NUMERICAL COMPUTATIONS LEADING TO LOCALIZATION OF ACOUSTIC EMISSION USING GEODESIC CURVES

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Abstract
We deal with the exact model of localization of acoustic emission (AE) sources on real complex solid bodies. Our approach is based on numerical computations of precise geodesic curves on 3D vessels which can be composed from several parametrized surfaces with multiple intersections. Instead of solving the geodesic differential equations under specific Christoffel symbols, we compute our geodesics numerically by Finite Difference, Newton–Raphson, and Fixed Point Iteration methods. For faster calculations we propose technical improvements and optimizations. We also consider the case when the surface contains the holes, i.e. the exact geodesic curve has to bypass a given obstacles. These techniques are used in real experiment on the galvanized steel watering can that represents a vessel with higher geometrical complexity. The layout of piezoelectric acoustic emission sensors supposes that the main part of the body can be inaccessible due to high temperature or radioactivity, such as in the case of nuclear power station health monitoring. Finally, we present the results of AE localization principle using length (or time) differences measured by means of geodesics found on the watering can while minimizing its distinctions from the real time differences measured.

Keywords: Acoustic emission, non-destructive defectoscopy, geodesic curves, parametrized surfaces, $\Delta t$ – exact localization

1 Acoustic emission and $\Delta t$ localization

Acoustic emission is a physical phenomenon and a diagnostic method [1]. By a phenomenon of AE there are denoted the elastic waves generated by a sudden release of mechanical tension inside the body or other processes causing tension surface waves in a material structure under consideration, for example a crack of withered branch or a knock on the surface. AE method is a way of detection and monitoring the AE phenomenon or other similar processes, e.g. leaks of an agent from pressured container. A set of cycles called hits is the result of an event of AE, i.e. the dynamical impact initiating AE. If the hits can be distinguished in time-line then we deal with cracking AE. On the other hand, if the acoustic waves are overlapping and cannot be distinguished, we are dealing with continuous AE.

In this work we localize the sources of AE on a curved surfaces. For the purpose of localization via geodesics, i.e. the shortest possible paths between two points, we make use of $\Delta t$ localization method based on knowledge of time differences between arrivals of signals to each sensor asymmetrically located on the surfaces.
For the three sensors in a plane (Fig. 1a), let us denote time difference between arrivals by $\Delta t_{12} = t_1 - t_2$, $\Delta t_{23} = t_2 - t_3$, and $\Delta t_{31} = t_3 - t_1$, where $t_1, t_2, t_3$ are time arrivals to the three sensors, respectively. Under uniform wave propagation velocity $c$, we get the corresponding length differences $c\Delta t_{12} = l_1 - l_2$, $c\Delta t_{23} = l_2 - l_3$, and $c\Delta t_{31} = l_3 - l_1$, which define hyperbolas whose intersection determine the point of AE source (Fig. 1b). Note that for the reasonable uniqueness we need at least three sensors. This concept can be generalized for curved surfaces, thus we need to obtain the shortest path between any two points on a general surface.

2 Geodesics and numerical computations

Briefly, geodesic is the curve between two points which has the shortest length of all curves connecting these two points. More generally, it is the curve whose geodesic curvature $k_g$ is zero in all its points [3]. A geodesic curvature $k_g$ is a magnitude of a projection of normal vector $n$ to the tangent plane of the surface $S$ in a point $P$ (see Fig. 2a).

Note that such a curve with zero curvature $k_g$ in all its points does not need to be the shortest possible due to parametric domains, see later in Section 3. In this paper, geodesic curve on a surface is obtained by solving the system of two non-linear ordinary differential equations of second order [3]

$$\ddot{u} + \Gamma_{11}^1 \dot{u}^2 + 2\Gamma_{12}^1 \dot{u} \dot{v} + \Gamma_{22}^1 \dot{v}^2 = 0,$$

$$\ddot{v} + \Gamma_{11}^2 \dot{u}^2 + 2\Gamma_{12}^2 \dot{u} \dot{v} + \Gamma_{22}^2 \dot{v}^2 = 0,$$

(1)
where $u, v$ are coordinates in parametric domain of chosen parametrization $X$ of surface $S$ and $\Gamma^k_{ij}(u,v)$ are so called Christoffel’s symbols which represent metric topology of the surface and they can be obtained from $X$. Instead of solving (1) analytically, we compute geodesics by means of finite difference method.

**Finite difference method** replaces the derivatives in (1) by the corresponding differences and, after arrangements, we get the final system of $N$ nonlinear equations

\[
\frac{\omega_{1}^{k+1} - 2\omega_{1}^{k} + \omega_{1}^{k-1}}{h^2} + \frac{1}{4h^2} \sum_{i,j=1}^{2} \Gamma^1_{ij}(\omega_{1}^{k}, \omega_{2}^{k}) \Delta \omega_{i}^{k} \Delta \omega_{j}^{k} = 0,
\]

\[
\frac{\omega_{2}^{k+1} - 2\omega_{2}^{k} + \omega_{2}^{k-1}}{h^2} + \frac{1}{4h^2} \sum_{i,j=1}^{2} \Gamma^2_{ij}(\omega_{1}^{k}, \omega_{2}^{k}) \Delta \omega_{i}^{k} \Delta \omega_{j}^{k} = 0,
\]

where $h = 1/N$, $\omega_1 := u$, $\omega_2 := v$ and $\Delta \omega_i^k := \omega_i^{k+1} - \omega_i^{k-1}$ for $k = 1, ..., N - 1$. Integer $N$ denotes the number of points in meshgrid. This system is solved by following iterative methods [7].

**Fixed point iteration method** procedure is used in general iterative form

\[
\mathbf{u}_{n+1} = \overrightarrow{\varphi}(\mathbf{u}_n, \mathbf{v}_n), \quad \mathbf{v}_{n+1} = \overrightarrow{\psi}(\mathbf{u}_n, \mathbf{v}_n),
\]

where $\mathbf{u}_n = (u_1, ..., u_N)$, $\mathbf{v}_n = (v_1, ..., v_N)$.

**Newton method** is consecutively applied through

\[
dF(x_n) \cdot \Delta x_n = -F(x_n),
\]

\[
x_{n+1} = x_n + \Delta x_n,
\]

where $x_n = (u_1, ..., u_N, v_1, ..., v_N)$ for every iteration index $n$.

### 3 Computations on compound bodies

Since we are dealing with AE source localization on real bodies we need to decompose them to elementary surfaces so that we can describe them parametrically and easily obtain its Christoffel’s symbols. Now, specific problem arises how to compute the geodesics on such a fragmentized body. We compute geodesics on each of the surfaces between start/end points by using one “connection point” lying at the intersection of two surface sections (Fig. 3). Then we iterate through all the points of intersection and minimize the sum of lengths of sectional geodesics.

![Figure 3. Finding minimum of sum of lengths through intersections](image-url)
To fasten the computation through intersections, the iterative *Sequential algorithm* (SA) was developed [6]. We choose a number of points through which SA will iterate in every step, let’s say three as in Fig. 4a. Then we compute lengths of geodesics through these points and choose two points in which geodesics have the minimal length, i.e. the points 1 and 2 in Fig. 4a. Finally, the (shorter) interval between these two points is split uniformly with the new point 3’ and the next step considers 1, 2, 3’ to be new initial points. This procedure can be easily generalized to more then three points, see Fig. 4b.

![Figure 4. a) SA with three points b) SA with four points](image)

As we mentioned earlier, the curves with zero geodesic curvature $k_g$ in all points need not be the shortest. This is caused by the choice of parametric domain. In practical applications, we work with solid vessels whose elementary parts are closed rotary surfaces. It means that at least one of coordinates $u, v$ of parametrization domain takes values in $(0, 2\pi)$ or $(-\pi, +\pi)$. These intervals are mathematically equivalent apart from a discontinuity at $2\pi \leftrightarrow 0$ or $\pi \leftrightarrow -\pi$. Since geodesics computed from (1) cannot „go“ through this discontinuity we do not sometimes get the shortest possible curve, see the curves in Fig. 5.

![Figure 5. Different geodesics based on different parametric domains](image)
The shortest curves in Fig. 5 were obtained by changing the domain interval from \((-\pi, +\pi)\) to \((0, 2\pi)\). This problem is evidenced in Fig. 6. A simple criterion for changing the domain intervals (i.e. the discontinuity position) is to check if the arc length of geodesic in parametric domain is more than \(\pi\). Unfortunately, this criterion cannot be used generally, it works well in cases of regular surfaces such as planes, cylinders, cones, paraboloids, spheres, or surfaces as in Fig. 7.

Another problem we deal with is that some topologically complex surfaces, e.g. as in Fig. 7, have very complicated Christoffel’s symbols \(\Gamma^k_{ij}\) and it may result in failures of Newton method. Moreover, we choose the straight line between two points in parametric domain as the initial guess for the root of NM iterative method. In order to improve the convergence of NM we make our initial guess more accurate applying the FPI method.

### 4 Geodesics bypassing obstacles

In order to get more precise localization of AE sources, we have developed a method, which can be used to compute geodesics highly respecting topology of the surface. In the Figure 8 we can see a geodesic passing through the hole (intersection with the spout) in the container of watering can. Physically it is impossible that an acoustic signal could propagate this way. If we were able to compute geodesics bypassing the obstacle we should obtain better results in the localization.
We propose the following method:

1. Firstly we approximate the hole in the parametric domain with a polygon, compute geodesic and check if it is passing through the hole (see Fig. 8 right). We are dealing with finite number of points, it is sufficient to check some of the points of the geodesic individually with the *winding number* (WN) algorithm [9].

2. Then the intersections of the geodesic and the border is found, i.d. last points of the geodesic computed respectively from A and B, which are not lying in the polygon (points $p_1, p_2$ in the Figure 8). For these points $p_1, p_2$ we find the closest points on the polygon, denote them $q_1, q_2$.

3. Now we compute geodesics from the point A to the points on the polygon counter clock-wise starting at $q_1$, until any of these geodesics lies in the polygon. The last point on the polygon for which the geodesic is not lying in the polygon we denote $q_{1_{max}}$. The same is done clock-wisely from $q_1$ and denote the last point $q_{1_{min}}$ (see Fig. 9). The same procedure for the point B results in points $q_{2_{max}}$ and $q_{2_{min}}$.

4. Finally we obtain sets of points on polygon $M_A, M_B$ (Fig. 9 right), through which we minimize the sum of lengths of geodesics by means of methods described in section 3. The reason for introduction of $q_1, q_2$ is to fasten the computation - we do need not to check all of the points on polygon to determine sets $M_A, M_B$ thus we can avoid of computation of many redundant geodesics.

Figure 8. Geodesic (black) passing through the hole denoted by red intersection curve (left). The same scene in the parametric domain (right)

Figure 9. Illustration of the computation of geodesics along the hole
Furthermore we have to get rid of the geodesics which are in the “forbidden” region, namely we have to dispose of the yellow geodesic in the Figure 10 - in the real watering can, there is not any part of spout in the container. From the remaining allowed geodesics we choose the shortest. In this case it is the violet one.

By comparison of Figures 10 and 8 it is evident that we should get more realistic path of the propagating signal. A drawback of this optimalization is that it is much more computationally intensive.

5 Localization of the sources of AE

The goal is to find a point on the surface where the computed time (length) differences are identical to the measured time differences. To localize source of AE means to find a minimum of a certain functional $F$ which can be define by two possible approaches, which are mathematically equivalent but not equivalent practically. Suppose we have measured time differences $\Delta t_{ij}$ among sensors. From the tested point $p = (u, v) \in U$ we compute lengths of geodesics $l_{pi}$, $i = 1, ..., s$ where $s$ denotes number of sensors in action.

Comparing time differences
Because the geodesic is composed from the geodesics on each of the part of the body, it holds $l_{pi} = l_{p1} + l_{p2} + ... + l_{pm}$, where $m$ denotes number of parts where the whole geodesic lies. From the known propagating speed of acoustic signal on each of the components $c_j$, $j = 1, ..., m$ we get $t_{pi} = \frac{l_{p1}}{c_1} + \frac{l_{p2}}{c_2} + ... + \frac{l_{pm}}{c_m}$. Finally time differences $\Delta T_{ij}$ are obtained. Then the functional $F$ has form

$$F(p) = \sum_{i<j} |\Delta t_{ij} - \Delta T_{ij}|.$$  

Comparing length differences
Multiplying time differences $\Delta t_{ij}$ by known velocities $c_{ij}$ between each of of components (velocities through intersections) we get measured length differences $c_{ij}\Delta t_{ij}$. From the tested point $p$ we obtain lengths of geodesics $l_{pi}$ and consequently length differences $\Delta l_{ij}$. Then the functional $F$ has form

$$F(p) = \sum_{i<j} |c_{ij}\Delta t_{ij} - \Delta l_{ij}|.$$  

Velocities $c_{ij}$ and $c_i$ may be very different in practice so the values of $F$ differ which means we get quite different localization of the source.
6 Compass optimization algorithm

In order to localize source of AE we seek for the point on the surface, where the functional $F$ from previous section attains its minimal value. Instead of testing every potential point on the given surface, we use the iterative method called *Compass algorithm* [5].

1. choose an initial (basis) point $(u_B^{(0)}, v_B^{(0)})$ in the parametric domain and then also initial step $(h_u^{(0)}, h_v^{(0)})$, 

2. for $k = 0, 1, ...$

   • compute values $F^{(k)}(u_B^{(k)}, v_B^{(k)})$, $F^{(k)}(u_B^{(k)} \pm h_u^{(k)}, v_B^{(k)})$, $F^{(k)}(u_B^{(k)}, v_B^{(k)} \pm h_v^{(k)})$, $F^{(k)}(u_B^{(k)} \pm h_u^{(k)}, v_B^{(k)} \pm h_v^{(k)})$, 

   • pick up that point, where $F$ attains its minimum and set it as new basis point $(u_B^{(k+1)}, v_B^{(k+1)})$, 

   • if $(u_B^{(k+1)}, v_B^{(k+1)}) \equiv (u_B^{(k)}, v_B^{(k)})$ then set $h^{(k+1)} = h^{(k)}/2$, else change step to $h^{(k+1)} = h^{(k)}F^{(k)}(u_B^{(k+1)}, v_B^{(k+1)})/F^{(k)}(u_B^{(k)}, v_B^{(k)})$

The application of Compass algorithm on the main body of real galvanized watering can (Fig. 11a) is illustrated in Fig. 11b, where the successive reduction of scanning area is visible.

![Figure 11. a) Experimental vessel with AE sensors  b) Compass algorithm convergence](image)

7 Galvanized watering can experiment

The numerical algorithms and optimization methods are used for the real galvanized watering can with complex geometry. The placement of 4 piezoceramic acoustic emission sensors is provided in Fig. 11a. The experimental vessel is the composition of five sub-bodies, thus we have to deal with several surfaces and its intersections. Propagating velocities of acoustic signals providing different types of functional $F$ are stated in Tables 1 and 2.
Table 1. Average velocities on each component of the watering can

<table>
<thead>
<tr>
<th></th>
<th>Container</th>
<th>Bottom</th>
<th>Spout</th>
<th>Handle</th>
</tr>
</thead>
<tbody>
<tr>
<td>Velocity</td>
<td>5374.5 m/s</td>
<td>5236.8 m/s</td>
<td>5373.5 m/s</td>
<td>5349.8 m/s</td>
</tr>
</tbody>
</table>

Table 2. Average velocities between each component of the watering can. Numbers 1,2,3,4 refers to position of sensors in Fig. 11b (sensor 3 is on the bottom)

<table>
<thead>
<tr>
<th></th>
<th>1-2</th>
<th>1-3</th>
<th>1-4</th>
<th>2-3</th>
<th>2-4</th>
<th>3-4</th>
</tr>
</thead>
<tbody>
<tr>
<td>Velocity</td>
<td>5263.8 m/s</td>
<td>4678.1 m/s</td>
<td>4766.7 m/s</td>
<td>5028 m/s</td>
<td>4802 m/s</td>
<td>4353.4 m/s</td>
</tr>
</tbody>
</table>

The classical repeated pentests in the three different positions on the main cone-shaped surface of watering can were carried out and then we localized the pentests’ sources by our exact geodetics based iterative methods. The Fig. 14 presents the final results, i.e. three localization maps for 3 different acoustic emission excitation areas. The localization clusters achieved are quite well concentrated and the deviations from the true AE pentest excitation sources are up to 0.5cm in most cases. This satisfactory result indicates very promising concept of proposed geodetic AE sources localization in the case of topologically complicated material structures.

Figure 14. Pentests on watering can and its individual localizations (the sensor positions are denoted by black dots)
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Reference


