A Study of Attenuation and Acoustic Energy Anisotropy of Lamb Waves in Multilayered Anisotropic Media for NDT and SHM applications

Miguel A. TORRES-ARREDONDO 1,2, Henning JUNG 2, Claus-Peter FRITZEN 1,2

1 University of Siegen, Centre for Sensor Systems; Siegen, Germany
2 University of Siegen, Institute of Mechanics and Control Engineering-Mechatronics; Siegen, Germany
Phone: +49 271 7402132, Fax: +49 271 7402769; e-mail: torres, jung, fritzen@imr.mbr.uni-siegen.de

Abstract
Guided ultrasonic waves have many useful properties that can be exploited for non-destructive testing (NDT) and structural health monitoring (SHM) applications. However, for a reliable fault monitoring, much information and an improved understanding of the sources and propagation of these waves is required. The knowledge of characteristics such as attenuation, energy focusing of Lamb waves and wave velocity is then vital for the selection of optimal inspection frequencies, the optimization of sensor networks and a reliable damage localization. On that account, the present paper develops a higher order plate theory for modelling disperse solutions in viscoelastic fibre-reinforced composites in order to investigate the radiation and attenuation of Lamb waves in anisotropic media. Numerical simulations and experiments demonstrate that material anisotropy has a strong influence on the velocities, attenuation and acoustic energy for the different modes of propagation, and that they are frequency and angle dependent. Simulations with the spectral element method (SEM) will show the wave propagation in an anisotropic plate. Additionally, a comparison with experimental data validates the model. Finally, the proposed approaches offer a higher computational efficiency and simplicity in comparison to traditional methods, making them more suitable for their application in Lamb wave based damage detection systems.

Keywords: Lamb Waves, Viscoelasticity, Fibre Composite, Attenuation, Energy Focusing

1. Introduction

During the last decade many SHM approaches based on ultrasonic waves have been developed in order to detect structural defects like cracks, corrosion damage and delaminations [1]. Guided ultrasonic waves are a valuable tool in order to get information regarding the origin and importance of a discontinuity in a structure for a longer safe life and lower operation costs [2]. However, it is only possible to benefit from these advantages once the complexity of guided wave propagation are disclosed. To set up an SHM or NDT system for a real-world structure, a deep knowledge of wave propagation phenomena including effects of material damping, beam spreading effects, energy focusing and wave scattering is necessary. Thus, the understanding of dispersion characteristic is of great importance since it plays a critical role in the selection of the optimal inspection frequencies for the improvement of the sensitivity, optimization of sensor networks in terms of sensor placement and number of sensors, and for the modal analysis and localization of the propagating waves [3].

The modelling of wave propagation in multilayered anisotropic structures has been extensively studied by several researchers and a considerable amount of literature has been published on this topic [4]. Analyzing guided waves in these structures is often categorized into three methods. There are methods based on exact three dimensional elasticity, waveguide finite element methods and laminated plate theories of different orders. Exact methods are based on the superposition of bulk waves that include the popular matrix based methods [5]. Waveguide finite element methods (FEM) have appeared for modelling the guided wave propagation numerically as an alternative to exact methods [6]. However, in order to detect small damage, high frequency excitation signals are generally required owing to the fact that the size of the defects should be similar to the wavelength of the propagating waves. Therefore, a very dense finite element mesh is inevitable to accurately simulate the wave
propagation, i.e. including the effects of wave scattering from structural discontinuities, at the increase of computational cost. A more promising method is the spectral element method in the time domain that was first proposed by Patera [7]. It combines the accuracy of global pseudo-spectral methods with the flexibility of the FEM. An alternative providing also simplicity and low computational cost in comparison to other techniques are laminated plate theories. These theories expand the displacement fields in terms of the thickness to any desired degree and reduce the 3-D continuum problem to a 2-D problem [8]. This contribution presents two approaches in order to analyze the wave propagation phenomena in multilayered anisotropic and viscoelastic structures. First, the formulation of spectral elements for flat shells based on first order shear deformation (FSDT) theory is presented for the modelling and simulation of wave propagation. Second, a third order plate theory that can approximate five symmetric and six antisymmetric Lamb wave modes is proposed in order to obtain the complex dispersion relations. The motivation for expanding the displacement field up to the cubic term in the thickness is to provide better kinematics and accurate interlaminar stress distributions [9]. Moreover, the utilization of a 2D approach is justified by comparison of dispersion curves to exact 3D solutions. Additionally, the two classical models of viscoelastic attenuation are considered. The purpose of this work is to present theoretical developments, numerical and experimental results on the effects of material layup on wave propagation, Lamb wave energy focusing and attenuation, and their importance for the proper development of on-line systems. Since the use of composite materials has extensively increased in the design of existing engineering structures, what also increases the analysis complexity of such structures, this poses a necessity for fast modelling tools that can be used for a rapid and reliable analysis such as the ones presented here.

2. Spectral Elements for Flat Shells

The spectral element discretization based upon quadrangular elements is quite similar to classical FE in many points, but a few variations in construction result in significantly improved element properties. A mesh of \( n_e \) non-overlapping elements \( \Omega_e \) is defined on the domain \( \Omega \). By using an invertible local mapping these elements are subsequently mapped individually on a reference element \( \Omega^{ref} : [\xi, \eta] \subset [-1,1] \times [-1,1] \). On each element a set of Gauss-Lobatto-Legendre (GLL) nodes is defined. Within the reference element these nodes are the \((N+1)\) roots of the polynomials 

\[
(1-\xi^2)P^N_{N-1}(\xi) = 0 \quad \text{and} \quad (1-\eta^2)P^N_{N-1}(\eta) = 0
\]

where the Lobatto polynomial \( P^N_{N-1} \) denotes the first derivative of the \((N-1)\)-th order Legendre polynomial. In contrast to classical lower order finite elements, the distribution of nodes is irregular. In Figure 1 a 2D mesh with 5 and 9 nodes, respectively per element edge is depicted.

![Figure 1. Examples of nodal distribution within the reference element for 25 (left) and 81 (right) nodes](image-url)
The spectral shell element is based on the first order shear deformation theory (FSDT) with out-of-plane displacement $w$, independent rotations $\psi_x$ and $\psi_y$ and in-plane displacements $u$ and $v$ (see Figure 3).

Lagrange interpolation polynomials can be used as shape functions, leading to an expression of the displacement field in the following form:

$$
\begin{bmatrix}
\psi(x,\eta)

\psi_y(x,\eta)

u(x,\eta)

v(x,\eta)
\end{bmatrix}
= \sum_{i=1}^{N+1} \psi_i(x,\eta) \sum_{j=1}^{N+1} \psi'_j(x,\eta) \hat{q}_{ij}^{(e)} = \sum_{i=1}^{N+1} \sum_{j=1}^{N+1} \psi_i(x,\eta) \cdot \psi'_j(x,\eta) \cdot \hat{q}_{ij}^{(e)},
$$

where $\psi_i(x,\eta)$ denotes the $i$-th 1D Lagrange interpolation function. The nodal degrees of freedom are labelled with a hat and arranged in vector $\hat{q}$. An important property of these interpolation polynomials is the discrete orthogonality $\psi_i(x,\eta) \cdot \psi'_j(x,\eta) = \delta_{ij}$, where $\delta_{ij}$ denotes the Kronecker delta. Examples of selected shape functions can be seen in Figure 2.

![Figure 2. Example of three typical shape functions of a spectral element with 25 nodes](image)

Utilizing this kind of shape functions based on the GLL-nodes leads to the highest interpolation accuracy [10]. This leads to the advantage that only five to six nodes, depending on the degree of the interpolation polynomial, per shortest wavelength of the excited frequency range are necessary to capture the structural behaviour with the same accuracy as using a lower order FE with 15-30 nodes [11]. The derivation of the weak form and the assembly of the mass -, stiffness- and damping matrix $M$, $K$ and $C$ follows standard FE procedures and leads to a linear system of 2nd-order differential equations. They are given by:

$$
M \ddot{q} + C \dot{q} + K q = F.
$$

The mass matrix $M$, the stiffness Matrix $C$, the damping Matrix $C$ and the vector $F$ including the external forces are defined as follows:

$$
K^{(e)} = \int_{Gr} [B(x,y)]^T D B(x,y) \det(J) d\Omega \approx \sum_{i=1}^{N+1} \sum_{j=1}^{N+1} w_i \cdot w_j [B(x_i,y_j)]^T D B(x_i,y_j) \det(J),
$$

$$
M^{(e)} = \int_{Gr} \Psi(x,y)^T H \Psi(x,y) \det(J) d\Omega \approx \sum_{i=1}^{N+1} \sum_{j=1}^{N+1} \hat{w}_i \cdot \hat{w}_j \Psi(x_i,y_j)^T H \Psi(x_i,y_j) \det(J),
$$
\[
C^{(e)} = \int_{\Omega^e} \left[ \Psi(x, y) \right]^T C_m \Psi(x, y) \det(J) d\Omega \approx \sum_{i=1}^{N+1} \sum_{j=1}^{N+1} \tilde{w}_j \tilde{w}_i \left[ \Psi(x_i, y_j) \right]^T C_m \Psi(x_i, y_j) \det(J),
\]

where \( \tilde{w}_i \) denotes the integration weights according to the Gauss-Lobatto integration rule. Equations (3) - (5) contains the strain-displacement matrix \( B \), the material stiffness matrix \( D \), the matrix of shape functions \( \Psi \) and the matrix \( H \) with the inertia terms. A detailed explanation of the assembly of these matrixes can be found in [12]. After the assembly of all elements, this results in a linear system of 2nd-order differential equations that is very similar to a dynamic system resulting from conventional FE but with the advantageous property of diagonal mass- and damping matrices for laminates with symmetrical layup.

By using the central difference scheme, the resulting system of equations (2) can be solved very fast. Moreover, by incorporating the electromechanical coupling of piezoelectric elements as actuators and sensors into the model, it is possible to simulate an NDT/SHM-system from a given input voltage up to the resulting sensor output voltage. The coupling, that is based on the fundamental piezoelectric equations, is not discussed here. The detailed equations can be found in [13].

3. Mathematical Framework for the Plate Theory

The model considers a linearly viscoelastic, non-piezoelectric layer of material subjected to a complex stress system in three dimensions. The material is considered to have a monoclinic symmetry. Figure 3 depicts the definition of stress resultants \((N, M, Q)\) in the three dimensional system for a given propagation direction \(\alpha\) and fiber orientation \(\phi\).

![Figure 3. Complex Stress System in Three Dimensions](image)

The approximated displacement fields are given by

\[
\begin{align*}
    u &= u_0(x, y, t) + z\psi_x(x, y, t) + z^2\phi_x(x, y, t) + z^3\lambda_x(x, y, t) \\
    v &= v_0(x, y, t) + z\psi_y(x, y, t) + z^2\phi_y(x, y, t) + z^3\lambda_y(x, y, t) \\
    w &= w_0(x, y, t) + z\psi_z(x, y, t) + z^2\phi_z(x, y, t)
\end{align*}
\]

(6)

where \(u\), \(v\), and \(w\) are the displacement components in \(x\), \(y\) and \(z\) directions, \(\psi_x\) and \(\psi_y\) represent rotations having the same meaning as in the first order shear deformation theory.
The additional terms expand the displacement field. The strain energy of each layer can be represented as

$$U = \frac{1}{2} \int \left( C_{11} e_x^2 + 2 C_{12} e_x e_y + 2 C_{13} e_x e_z + 2 C_{16} e_x \gamma_{xy} + C_{22} e_y^2 + 2 C_{23} e_y e_z + 2 C_{26} e_y \gamma_{xy} + C_{33} e_z^2 + 2 C_{36} e_z \gamma_{xy} + C_{66} \gamma_{xy}^2 + C_{44} \gamma_{yz}^2 + 2 C_{45} \gamma_{yz} \gamma_{xz} + C_{55} \gamma_{xz}^2 \right) dV. \quad (7)$$

Additionally,

$$\sigma_x = \frac{\partial U}{\partial e_x}, \ldots, \tau_{xy} = \frac{\partial U}{\partial \gamma_{xy}}. \quad (8)$$

If the global coordinate system does not coincide with the material coordinate system, but is rotated by an angle $\phi$ around the $z$ axis, a coordinate transformation of the elastic stiffness matrix is required so that the axes of the anisotropic medium coincide with the chosen global coordinate axes [15]. Viscoelastic layers can be simulated by allowing the stiffness matrix to be complex.

The plate constitutive equations may be derived from the strain energy density in the 3-D elasticity theory and the linear elastic stress-strain and strain-displacement relations. The equations of motion may be derived from the dynamic version of the principle of virtual displacements. Consequently, the system can be expressed in a matrix form, and by imposing boundary conditions and setting its determinant to zero, a characteristic function relating the angular frequency to the wavenumber is obtained. For the case of symmetric laminates, the system of equations can be decoupled into two independent systems of equations for the symmetric and antisymmetric modes of propagation. A complete description of the numerical strategy for the tracing of the dispersion solutions and the complete analytical expressions are presented by the authors in [16-17].

In Figure 4 the velocities of the different modes of propagation are plotted over the frequency for the 1.5mm thick glass fibre reinforced plastic (GRFP) plate with properties listed in Table 1 for $\alpha = 30^\circ$. Curves depicted in continuous lines are calculated using exact three dimensional theory and dashed lines using the proposed third order plate theory. For a frequency up to 800 kHz, what corresponds to a frequency-thickness product of 1.2 MHz mm, the error in comparison to group velocity of $S_{0y}$-mode is below 3%. Just as well is the conformity of the out-of-plane mode $A_{0y}$-mode.

Figure 4. Comparison between the exact elasticity theory and the third order plate theory; Phase Velocity (left) and Group Velocity (right)
4. Material Damping Models for Wave Propagation

The mathematical modelling of viscoelasticity using ideas from elasticity has attracted the attention of a large number of investigators over the past century. In order to account for material damping, the stiffness matrix is represented by a complex quantity. The real part $C$ of this complex term relates to the elastic behaviour of the material and defines the stiffness [18]. The imaginary component $\eta$ relates to the material viscous behaviour and defines the energy dissipative ability of the material. Two models are often used to describe viscoelastic behaviour. The first model is called the hysteretic model whose complex stiffness matrix is given by

$$\tilde{C} = C + i\eta.$$ \hspace{1cm} (9)

The hysteretic model assumes no frequency dependence of the viscoelastic constants. The second model is the Kelvin–Voigt model and assumes a linear dependence of the viscoelastic coefficients. The complex stiffness matrix is expressed as

$$\tilde{C} = C + i\frac{\omega}{\omega_0} \eta$$ \hspace{1cm} (10)

where $\omega$ is the angular frequency and $\omega_0$ is the frequency of characterization. The influence in the attenuation predicted by both methods for the fundamental modes of propagation for a 3.6mm thick unidirectional carbon-epoxy plate is depicted in Figure 5. The characterization frequency is 2MHz.

![Figure 5. Comparison of attenuation between the hysteretic and Kelvin-Voigt models as a function of frequency](image)

From the previous picture it can be clearly seen that the attenuation is a linear function of the frequency in the case of the hysteretic model and a quadratic function of the frequency in the case of the Kelvin–Voigt model. Additionally, both models are just coincident in the frequency of characterization and away from this point, the deviation in the prediction of both models is noteworthy. The hysteretic model is used for the analysis presented in the next sections. This is motivated by the fact that this model is more frequently used in the NDT and SHM literature. However, it is worth mentioning that some authors choose the attenuation model depending on the nature of the mode [19].
5. Energy Focusing of Lamb waves

Although the problem of wave propagation in multilayered composite materials has been addressed relatively well in the literature, few studies of the focusing effect of Lamb waves exist in the SHM and NDT scientific literature [20]. It seems to be related with the fact that this effect is hardly encountered experimentally due to the very low amplitude of the recorded modes having this behaviour. Additionally, measurements, which are normally done at the structure surface, are more sensible to the out of plane particle motion modes which only present this behaviour at fast-attenuative high frequencies. It is worth to note that the direction of phase (α) and group velocity (ϑ) is not general the same in anisotropic media. The energy focusing effect can be very strong in some anisotropic materials when the group velocity direction remains the same, while the phase velocity direction varies [21]. This phenomenon of energy focusing, analogue to the phonon focusing effect, in which the energy flux is much more concentrated in some directions can be analyzed by the focusing factor [22]

\[ A(\vartheta) = \left| \frac{d\vartheta}{d\alpha} \right|^{-1} = \left[ s^2 + \left( \frac{ds}{d\alpha} \right)^2 \right]^{-\frac{1}{2}} |K_s|^{-1} \]  

(11)

where \( s \) is the slowness surface and \( K_s \) its curvature. The focusing is capable of rendering a superposition of waves into a flux pattern containing caustics, i.e. points of zero curvature correspond to caustics in the acoustic intensity.

6. Results

The proposed viscoelastic plate theory formulation and SEM are applied to several examples including an anisotropic elastic plate and two multilayered anisotropic viscoelastic carbon fibre reinforced plastic (CFRP) plates for the analysis of the effects of material anisotropy and material layup on wave propagation, Lamb wave energy focusing and attenuation.

6.1 Elastic Glass Fibre Reinforced Plastic Plate

In order to validate the modelling approach, a case study has been conducted on a unidirectional glass-fibre reinforced plastic (GFRP) plate. A single-layered specimen was selected because of its highly anisotropic character. The elastic material properties provided by the manufacturer are given in Table 1. The fibres are oriented in y direction. Figure 6(a) shows the structure that has the dimensions 800mm×800mm and a thickness of approximately 1.5mm. Nine piezoelectric transducers are attached to the surface of the structure.

<table>
<thead>
<tr>
<th>( E_1 ) (GPa)</th>
<th>( E_2 ) (GPa)</th>
<th>( E_3 ) (GPa)</th>
<th>( G_{12} ) (GPa)</th>
<th>( G_{13} ) (GPa)</th>
<th>( G_{23} ) (GPa)</th>
<th>( v_{12}^{\text{re}} )</th>
<th>( v_{13}^{\text{re}} )</th>
<th>( v_{23} )</th>
<th>( \rho ) (kg/m³)</th>
</tr>
</thead>
<tbody>
<tr>
<td>30.7</td>
<td>15.2</td>
<td>10</td>
<td>4</td>
<td>3.1</td>
<td>2.75</td>
<td>0.3</td>
<td>1700</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

The experimental group velocities were determined in the defined frequency by means of time-delay measurements. Numerical results for the group velocities for the fundamental modes of propagation at a central frequency of 200kHz are depicted in Figure 6(b). The wave surface for the \( S_0 \) mode at 200kHz is shown in comparison with some measured values at discrete angular points (magenta circles) in order to validate the analytical model with experimental data. It can be seen that the estimated group velocity matches the theoretical
curve very well. The proposed spectral element approach in combination with the electro-mechanical coupling of piezo patches is demonstrated at a simulation example in Figures 6(c) and (d). The structure is excited by a piezoelectric patch located in the middle of the structure. The excitation voltage signal is a Hann-windowed toneburst with a carrier frequency of 200kHz with 5 cycles. In Figure 6(b) and (c), the caustics of the SH\(_0\) mode indicate the energy focusing in these directions for this wave mode. It can be also seen how the energy of the S\(_0\) and A\(_0\) modes is highly concentrated in the fibre direction.

Figure 6. Propagation Modes: (a) Experimental Setup, (b) Wave Curve for the 1.5\(mm\) thick GFRP in vacuum, (c) SEM Snapshot of In-Plane Motion after 0.08\(ms\) and (d) SEM Snapshot of Out-of-Plane Motion after 0.29\(ms\)

6.2 Carbon Fibre Reinforced Plastic with Unidirectional Woven Fabric Reinforcement

A viscoelastic unidirectional lamina of 5.1\(mm\) of thickness, a density of 1500kg/m\(^3\) and a [0\(°\)]\(_{18}\) layup is analyzed in this section. The elastic and viscoelastic material properties are provided in Table 2. The structure is excited by a piezoelectric patch located in the middle of the structure using a Hann-windowed toneburst with a carrier frequency of 95kHz with 5 cycles.

Table 2. Material properties of unidirectional CFRP 5.1\(mm\) thick plate (units in GPa)

<table>
<thead>
<tr>
<th>(C_{ij})</th>
<th>(\eta_{ij})</th>
</tr>
</thead>
<tbody>
<tr>
<td>125</td>
<td>3</td>
</tr>
<tr>
<td>6.3</td>
<td>0.9</td>
</tr>
<tr>
<td>5.4</td>
<td>0.4</td>
</tr>
<tr>
<td>13.9</td>
<td>0.6</td>
</tr>
<tr>
<td>7.1</td>
<td>0.23</td>
</tr>
<tr>
<td>14.5</td>
<td>0.6</td>
</tr>
<tr>
<td>3.7</td>
<td>0.12</td>
</tr>
<tr>
<td>5.4</td>
<td>0.3</td>
</tr>
<tr>
<td>5.4</td>
<td>0.5</td>
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</tbody>
</table>
Results for this material are presented in Figure 7. Following a similar behaviour as for the previous case, it can be seen from Figures 7(a) to (d) how the fibres act as a guide for the energy in their direction and as scatterers in the perpendicular direction for the $A_0$ and $S_0$ mode of propagation. The attenuation is also affected a great deal by the anisotropy of the material (see Figure 7(e)). This is a common characteristic of anisotropic materials where the velocity, attenuation and energy of propagation of the multiple modes are both frequency and angle dependent. This dependence plays a major role in the complexity of the mode shapes of propagating modes, what also affects their detectability. Figure 7(f) shows the capabilities of the phonon focusing factor to precisely track the angular dependent energy concentration effect. The cuspidal regions in the $SH_0$ mode explains the energy patterns containing caustics.

Figure 7. Modes at 95kHz: (a) Velocity Curve, (b) Wave Curve, (c) Snapshot of In-Plane Motion after 0.058ms, (d) Snapshot of Out-of-Plane Motion after 0.158ms, (e) Attenuation Curve and (f) Focusing Curve
6.3 Carbon Fibre Reinforced Plastic with Non Crimp Fabric Reinforcement

A viscoelastic orthotropic multilayered plate of 4.7\textit{mm} thickness and a density of 1500\textit{kg/m}^3 is analyzed in this section. Numerical results analyzing the effects of material layup on wave propagation, Lamb wave energy focusing and attenuation are presented in Figure 8. The elastic and viscoelastic material properties of the plate analyzed here are listed in Table 3.

![Figure 8. Modes at 95kHz: (a) Velocity Curve, (b) Wave Curve, (c) Snapshot of In-Plane Motion after 0.045ms, (d) Snapshot of Out-of-Plane Motion after 0.113ms, (e) Attenuation Curve and (f) Focusing Curve](image)

Table 3. Material properties of orthotropic CFRP 4.7\textit{mm} thick plate (units in GPa)

<table>
<thead>
<tr>
<th>C_{11}</th>
<th>C_{12}</th>
<th>C_{13}</th>
<th>C_{22}</th>
<th>C_{23}</th>
<th>C_{33}</th>
<th>C_{44}</th>
<th>C_{45}</th>
<th>C_{66}</th>
</tr>
</thead>
<tbody>
<tr>
<td>70</td>
<td>23.9</td>
<td>6.2</td>
<td>33</td>
<td>6.8</td>
<td>14.7</td>
<td>4.2</td>
<td>4.7</td>
<td>21.9</td>
</tr>
<tr>
<td>\eta_{11}</td>
<td>\eta_{12}</td>
<td>\eta_{13}</td>
<td>\eta_{22}</td>
<td>\eta_{23}</td>
<td>\eta_{33}</td>
<td>\eta_{44}</td>
<td>\eta_{55}</td>
<td>\eta_{66}</td>
</tr>
<tr>
<td>1.8</td>
<td>0.9</td>
<td>0.3</td>
<td>1.4</td>
<td>0.2</td>
<td>0.5</td>
<td>0.17</td>
<td>0.2</td>
<td>0.5</td>
</tr>
</tbody>
</table>
This viscoelastic CFRP plate is made of 18 equal layers resulting in a total thickness of 4.7 mm with a stacking of \([-45^\circ \ 0^\circ \ 45^\circ \ 0^\circ \ -45^\circ \ -45^\circ \ 0^\circ \ 45^\circ\]s. In a similar fashion as before, the structure is excited by a piezoelectric circular patch located in the middle of the structure using a Hann-windowed toneburst with a carrier frequency of 95 kHz with 5 cycles. The piezo patch has a diameter of 10 mm and a thickness of 0.25 mm. The plate is meshed with 80×80 spectral shell elements using 36 nodes per element. From Figures 8(a) to (e) it can be seen that the mode wave velocities and attenuation are not strongly related to the frequency and orientation of propagation. This is given by the fact that the multilayered and multi-oriented composition of the structure mitigates the anisotropic impact of each layer. For the A_0 and S_0 fundamental modes of propagation the focusing factor is highest in the direction \(\alpha = 0^\circ\); however, a second maximum also occurs at \(\alpha = 90^\circ\). Although not depicted here, the focusing effect becomes more pronounced as one moves along the frequency axis to higher frequencies for the fundamental symmetric modes. This effect is less significant for the fundamental antisymmetric mode. In this example, the caustics present before in the SH_0 mode are not any more present. However, the cuspidal regions of the SH_0 in the focusing curve indicate the energy concentration in the four quadrants at approximately \(\alpha = 48^\circ, 138^\circ, 228^\circ\) and \(318^\circ\).

5. Discussion and Conclusions

In this paper two different approaches are proposed as useful tools for the investigation and design of wave propagation based systems established upon modal analysis. First, an approach for the modelling of guided wave-based SHM and NDT systems for thin shells has been presented. Second, a coupling between viscoelasticity theory and a laminated plate theory has been suggested which is applicable to viscoelastic fibre reinforced composite materials for the calculation of wave velocities, attenuation and energy focusing for the different modes of propagation. Comparisons to experimental data have been presented in order to validate the models. It was shown that the proposed methods provided accurate estimates of velocity and attenuation in anisotropic laminates in the frequency range of Lamb wave applications. Furthermore, the focusing of Lamb waves has been numerically depicted and its importance brought into consideration for the development of the afore mentioned systems. From numerical calculations it was illustrated that this phenomenon rendered a superposition of waves into a flux pattern containing caustics.

Nevertheless, it is also worth mentioning that the models suffer from some limitations that prevent them from being used to solve the whole spectrum of composite laminate problems, i.e. for high thickness-frequency products. It is well known from literature that higher order theories accuracy deteriorates as the laminate becomes thicker; in our case an error below 3% in comparison to the exact 3D elasticity theory was obtained for a frequency-thickness product of 1.2 MHzmm. The proposed spectral element modelling approach in combination with the electromechanical coupling of piezo patches was also demonstrated by three simulation examples in composite materials and then compared and validated with the developed higher order plate theory. To the authors opinion, the developed methodologies allowed to simulate wave propagation phenomena with much higher efficiency compared to classical methods while maintaining good accuracy in the results. They are intended to shorten time consumption and cost of setting up wave propagation based systems. Additionally, it has been demonstrated that the knowledge of factors like attenuation, wave velocity and energy focusing of Lamb waves provides a better understanding of the wave propagation phenomena and allows to analyse in depth the influence of important parameters like actuator/sensor positions and to improve the probability of detection in case of passive monitoring methods.
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References


