Ultrasonic Testing and Entropy

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Abstract
Attempt is described to correlate results of the UT inspection of the object (i.e. object condition and defectivity) with its entropy. The system condition, based on the ultrasonic inspection, is estimated by using numbers of pixels with various flaw response amplitudes. Calculation of the system entropy is based on the statistical weight. Formulae for entropy calculation, based on numbers of different pixels, are presented. Pressure tubes of the CANDU reactors are used to illustrate the efficiency of the proposed statistical method of the object condition assessment.

Keywords: ultrasonic testing, entropy, statistical weight, statistical assessment.

1. Introduction

The goal of the paper is to correlate the results of ultrasonic (UT) non-destructive evaluation (NDE) of the object (system) with entropy of this system. The UT and other NDE methods allow detecting, characterizing and sizing various flaws in different objects. After that, the object defectivity, reliability and remaining life time can be determined. At the same time, the entropy of the system characterizes the degree of chaos (level of defectivity or destruction) of the system. Knowing it, the condition of the system and its life time can be determined. Therefore, it is reasonable to suggest that results of the UT testing of a system and entropy of this system should be correlated. This correlation can be used to for a statistical assessment of object condition.

2. Object condition based on the UT testing

First of all, it is necessary to introduce some criterion characterizing the degree of defectivity of the object under test. Probably, the simplest and the most natural criterion of the defectivity of the object is the 4-dimensional volume $V$ of this object, occupied by flaw responses. This 4D volume $V$ consists of many elemental 3D spatial cells (or pixels), covering the 3D volume, occupied by the UT responses of flaws. Respectively, the 3D spatial volume $\Delta V$ of each pixel is a product of three components: pixel length, pixel width and pixel depth. Three pixel dimensions depend on the inspection system parameters: scanning rasters and digitization rate.

A pixel can be considered a single “physical point” or the smallest addressable and resolvable element that can be distinguished. Each pixel has its own spatial address, which corresponds to its three spatial coordinates. However, three spatial coordinates of each pixel within the flaw response are not sufficient to characterize this pixel. Since the response amplitude varies from one pixel to the other, it is necessary to introduce the fourth “dimension” of each pixel – the amplitude $A$ of the UT response in this pixel. Let us consider for simplicity, that there are only four types of possible response amplitudes $A$:

- $A = 0 - 10\%$ (no flaw response).
- $A = 10 - 40\%$ (a weak flaw response).
- $A = 40 - 70\%$ (a medium flaw response).
- $A = 70 - 100\%$ (a strong flaw response).
Note that strictly speaking, the criterion of defectivity should include all measurable parameters of the flaw response: dimensions, amplitude, phase, peak frequency of spectrum, bandwidth, etc. However, further for simplicity we will use only mentioned above 4D volume, covering the flaw responses and consisting of the 3D spatial pixels with specified response amplitude $A$ per each pixel.

Generically, the 4D volume $V$, containing flaw responses, can be characterized by spatial address of each pixel and four numbers: $n_{0-10}$, $n_{11-40}$, $n_{41-70}$, and $n_{71-100}$, where $n_{0-10}$ is the number of 3D spatial pixels in the object with almost zero amplitude $A$ of the response varying from 0% to 10% full screen height (FSH); $n_{11-40}$ is the number of 3D spatial pixels in the object, which cover the flaw response areas with weak response amplitude $A$ varying from 11% to 40% FSH; $n_{41-70}$ is the number of 3D spatial pixels in the object, which cover the flaw response areas with medium $A$ varying from 41% to 70% FSH; and $n_{71-100}$ is the number of 3D spatial pixels in the object, which cover the flaw response areas with strong $A$ varying from 71% to 100% FSH.

At the first sight, it seems that four numbers: $n_{0-10}$, $n_{11-40}$, $n_{41-70}$, and $n_{71-100}$, are sufficient to statistically characterize the object defectivity. And it would be true, if all pixels covering the response areas were absolutely independent of each other. However, usually it is not a case. Pixels, covering the response areas, sometimes can exist only in aggregates: pixels with large amplitude $A$ are typically surrounded by pixels with smaller amplitudes. That is how the UT response of any flaw looks like, because of the physical nature of a flaw, which always develops from a weak flaw to the strong one. The number of pixels with small $A$ around pixel with a large $A$ depends on the degree of the flaw development. In our approach presented below, we neglect for simplicity this phenomenon and consider pixels with various response amplitudes independent of each other.

The maximum defectivity of the object from the statistical point of view can be achieved when flaws are distributed uniformly with equal representation of all four different types of pixels with various $A$. It means that the each one of these four types of pixels occupies the same 3D spatial volume, which equals $N/4$, where $N$ is the total number of 3D spatial pixels in the object.

There is also one more reason why four numbers: $n_{0-10}$, $n_{11-40}$, $n_{41-70}$, and $n_{71-100}$, are not sufficient to statistically characterize the object defectivity. Different flaws are not equally dangerous. As a matter of fact, flaws with different shapes, orientations and dimensions have different levels of danger, and respectively, they represent different levels of the object defectivity. For example, flaws oriented in the depth direction (direction with minimum size in the object), are the most dangerous, because they typically lead to the object cracking and failure (e.g. leakage or even destruction). To diminish the influence of this factor, one should analyze only parts of the object where flaws have been detected.

Let us illustrate this idea on the example of the UT inspection of pressure tubes (PT) in the CANDU reactors [1]. PT is ZrNb tube ~6000mm long with inside diameter ~100mm and wall thickness ~4mm. The minimum dimension of the PT is of course the wall thickness, which lies in the radial direction. Therefore, flaws oriented in the radial direction, are the most dangerous, because they can lead to the PT cracking through the wall, leakage and destruction. Typically the flaws in the PT are located on the inside surface of the PT wall and are oriented in the axial
direction: they are ~2-30mm long, ~1-6mm wide in the circumferential direction and ~0.1-0.7mm deep in the radial direction.

During the UT inspection of PT [1], the scanning raster in the axial direction is typically 0.4mm, raster in the circumferential direction is 0.1mm, and resolution in the radial direction is 0.023mm. As a result, the total number of pixels in the PT axial direction is ~15,000, in the circumferential direction ~4,000, and in the radial direction ~160. In total, it means that there are ~10^10 3D spatial pixels in the PT.

Of course, different tubes, depending on their defectivity, have different numbers of flaw indications. For example, there are always so called “good” tubes, which typically have 20-40 indications with small and medium response amplitudes, located at the PT inside surface and ~0.1-0.2mm deep. Analysis of the inspection results of a typical “good” PT shows that flaw responses are usually located within the PT wall layer 0.2mm thick at the inside surface. These flaw responses cover ~2·10^5 3D spatial pixels out of the total N ≈ 5·10^8 3D spatial pixels. Generically, for this 0.2mm thick layer of the “good” PT, where responses of the flaws were detected, N=n_{0-10}+n_{11-40}+n_{41-70}=5·10^8, n_{0-10}=4.998·10^8, n_{11-40}=1.3·10^5, and n_{41-70}=0.7·10^5.

However there are also “bad” tubes, which have a few hundreds indications with various response amplitudes at the inside surface ~0.1-0.7mm deep. Analysis of inspection results of a typical “bad” PT shows that flaw responses are usually located within the PT wall layer 0.7mm thick at the inside surface. These flaw responses cover ~3·10^6 3D spatial pixels out of the total N ≈ 1.5·10^9 3D spatial pixels. In general, for this part of the PT (0.7mm thick layer), where responses of the flaws were detected, N=n_{0-10}+n_{11-40}+n_{41-70}+n_{71-100}=1.5·10^9, n_{0-10}=1.497·10^9, n_{11-40}=2·10^6, n_{41-70}=0.7·10^6, and n_{71-100}=0.3·10^6.

Obviously, one should use all available information from different probes [1] used for inspection and providing flaw responses. Therefore, it is necessary to determine the most efficient way to somehow “combine” their responses. Emphasize, that under the term “flaw” we understand not only the flaw itself (crack, fret, corrosion, disbond, delamination, porosity, etc), but also the stress concentration area, the region with changed material parameters, etc.

Also note that sometimes our measurements provide only a few approximate parameters of the flaw. In such a case, the judgment is based only on this available incomplete assessment of the flaw. It might happen that inspection system cannot, for example, detect such flaws as stress concentration areas or areas with changed material parameters. Then, of course, the “degree of destruction” of the object, which we obtain based on our measurements, is lower than it is in reality. The cause of this error is not the proposed statistical assessment of the object condition and defectivity based on the detected flaws, but the low sensitivity of the inspection system, which cannot detect some types of flaws or cannot measure some parameters of the flaw.

3. **Object condition based on the statistical weight and entropy**

Entropy is essentially a measure of the number of ways in which a system may be arranged, often taken to be a measure of the "disorder" [2]. Specifically, this definition describes the entropy as being proportional to the logarithm of the number of possible microscopic
configurations of the individual “elements” of the system (microstates) which could give rise to the observed macroscopic state (macrostate) of the system.

For a given set of macroscopic variables, the entropy measures the degree to which the probability of the system is spread out over different possible microstates. In contrast to the macrostate, which characterizes plainly observable average quantities, a microstate specifies all details about the system “elements”. The more such states available to the system with appreciable probability, the greater the entropy.

According to the statistical definition [2] of the entropy $S$ of the system, it equals

$$S = k_b \ln \Delta \Gamma \quad \text{...............................................}(1)$$

where $k_b=1.38065 \times 10^{-23}$ J/K is the Boltzmann constant and $\Delta \Gamma$ is the thermodynamic probability or statistical weight, which in turn equals to the total number of the microscopic states (microscopic complexions), compatible with the required macroscopic state. In other words, the statistical weight $\Delta \Gamma$ is the number of the microscopic states, which can realize the required macroscopic state.

More specifically, entropy is a logarithmic measure of the density of states:

$$S = k_b \sum_i \left( P_i \ln P_i \right) \quad \text{...............................................}(2)$$

where the summation is performed over all the microstates the system can be in, and $P_i$ is the probability that the system is in the $i^{th}$ microstate. For almost all practical purposes, this can be taken as the fundamental definition of entropy since all other formulas for $S$ can be mathematically derived from it, but not vice versa.

In what has been called the fundamental assumption of statistical thermodynamics (or the fundamental postulate in statistical mechanics), the occupation of any microstate is assumed to be equally probable (i.e. $P_i = 1/\Delta \Gamma$ since $\Delta \Gamma$ is the number of microstates). This assumption is usually justified for an isolated system in equilibrium. Then the previous equation reduces to (1). We are interested in the disorder in the distribution of the system over the permissible microstates. This measure of disorder is provided by (1). Each specified macroscopic state can be represented by many microscopic states (similar to any “degenerate” state in quantum mechanics), which differ by some parameters, but occupy the same “phase space” volume. The larger the “phase space” of the flaw, the more damaged object is and, respectively, the greater its entropy is.

Let us consider the system (or object) consisting of $N$ elements, classified into $n$ classes (e.g. response amplitudes). Let $m_i$ be the occupation number of the $i^{th}$ class (i.e. this class can be differently represented $m_i$ times). The macroscopic state of the system is given by the set of occupation number $A_n=(m_1, m_2, ..., m_n)$. Then, the statistical weight or degree of disorder of the macrostate $A_n$ is given by formula containing the permutations
\[ \Delta \Gamma(A_n) = \frac{N!}{\prod_{i=1}^{n} m_i!} \] .................................(3)

where any number with a factorial sign determines the number of the respective permutations. The quantity \( \Delta \Gamma(A_n) \) represents the total number of the microscopic states (or complexions or elements) compatible with the constrains the system is subjected to.

In other words, the statistical weight is a coefficient of a multi-set permutation. It means that if the multiplicities of the elements (taken in some order) are \( m_1, m_2, \ldots, m_n \) and their sum equals \( m_1 + m_2 + \ldots + m_n = N \), then the number of multi-set permutations of \( N \) is given by the multinomial coefficient \( \Delta \Gamma \). Respectively, substituting (3) in (1), we obtain that entropy \( S \) is equal to

\[
S = k_B \ln \Delta \Gamma = k_B \ln \left( \frac{N!}{m_1! m_2! \ldots m_n!} \right) .................................(4)
\]

For example, let us analyze a wall consisting of \( N \) identical white bricks. This wall (or macrostate) can be realized by only one possible microstate, because if we exchange positions of any two absolutely identical white bricks, then the obtained microstate will be indistinguishable from the previous one. Now, if there is one black brick in this wall, then this wall (or macrostate) can be realized by \( N \) possible various positions of this black brick (i.e. by \( N \) different microstates). If there are two black bricks in this wall, then this wall (or macrostate) can be realized by \( N(N-1) \) possible various positions of these black bricks (i.e. by \( N(N-1) \) different microstates). And so on. It means that the statistical weight, determined by formula (4), really represents the “measure” of the “color disorder” which this brick wall has.

Now let us return to our case, i.e. the UT testing of the object. For this case, the each “element” (i.e. pixel) of the flaw response has four dimensions: three spatial coordinates (pixel address) and flaw response amplitude in this pixel.

Determine the number of all possible permutations, representing all possible distributions of \( N \) pixels within the PT wall, by applying formula (4). We have four different possible amplitudes (see section 2), i.e. amplitude of each pixel equals \( A_i \), where \( i=1, 2, 3, 4 \). The number of pixels with amplitude \( A_i \) is \( m_i \), i.e. pixel with amplitude \( A_i \) is represented in our flaw response \( m_i \) times. Then the statistical weight \( \Delta \Gamma \) (or degree of disorder) of this flaw response will be equal to (4). The value \( \Delta \Gamma \) will include all possible variations, because it equals to number of permutations of \( N \) spatial pixels, divided by product of the numbers of permutations within the each type of pixels. Then for two examples, analyzed in section 2 (“good” tube and “bad” tube), we obtain:

\[
S_{\text{good}} = k_B \ln \{ (5 \cdot 10^8)!/ [(4.998 \cdot 10^8)! \cdot (1.3 \cdot 10^8)! \cdot (0.7 \cdot 10^8)!] \} \approx k_B \ln (2.8 \cdot 10^{801538}) \approx 1.3 \cdot 10^6 \cdot k_B .................................(5)
\]
\[ S_{\text{bad}} = k_B \ln\left(\frac{(1.5 \cdot 10^9)!}{(1.497 \cdot 10^9)! \cdot (2 \cdot 10^6)! \cdot (0.7 \cdot 10^6)! \cdot (0.3 \cdot 10^6)!}\right) \approx k_B \ln\left(\frac{(7.45 \cdot 10^{26904813})}{(5.75 \cdot 10^9)!}\right) \approx 9.2 \cdot 10^7 \cdot k_B \] 

The obtained results mean that for the “bad” tube (where there are much more flaws than in the “good” tube, and also these flaws are larger and deeper), the entropy is about two orders of magnitude greater than for the “good” tube. At the same time, the absolute value of entropy has a little meaning for the object defectivity estimate. To obtain such assessment, one should compare the calculated value of the object entropy with maximum possible value of entropy, which can be achieved for our example when flaws are distributed uniformly with equal representation of all four different types of pixels with various amplitudes \( A \). Recall that this maximum value of entropy is related to the maximum object defectivity.

Calculating the maximum possible entropy values for the respective PT layers of the “good” and “bad” tubes, we obtain the following

\[ S_{\text{max\ good}} = k_B \ln\left(\frac{(5 \cdot 10^8)!}{(1.25 \cdot 10^8)!}\right) \approx k_B \ln(9.75 \cdot 10^{90095628}) \approx 8.2 \cdot 10^8 \cdot k_B \] 

\[ S_{\text{max\ bad}} = k_B \ln\left(\frac{(1.5 \cdot 10^9)!}{(0.375 \cdot 10^9)!}\right) \approx k_B \ln(8.87 \cdot 10^{903015633}) \approx 9.4 \cdot 10^9 \cdot k_B \]

First of all, these results show that maximum possible value of entropy depends, of course, on the thickness of the PT layer, where flaws were detected: for 0.2mm thick layer the maximum entropy is about ten-fold less than for 0.7mm thick layer. For the whole tube, the maximum possible entropy, related to the maximum object defectivity, will be probably about \( 10^{15} \cdot k_B \).

Secondly, for the “good” tube the calculated entropy is approximately three orders of magnitude lower than maximum possible amplitude, while for the “bad” tube the calculated entropy is approximately only two orders of magnitude lower than maximum possible amplitude. It means that some threshold (criterion) of the entropy value (and respectively, the threshold of the object defectivity) can be introduced. Let for example, the entropy, calculated using the obtained UT flaw responses, is about one order of magnitude lower than maximum possible entropy of this tube. It probably means that defectivity of this tube is already very high, it is dangerous to use it further, and therefore it should be replaced.

Respectively, the following steps of the statistical assessment method of the object condition and defectivity can be proposed:

1. Perform UT inspection of the object.
2. Determine the object area where flaws were detected.
3. Determine numbers of pixels with various flaw response amplitudes.
4. Calculate value of entropy of the object area with flaws using formula (4).
5. Calculate maximum possible value of entropy of the same area of the object.
6. Compare these two values and make a decision regarding object’s level of defectivity.

Of course, all numbers in (5)-(8) represent only very approximate entropy estimates, because the general approach, numerous simplifications, calculation technique, basic formulae, and other factors are just the experimental ones, which were used for investigation and testing of the idea. However, even this experimental approach is useful. It might happened that if, for example, the
entropy of the whole tube exceeds some threshold value, then it means that this PT is already severely damaged and should be replaced, because it is dangerous to continue using it. Recall, that proposed approach provides of course only statistical (probabilistic) assessment of condition (defectivity) of the object and its remaining life time.

The general approach, the calculation technique, the simplifications, and all other components of the proposed method should and will be improved and clarified, when more experience on the correlation between the flaw responses and entropy of the respective object is acquired.

In general, the entropy may serve as a damage evaluation criterion of the object for the statistical assessment of the object condition. Various ideas, concerning this approach for estimation of different objects (industrial and biological) have been addressed by many authors.

It is obvious, that in order to properly characterize the object, we should know not only the entropy \( S \) at one fixed moment of time, but the function \( S(t) \) or its time derivatives \( dS/dt \) and may be even \( d^2S/dt^2 \). In such a case, we will know deterioration rate and, respectively, life time of the object (technical or biological). Note that derivative \( dS/dt \) can be approximately obtained by determining the entropy \( S \) at different moments of time, for example, during the periodical monitoring of the object, e.g. periodical inspection campaigns of the PT [1].

Recall, that in the end, we are interested only in some “integral” assessment of the object. For a practical application usually it is not important, what flaws are in the object and where they are located; only structure integrity and remaining life time of the object are important. In this sense, the entropy might be a very useful and convenient criterion of such “integral” assessment.

5. Conclusions

- An attempt is made to correlate the results of the UT inspection of the object (i.e. object defectivity) with its entropy.
- Object defectivity, based on the UT evaluation, is estimated by using numbers of pixels with various flaw response amplitudes.
- Entropy of the object is calculated based on the statistical weight, which is number of the microscopic states of the object, compatible with the required macroscopic state of this object.
- Formulae for entropy calculation, based on the numbers of different pixels, are presented.
- Pressure tubes of CANDU reactors are used to illustrate the efficiency of the proposed statistical assessment method of object condition and defectivity.
- Merits, limitations and possible improvements of the proposed method are discussed.

References