AN INVESTIGATION OF FOCUSING AND ANGULAR TECHNIQUES FOR VOLUMETRIC IMAGES BY USING THE 2D CIRCULAR ULTRASONIC PHASED ARRAY

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Abstract. New array instrumentation allows ultrasonic arrays to be used with increasing numbers of elements. This leads to the possibility of volumetric imaging of engineering components. In this paper, two dimensional (2D) ultrasonic transducer arrays are used to generate beams that can be steered and focused throughout a three-dimensional (3D) volume from a single location. A model is developed to generate time signals for all combinations of transmit and receive elements. Based on this model, a theoretical investigation of the focusing and steering properties of 2D circular arrays is described. The imaging performance of a circular array is then quantified from their point spread functions.

1 Introduction

Many different techniques have been recently developed for volumetric imaging [1-3]. They can be classified into two categories: mechanical scanners and 2D ultrasonic phased arrays. In a mechanical scan, a single probe at a fixed angle is mechanically scanned over a component and for a defect to be detected its orientation must be favourable with respect to the incident beam. In some cases, the orientation of likely defects can be estimated and the probe orientation chosen accordingly. However, in cases such as the detection of flattened inclusions in forged parts, the orientation of potential defects cannot be reliably anticipated and reliable detection requires many scans using probes at different orientations. However, with a 2D phased array the complete data set can be extracted from the array at a given location and a volume of the specimen can then be probed using offline post processing. Not only will this enable sensitivity to defects at unfavourable angles to be increased, but also once a defect had been identified, its 3D reflectivity function can be computed from the acquired data.

In this paper two post processing algorithms, termed the Focusing Method (FM) and Angular Method (AM) scan are presented. In the FM algorithm, an image is created in which the beam has been focused, in turn, on every point within the field of view [4]. In the AM scan algorithm, an image is created in which the beam has been steered through a range of angles to inspect the volume of interest [5,6].

In order to quantify the performance of these two imaging algorithms, a parameter called Array Performance Indicator (API) is used. The performance of the FM and AM scan imaging algorithms are also compared using experimental data. These experimental data are obtained by a 2D prototype circular phased array from flat bottom holes in aluminum.
2 Simulation of array data

The array data are simulated by the response of a point reflector that can be located anywhere in the +z halfspace, as indicated in Figure 1. Array elements are uniformly spaced on the circumference of a circle of radius, r, in the x-y plane. A simulation program (written in Matlab) is used to generate time domain signals (time-traces) for all combinations of transmit and receive elements. For example, if n is the number of elements in the array then a maximum of \(n \times n\) possible time-traces are available for all transmitter-receiver combinations. However, the number of independent time-traces that actually need to be computed for the simulation is only \(\frac{n(n+1)}{2}\), because the time-trace when element 1 transmits to element 2 is identical to the time-trace when element 2 transmits to element 1 due to reciprocity.

The first step in the simulation is to define a transmitted signal \(f_0(t)\) in the time, \(t\), domain. Typically this is a Hanning windowed tone burst containing five cycles.

The second step in the simulation is to calculate the propagation distance associated with the reflected signal for each time-trace. For the tx - rx time trace (tx – transmitter and rx – receiver), the propagation distance, \(d_{tx,rx}\) from the transmitter to the reflector and back to the receiver as shown in Figure 1 is calculated as follows:

\[
d_{tx,rx} = \sqrt{(x_{ref} - x_{tx})^2 + (y_{ref} - y_{tx})^2 + z_{ref}^2} + \sqrt{(x_{ref} - x_{rx})^2 + (y_{ref} - y_{rx})^2 + z_{ref}^2}\]

where \(x_{ref}, y_{ref}\), and \(z_{ref}\) are the coordinates of the reflector and \(x_{tx}, x_{rx}, y_{tx}\), and \(y_{rx}\) are the coordinates of the transmit and receive array elements respectively.

The third step in the simulation is to apply phase shifts to the spectrum of the input signal to simulate the propagation delays and compute the phase shifted spectrum, \(G_{tx,rx}(\omega)\). By ignoring the attenuation of the medium and the element directivity effects it can be written:

\[
G_{tx,rx}(\omega) = q F_0(\omega) e^{-ikd_{tx,rx}}
\]
where $q$ is a factor to account for the missing pitch-catch information ($q = 1$ for pulse-echo time-traces where the transmitter and receiver are the same and $q = 2$ for pitch-catch time-traces), $k$ is the circular wave number and $F_0(\omega)$ is the spectrum of the transmitted signal. Finally, the spectra $G_{tx,rx}(\omega)$ is returned to the time-domain to yield the time trace $g_{tx,rx}(t)$.

3 Array data processing

Two post processing algorithms, termed the Focusing Method (FM) and the Angular Method (AM), were then used to generate the images. The FM algorithm proceeds by first discretising any region of space in front of the array into a grid as shown in Figure 2(a). The beam is then focused at every point in the grid to generate a fully focused image. The intensity of the image, $I(x,y,z)$ for the FM at any point in the grid is calculated by:

$$I(x,y,z) = \sum g_{tx,rx}\left(\frac{d_{tx,rx}}{c_l}\right)$$

where $c_l$ is the bulk longitudinal velocity and

$$d_{tx,rx} = \sqrt{(x-x_{tx})^2 + (y-y_{tx})^2 + z^2} + \sqrt{(x-x_{rx})^2 + (y-y_{rx})^2 + z^2}$$

The AM scan algorithm proceeds by steering the beam through a range of angles, as shown in Figure 2(b). The intensity of the image, for the angular scan method is calculated by:

$$I(x,y,z) = \sum g_{tx,rx}(\Delta t_{tx,rx})$$

where

$$\Delta t_{tx,rx} = \frac{1}{c_l} \left( \frac{z}{\cos \theta} - r \left( \cos \left( \phi - \frac{2\pi n_{tx}}{N} \right) + \cos \left( \phi - \frac{2\pi n_{rx}}{N} \right) \right) \sin \theta \right)$$

The $n_{tx}$ and $n_{rx}$ are the indices of the circular array element and $N$ is the total number of elements. The radius of the circular array is $r$, the elevation angle is $\theta$ and the azimuth angle is $\phi$ as shown in Figure 1.

Figure 2: Imaging algorithms a) TFM scan  b) Angular scan
4 Comparison of FM and AM scan performance

In order to quantify the performance of each scan algorithm a parameter called the Array Performance Indicator (API) is defined. The API is a dimensionless measure of the volume of the 3D point spread function (PSF):

$$API = \frac{V_{-6dB}}{\lambda^3}$$  \hspace{1cm} (7)

where $V_{-6dB}$ is the volume of PSF that is within –6dB of its maximum value. A low API value indicates that the beam from an array is well focused.

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**Figure 3:** B scan images when the reflector point at $x = 3.3\lambda$, $y = 0$, and $z = 10\lambda$: a) TFM scan image b) Angular scan image. The scale shown to right of each image is in dB.

**Figure 4:** Comparison of TFM and Angular scan model data using API value a) $API$ vs. reflector $z$ position; b) $API$ vs. reflector steering angle at $z = 16.6\lambda$ & $z = 25\lambda$. 

The volume of the PSF is calculated by first computing \( I(x,y,z) \) over a 3D mesh of points around the reflector location and then counting the number of voxels (a voxel is a three dimensional pixel) within which the value of the PSF is within \(-6\)dB of its maximum value. Even with current computing power, the calculation of intensity over a 3D mesh of points is computationally intensive and, therefore, it is desirable for the pitch of the mesh and hence the voxel size to be as large as possible [7].

Figures 3a and 3b show the \( x-z \) plane images of the FM and AM scan respectively, from a single reflector located at a distance of \( x = 3.3\lambda \) and \( z = 10\lambda \). It can be seen by comparing Figure 3a with Figure 3b that the image resolution is increased when the FM algorithm used. The API value for the FM and AM scan was 0.95 and 1.56 respectively.

Further comparison of the FM and AM scan was carried out. Firstly, a point reflector was moved along the \( z \) axis from near to far field while the \( x \) and \( y \) positions remain constant and equal to zero. The API was calculated at each reflector point. Model data from the two imaging algorithms are shown in Figure 4(a). The FM scan always demonstrates the lowest API value for reflectors on the \( z \) axis (specially in the near field). Secondly, the point reflector was moved along an arcs of radius \( 16.6\lambda \) and \( 25\lambda \). Model data from the two imaging algorithms are shown in Figure 4(b). It again demonstrates lower API values for the FM.

6 Conclusion

This paper has described an investigation into the performance of two imaging algorithms termed the FM and AM scan for post processing ultrasonic array data. The imaging performance of the FM and AM scan algorithm has been quantified by using the API value. Simulation results indicate that the image resolution is increased when the FM scan used and always demonstrated the lower API value than the AM scan, in both cases when the point reflector was on the \( z \) axis (specially in the near field) and moved along an arcs of radius \( 16.6\lambda \) and \( 25\lambda \).

References