumed values of the stiffness. The computed FRFs are then optimized to match measured FRFs by adjusting the assumed stiffness [9]. The measurements of the FRFs are performed by applying an impact load to a specimen with free boundary conditions while measuring the response of the specimen with an accelerometer. This paper presents a comparison of the complex modulus (E*) and complex Poisson’s ratio (v*) determined by modal testing and by conventional cyclic loading.

2. Methodology

The modal testing have been compared to results from conventional cyclic tension-compression testing performed to cylindrical specimens. The modal and the cyclic loading testing have been performed at the University of Lyon, Ecole Nationale des TPE, Laboratory of Tribology & System Dynamics. Four different specimens of the same asphalt mixture have been tested, where one specimen was tested by both the modal and the cyclic measurements. The asphalt concrete consists of 4.5 percent binder by weight with a penetration grade of 35/50. Further details of the specimens are presented in Table 1. Note the different dimensions of the tested specimens. The cyclic tension-compression (TC) test method have been thoroughly described in work by e.g. Di Benedetto et al. [12], Nguyen et al. [13] and Nguyen et al. [14].

<table>
<thead>
<tr>
<th>Specimen</th>
<th>Test method</th>
<th>Test ID</th>
<th>Mass (g)</th>
<th>Height (mm)</th>
<th>Diameter (mm)</th>
<th>Density (kg/m³)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>TC</td>
<td>TC – s.1</td>
<td>1572</td>
<td>140.9</td>
<td>73.8</td>
<td>2609.7</td>
</tr>
<tr>
<td>2</td>
<td>TC</td>
<td>TC – s.2</td>
<td>1564</td>
<td>140.1</td>
<td>73.7</td>
<td>2613.6</td>
</tr>
<tr>
<td>3</td>
<td>TC and FRFs</td>
<td>TC – s.3</td>
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<td>122.6</td>
<td>74.1</td>
<td>2606.8</td>
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<tr>
<td></td>
<td></td>
<td>FRF – s.3</td>
<td></td>
<td></td>
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<td></td>
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<tr>
<td>4</td>
<td>FRFs</td>
<td>FRF – s.4</td>
<td>1198</td>
<td>140.7</td>
<td>64.6</td>
<td>2586.0</td>
</tr>
</tbody>
</table>

2.1 Modal testing

To measure the FRFs an instrumented impact hammer (PCB model 086E80) was used to excite the specimen and an accelerometer (PCB model 352B10) attached by wax was used to measure the response. The specimen was placed on soft foam to provide free boundary conditions. The weight of the accelerometer is 0.7 g and it is assumed to not affect the response of the specimen. The hammer and the accelerometer, a signal conditioner (PCB model 480B21), a data acquisition device (NI USB-6251 M Series), and a computer were connected according to the illustration in Figure 1. The measurements were recorded with a sampling frequency of 500 kHz and performed by using the data acquisition toolbox in MATLAB. The FRFs were determined from the measured hammer impact and the acceleration based on averaging five measurements (n = 5) in the complex domain at each frequency according to Eqn. 1,
\[
H(f) = \left( \frac{1}{n} \sum_{k=1}^{n} X_k(f) \cdot X^*_k(f) \right) / \left( \frac{1}{n} \sum_{k=1}^{n} X_k(f) \cdot X^*_k(f) \right),
\]

where \( H(f) \) is the frequency response function, \( Y(f) \) is the measured acceleration, \( X(f) \) is the measured applied force and \( X^*(f) \) is the complex conjugate of the applied force. The positions of the impact excitation for the longitudinal modes of vibration are illustrated in Figure 1. Note that the specimen was laying down with the height along the horizontal direction.

Figure 1 Modal test set-up [8]

Figure 2 present the measured response of specimen 4 at 14.2 °C. Figure 2a show the acceleration in time domain, Figure 2b show the averaged measured FRF and the measured coherence function is presented in Figure 2c.

The coherence function is a measure of the phase difference between the input and output where a value of one means that this difference is constant for the five different impacts. The coherence
function is used here as an indication of the quality of the measured FRFs and is calculated accordingly,

\[
CF(f) = \left[ \left( \frac{1}{n} \sum_{k=1}^{n} X_k^*(f) \cdot Y_k(f) \right)^2 \right]^{1/2} \left[ \left( \frac{1}{n} \sum_{k=1}^{n} X_k(f) \cdot X_k^*(f) \right) \left( \frac{1}{n} \sum_{k=1}^{n} Y_k(f) \cdot Y_k^*(f) \right) \right].
\]

(2)

where CF(f) is the coherence function, X*(f)·Y(f) is the cross power spectrum, X(f)·X*(f) is the auto power spectrum of the impulse and Y(f)·Y*(f) is the auto power spectrum of the response.

The measurements were repeated for nine temperatures between -19.1 to 47.1 °C. A temperature chamber was used to condition the specimens to the test temperatures and the conditioning time was at least 5 hours for each temperature.

2.2 Determination of E* and v*

The E* and v* has been derived from the modal testing by computing theoretical FRFs and optimizing them to the measurements. This procedure was performed by using a finite element program (COMSOL Multiphysics 4.3b) to compute the FRFs and MATLAB to automatically minimize the difference between the measured and calculated FRFs by adjusting the assumed E* and v*.

The computation of the FRFs was performed in a three-dimensional space and with free boundary conditions. The point load in the model of 1 N was applied to the point corresponding to the actual hammer impacts during the measurements of the longitudinal modes of vibration (see Figure 1). The acceleration was also determined in the point corresponding to the accelerometer placement during the measurements. The mesh of the model consists of tetrahedral elements with quadratic shape functions. A suitable mesh size for good accuracy and to limit the computational time was determined through a convergence study to a maximum element size of 2 cm. The patternsearch algorithm in MATLAB was used to optimize the computed FRFs since it has been found to be efficient in finding the global minimum [15]. The difference between measured and theoretical FRFs was calculated over 40 frequencies that were distributed over the resonance frequencies of the measured FRFs. Eqn. 3 presents the objective function used to minimize the difference between the theoretical and measured FRFs,

\[
Error = \sum_{i=1}^{N} \left( H_{MNorm} \cdot \frac{H_M - H_T}{H_M} \right). \tag{3}
\]

where H_{MNorm} is the normalized measured FRF used to weigh the frequencies around the resonances higher, H_M is the measured FRF, H_T is the theoretical FRF, N is the number of data points and i is the index of the data point.

The Havrilak-Negami (HN) model that can express E* and v* as a function of frequency was used to account for the viscoelasticity of asphalt concrete. The HN model has been widely used to characterize various viscoelastic materials including asphalt concrete [9-11,16-19]. There are also other models that can be used for this purpose. For example, the 2S2P1D model has been commonly applied to bituminous materials and can accurately characterize the complex behavior of both asphalt binders and mixtures [10, 12-14]. The E*(ω) and v*(ω) are expressed by the HN model according to Eqn. 4 and 5,
\[ E^*(\omega) = E_\infty + \frac{(E_0 - E_\infty)}{[1 + (i\omega\tau_\alpha)\alpha]^\beta}; \quad (4) \]

\[ \nu^*(\omega) = \nu_\infty + \frac{(\nu_0 - \nu_\infty)}{[1 + (i\omega\tau_\nu)\beta]}; \quad (5) \]

where \( E_0 \) and \( \nu_0 \) are the low frequency values of the modulus and Poisson’s ratio, \( E_\infty \) and \( \nu_\infty \) are the high frequency values of the modulus and Poisson’s ratio, \( \alpha \) governs the width of the loss factor peak, \( \beta \) governs the asymmetry of the loss factor peak, and \( \tau \) and \( \tau_\nu \) is the relaxation time of the complex modulus and complex Poisson’s ratio, respectively.

Equation 4 and 5 can be used to optimize the computed FRFs of each measurement temperature separately. However, in addition to the frequency dependency, the temperature dependency of the asphalt concrete can be accounted for by substituting the Williams-Landel-Ferry equation (see Eqn. 6) into the HN model [20]. This enables an optimization of the FRFs for all temperatures simultaneously to obtain one set of parameter values that describe all measured FRFs [8]. \( E^*(\omega,T) \) and \( \nu^*(\omega,T) \) can be characterized through Eqn. 6, 7 and 8 accordingly,

\[ \log \alpha_\tau(T) = \frac{-c_1(T - T_{\text{ref}})}{c_2 + T - T_{\text{ref}}}, \quad (6) \]

\[ E^*(\omega,T) = E_\infty + \frac{(E_0 - E_\infty)}{[1 + (i\omega\alpha_\tau(T)\tau_\alpha)\alpha]^\beta}, \quad (7) \]

\[ \nu^*(\omega,T) = \nu_\infty + \frac{(\nu_0 - \nu_\infty)}{[1 + (i\omega\alpha_\tau(T)\tau_\nu)\beta]}. \quad (8) \]

Six parameters of the HN model (\( E_\infty, \nu_\infty, \alpha, \beta, \tau \) and \( \tau_\nu \)) and two parameters of the WLF shift function (\( c_1 \) and \( c_2 \)) have been estimated in the optimization of the computed FRFs through the use of these equations. The FRFs are not sensitive to the low frequency parameters, \( E_0 \) and \( \nu_0 \), within a realistic range of values for asphalt concrete. Therefore, the parameters \( E_0 \) and \( \nu_0 \) have been assumed to 100 MPa and 0.5, respectively, which are typical values of asphalt concrete [10,12,21].

An application recently developed by using COMSOL Multiphysics 5.1 and the Application Builder simplify the practical use of this presented methodology [22]. A web browser can be used to access and open the application in which measured FRFs are uploaded and finite element computed FRFs are optimized to determine the \( E^* \) and \( \nu^* \).

### 3. Results

An example of the fit of the computed FRFs to the measured FRFs is shown in Figure 3. Normalized amplitudes of the measured and computed FRFs are presented to show the fit for all different temperatures in one single figure. The amplitudes of the FRFs reduces considerably with increasing temperatures due to increased intrinsic damping of the asphalt concrete. The
amplitudes have been divided with the maximum values of the measured FRFs between 0 to 16 kHz for each temperature. Figure 3 shows the fit for the FRF - s.4 specimen over the frequency range covering the fundamental longitudinal mode of vibration. Note the increase of the width of the resonance peaks with increasing temperatures, which show the significant increase of the damping in the material.

The maximum strains at the fundamental resonance frequency of the longitudinal mode of vibration are shown for the different measurement temperatures in Figure 4a. The strains have been approximated through the finite element method and the measured acceleration of FRF - s.4 at each measurement temperature. The strains were derived from a point in the middle of the specimen (the node), which is the location of where the maximum strains occurs for the fundamental longitudinal mode of vibration. In addition to the temperature and frequency dependency, the strains in Figure 4a are also a result of the magnitude of the applied force in the hammer impacts. Figure 4b show the strain normalized over the applied force to illustrate the effects of only the temperature and frequency on the strain level through modal testing.

Figure 3. Normalized measured and finite element computed FRFs of the fundamental longitudinal mode of vibration (FRF - s.4).

Figure 4. The maximum strain levels at the fundamental resonance frequency of the longitudinal mode of vibration (FRF - s.4).
Figure 5 presents the complex moduli master curves determined through the modal testing (FRF – s.3 and s.4) and through the cyclic tension-compression testing (TC – s.1 to s.3). The two master curves of the two specimens (with different dimensions) determined through modal testing are nearly identical. Also, the tension-compression testing of the three different specimens gave repetitive results of the complex moduli. The comparison between the FRF and the tension-compression determined complex moduli show that the low strain modal testing provides higher absolute values of the complex moduli and lower phase angle. However, the differences between the methods reduce as the frequency increases which indicate that the material is less strain level dependent at higher values of the modulus.

![Figure 5](image_url)

Figure 5. Comparison of the complex modulus master curves determined through FRF and tension-compression testing of four different specimens showing the absolute value, |$E^*$| (a), the phase angle (b), and loss and storage modulus (c).

Figure 6 presents the comparison of the tension-compression and the modal tested complex Poisson’s ratio. The difference between the Poisson’s ratios of the two specimens characterized through modal testing comes from that different amount of parameters in the applied HN model were optimized to match the theoretical and measured FRFs. The specimen FRF - s.3 has been estimated using the same $\tau$ value for both the complex modulus and complex Poisson’s ratio, while the specimen FRF - s.4 has been estimated with a different value of $\tau$ for Poisson’s ratio in the HN model. The latter approach slightly improved the fit of the computed FRF for the second longitudinal resonance frequency compared to using $\tau$ of the complex modulus. It can also be seen that complex Poisson’s ratio of the specimen FRF - s.4 agrees very well with two out of three presented tension-compression results of Poisson’s ratio. It is clear that the tension-compression determined Poisson’s ratio of the specimen TC - s.3 differs from all other results of the complex Poisson’s ratio presented in Figure 6. This is a strong indication of that the tension-compression results of TC - s.3 may not have been correctly measured. A computed
FRF based on the measured Poisson’s ratio of TC - s.3 strengthens this indication by resulting in a too large distance between the first and second resonance peaks compared to the measured FRFs of this specimen. This means that the tension-compression measured Poisson’s ratio of TC - s.3 is too low, which is also what can be seen by comparing the tension-compression results of the different specimens to each other. It should be noted that small measurements errors in the conventional test have a relatively large effect on the Poisson’s ratio. For example, an approximately 0.4 μm change of the specimen diameter corresponds to a change of Poisson’s ratio with ~0.1, which is about the difference shown in Figure 6a.

![Figure 6](image)

Figure 6. Comparison of the complex Poisson’s ratio master curves determined through FRF and tension-compression testing of four different specimens showing the absolute value, $|\nu^*|$ (a), the phase angle (b), and loss and storage modulus (c)

### 4. Conclusions

The two modal tested specimens have been shown to result in slightly higher absolute values of the complex moduli and lower phase angles compared to the tension-compression testing. The comparison of the complex moduli performed in this paper agrees with previous results presented by Gudmarsson et al. [10].

The modal and tension-compression testing gives similar results of the complex Poisson’s ratio, if the tension-compression complex Poisson’s ratio results of the specimen labeled TC - s.3 are ignored. This indicates that, in contrast to the complex moduli, the complex Poisson’s ratio may not depend on the different strain level range applied in this study. An improvement of the fit of the FRFs may be possible by using a value of the relaxation time for Poisson’s ratio different from the value used for the complex modulus.
Although the modal testing is simpler and more economic than the conventional cyclic testing, the analysis of the measurements may be complex. However, a newly developed application for the analysis of the modal measurements simplifies the determination of the complex modulus and complex Poisson’s ratio [22].

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**References**


