Differential inversion using surface wave methods for time-lapse monitoring
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Extended Abstract

Abstract
Surface wave methods are applied in this study for the purpose of time-lapse monitoring. This study proposes two approaches of Differential Inversion using surface wave methods, which uses data difference, instead of the data themselves, in the inversion process. The first approach uses an innovative method to calculate the difference between two phase velocity dispersion diagrams, which takes into account the difference and similarity at the same time. The second approach uses linear approximation of Rayleigh wave phase velocity, which relates the model variation with phase velocity variation directly. Both approaches are tested on laboratory-measured data.

Keywords: Time-lapse monitoring, Surface wave methods, Differential inversion

1 Introduction
Climate-changing, human activities or aging problems create variations in civil engineering structures and materials which, if monitored, bring useful information for maintenance. Time-lapse (TL) monitoring measures the present condition of a given material, named the baseline, and then repeat a successive series of measurements at different times, named repeatlines. By comparing the baseline with one of the repeatlines, material properties' variations, including weak ones, can be inferred. Surface wave (SW) methods are widely used for TL monitoring, because SW are very energetic and the inversion of their dispersion properties gives a depth-profile of some mechanical parameters. Classical SW inversion uses SW phase velocity (Vph) as inversion input data. It suffers from low sensitivity to the deep medium’s parameters due to high uncertainties in the measurements at low frequencies. TL monitoring can partially overcome the difficulties linked to the estimation of deep variations.

In this study, we propose a new strategy for TL monitoring using SW methods, by looking for the relation between the model parameter variations and the data difference within the frame of global inverse problems. For this purpose, we introduce differential inversion (DI) in surface wave methods. Instead of independent inversions of the baseline and repeatlines, DI uses the difference between measured data of baseline and repeatlines as inversion input data. Two approaches have been tested. The first one uses an innovative method to calculate the data difference, which is called the diagram difference (DD). The second approach uses a linear approximation of Rayleigh wave Vph in the inversion process, in order to relates the data difference with the model parameters variations by using the analytical equation of the sensitivity kernel.
2 Methodology on Differential Inversion

2.1 Diagram difference

The phase-shift method [1] is applied to extract the dispersion diagram and the corresponding phase velocity dispersion curve of the measured data. Considering the surface wave dispersion diagram as a histogram showing the velocity distribution as function of frequency, the difference between two dispersion diagrams (or histograms noted as H(A) and H(B) for baseline and repeatline) can be calculated as following [2]:

\[ D'_\text{ord}(A, B) = \sum_{i=0}^{q-1} \sum_{j=0}^{i} (H_j(A) - H_j(B)) \]

with i and j being the frequency index. Eq. (1) measures not only the difference but also the similarity between two dispersion diagrams [2]. The DD is then used in the DI approach.

2.2 Linear approximation of Rayleigh wave phase velocity

The linear approximation of \( V_{ph} \) is used in the second approach of DI, which uses the sensitivity kernel of \( V_{ph} \) to relates the model variation to the \( V_{ph} \) variation directly:

\[ V_{ph}(m_i) \approx V_{ph}(m_b) + \sum_{m_b} \left[ \frac{\partial V_{ph}}{\partial m_i} \right] (m_i - m_b) \]

With \( m_b \) being the model of baseline, \( m \) being the searched model and i is the layer number.

3 Experimental data and inversion results

3.1 DD approach using data of mortar-concrete slabs

Three mortar-concrete slabs (named D01, D06 and D08) are used and each slab is made of a layer of mortar, superimposed on the surface of a concrete slab. The mortar layers of the three slabs have different water-to-cement ratios that change the mechanical properties of each slab. The experimental set-up is composed of an ultrasonic source and a receiver probe with 16 receivers. Phase-shift method is applied on the recorded seismogram data and the extracted dispersion diagrams are presented in Fig. 1. DDs of two pairs (D06 and D01, D08 and D06) are calculated using Eq. (1) and are shown in Fig. 1 (d). One sees higher values of DD at higher frequencies, which are due to the variations in the mortar layer at shallow medium. Besides, differences between D08 and D06 are more significant than that between D06 and D01. Theses two DD curves are then used in the DI process to obtain the inverted models of D06 and D08 respectively.

Fig. 1 (e) and (f) show the inversion results of D06 and D08, where the results of DI (blue circles) are compared with classical Vph inversion (red dots), i.e. measured Vph is directly used for the inversion. Inversion results of DI are more concentrated than that of classical Vph inversion for both layers of D06 and D08. This proves that the DI using DD as inversion input data is capable to estimate medium’s variations for both layers.
Figure 1: Dispersion diagrams of (a) D01, (b) D06 and (c) D08. Red dots correspond to the maximum amplitude of dispersion diagrams at each frequency. (d) DDs between D06 and D01 in blue and between D08 and D06 in orange. Inversion results of (e) D06 and (f) D08 with blue circles for DI and red dots for classical Vph inversion (using measured Vph curve directly).

3.2 Linear approximation of Rayleigh wave $V_{ph}$

Three reduced-scale epoxy-resin models, named C25, C45 and C65, respectively, have been designed in order to simulate small variations in the medium to mimic time-varying models at three different times. The models have two layers and the variations occur at the deep layer. Fig. 2 (a) shows the measured dispersion curves of the three models, where variations are observed at low frequencies. Fig. 2 (b) shows the $V_{ph}$ variations between two pairs of models, C25 and C45, C45 and C65 respectively.

Eq. (2) is used in the DI process to estimate the variations between models. Inverted models is used to calculate the theoretical dispersion curves and is compared with measured dispersion curves (Fig. 2 (c) and (d) for models C45 and C65 respectively). $V_{ph}$ variations are also calculated between measured baseline and inverted repeatline and are shown in Fig. 2 (b). One sees that the calculated dispersion curve and $V_{ph}$ variation curve correspond to the measured data which shows the feasibility of this DI approach to estimate the model variations. The advantage of this DI approach is the use of linear approximation of Rayleigh wave $V_{ph}$ in the inversion approach, where the sensitivity kernel is calculated only once for the baseline. The calculation time is greatly reduced.
Figure 2: (a) Measured dispersion curves of three epoxy-resin models. (b) Relative variations for the measured phase velocities and the inverted phase velocities. (c) and (d): measured and inverted phase velocities of C45 and C65 models respectively.

4 Conclusion

In this study, we propose two DI approaches using SW methods, in order to estimate medium’s variation during the time-lapse monitoring. Experimental data are used for both approaches in order to test their feasibility. The first approach uses DD as inversion data, which contains the variation and similarity of two dispersion diagrams at the same time. The second method uses sensitivity kernel of Rayleigh wave $V_{ph}$ to relate the model variation with $V_{ph}$ and the low consuming time is its advantage.

References