

Using of non-stationary method for determination of thermal conductivity in a building structure

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Abstract

Non-stationary measuring equipment means a progress in methods of simple, reliable and quick determination of heat conductivity of building materials. Sometimes, it is not possible to use ordinary methods (stationary methods) for determination of thermal conductivity of some non-homogenous building components - for example in masonry ceramic construction or in building materials with moisture content. This paper describes a new method for determination of building materials heat conductivity coefficient including all involved procedures and evaluates the advantages connected with the use of this method.

1. Determination of the heat conduction coefficient by the hot wire method

The testing procedure of the heat conduction coefficient determination by the heat wire method is specified in the Czech Republic by the standards: CSN EN 993-14 a CSN EN 993-15. The measurement is carried out in the testing system with cross or parallel arrangement. The mostly used method for the determination of common building materials heat conduction coefficient is the modified method with the cross arrangement.

The suitable test pieces for the measurement have the minimum dimensions 40 x 100 mm and the minimum thickness about 20 mm. The surface area of test pieces which are in mutual contact should be ground so that the flatness difference of two points must not be greater than 0.2 mm. The circuit of the heating wire is during the measurement connected with the electrical source which is set up to secure the increase of the heating wire temperature with the maximum of 100°C in 15 minutes. The recording of the heating wire temperature starts at this moment. The measurement is finished after 10 ~ 15 minutes.

It is further necessary to wait till the stabilization of the temperature and the measurement is twice repeated. Three values of temperature rise rate of the heating wire are determined in this way under identical conditions. The scheme of the measuring device construction is presented in the following picture.

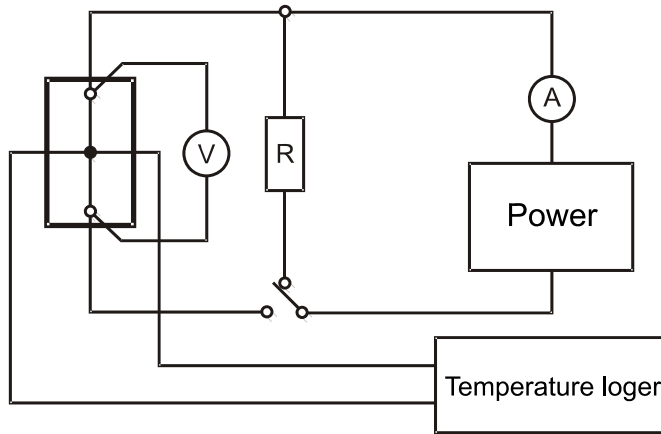


Figure 1. The arrangement of the measuring device following the standard CSN EN 993-14

The value of the heat conduction coefficient λ is calculated following the equation:

$$\lambda = \frac{I^2 \cdot R_m}{4 \cdot \pi} \cdot \frac{\ln\left(\frac{\tau_2}{\tau_1}\right)}{\Delta\theta_2 - \Delta\theta_1} \quad (1)$$

eventually

$$\lambda = \frac{I \cdot U_m}{4 \cdot \pi} \cdot \frac{\ln\left(\frac{\tau_2}{\tau_1}\right)}{\Delta\theta_2 - \Delta\theta_1} \quad (2)$$

where

I heating current [A],

U_m voltage drop on one length unit of the heating wire [$V \cdot m^{-1}$],

R_m electrical resistance on one heating wire length unit [$\Omega \cdot m^{-1}$],

τ_1, τ_2 times from closing the heating circuit [min.],

$\Delta\theta_1, \Delta\theta_2$ heating wire temperature rise after closing the circuit in times τ_1, τ_2 [$^{\circ}C$].

The hot wire method was used for the determination of the heat conduction. This method is the modification of the above mentioned method (cross arrangement) following the standard CSN EN 993-14. [3]

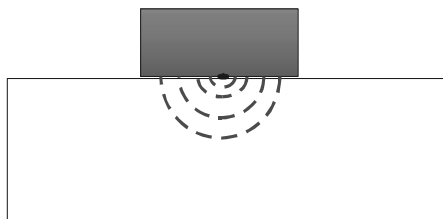


Figure 2. Determination of the heat conduction coefficient by the hot wire method

The thermal function of the line impulse source situated in the z axis of the coordinate system (cylindrical coordinates) has the following form [2]:

$$\theta(r, \tau) = \frac{Q}{4c \rho \pi a \tau} \exp\left(-\frac{r^2}{4 a \tau}\right)$$

where:

- Q total quantity of heat supplied by the length of the line temperature source to the test piece[J],
 r the distance from the line impulse source (normal to the axis) [m].

One half-space which constituted by the sample was substituted by the test probe in the case of this method. Because the properties of the test probe are during all measurements constant, these devices can be calibrated by two reference materials with the known heat conduction value λ_1 and λ_2 , whereas the value x represents the immediate regular speed of heating.

$$b = \frac{\lambda_2 - \lambda_1}{x_2 - x_1} \quad (3)$$

$$a = \lambda_1 - b \cdot x_1$$

The regular speed of heating x is determined from the measured course of temperature at the interface of the test probe and the sample in time dependence (logarithmical) by means of the least squares method:

$$x = a + b \cdot U^2 \frac{\left((j_2 - j_1 + 1) \cdot \sum_{j=j_1}^{j_2} \ln(\Delta \tau_j)^2 - \left(\sum_{j=j_1}^{j_2} \ln(\Delta \tau_j) \right)^2 \right)}{(j_2 - j_1 + 1) \cdot \sum_{j=j_1}^{j_2} \ln(\Delta \tau_j) \theta_{t,j} - \sum_{j=j_1}^{j_2} \ln(\Delta \tau_j) \sum_{j=j_1}^{j_2} \theta_{t,j}} \quad (4)$$

where:

$j_1 \dots j_2$, $j_1 < j_2$ are the limit indexes of the evaluated measuring points interval [-].

The value of the tested material heat conduction coefficient λ was determined by means of calibration constants following the equation:

$$\lambda = a + b \cdot x \quad (5)$$

2. The comparison of the hot wire method and the stationary plate method

Twenty test pieces were used for the determination of the heat conduction coefficient. These samples came from four different brick plants in the Czech Republic. The test pieces of classical burnt bricks had platy shape of non uniform dimensions. Before the measurement they were adapted to the dimension 300 x 300 [mm] in order to put them into the device for the determination of the heat conduction coefficient value. The thickness of test pieces was approximately in the range 40 – 60 mm.

The determination of the heat conduction coefficient was made with samples, which were before the measurement dried at the temperature + 110°C into constant weight. The apparatus Lambda 2300 of the Holometrix comp. was used for the determination of the heat conduction coefficient. The principle of this apparatus is based on the stationery plate method. Further the apparatus Shoterm QTM of the Shova Denco comp. was used,

which utilizes the non stationary hot wire method. Five measurements were realized altogether on each test sample in the case of the stationary method and in the case of the non-stationary method the number of measurements was doubled i.e. ten measurements were realized. The mean values $\bar{\lambda}$ and the standard deviations of mean values were calculated from the measured values for individual samples $s_{\bar{x}}$. The measured and the calculated values of the test samples are presented in tables below

Table 1. The mean values of heat conduction coefficients for samples from brick plant no.1

Quantity	Sample 1	Sample 2	Sample 3	Sample 4	Sample 5	Average
Hot wire method						
$\bar{\lambda}$ [W.m ⁻¹ .K ⁻¹]	0,4194	0,4257	0,4184	0,4277	0,4246	0,4231
s [W.m ⁻¹ .K ⁻¹]	0,0143	0,0231	0,0214	0,0225	0,0196	0,0202
$s_{\bar{x}}$ [W.m ⁻¹ .K ⁻¹]	0,0036					
Stationary method						
$\bar{\lambda}$ [W.m ⁻¹ .K ⁻¹]	0,4330	0,4318	0,4216	0,4214	0,4276	0,4271
s [W.m ⁻¹ .K ⁻¹]	0,0006	0,0010	0,0005	0,0010	0,0005	0,0007
$s_{\bar{x}}$ [W.m ⁻¹ .K ⁻¹]	0,0049					

Table 2. The mean values of heat conduction coefficients for samples from brick plant no.2

Quantity	Sample 1	Sample 2	Sample 3	Sample 4	Sample 5	Average
Hot wire method						
$\bar{\lambda}$ [W.m ⁻¹ .K ⁻¹]	0,4144	0,4486	0,4128	0,3983	0,4193	0,4187
s [W.m ⁻¹ .K ⁻¹]	0,0205	0,0174	0,0099	0,0349	0,0297	0,0225
$s_{\bar{x}}$ [W.m ⁻¹ .K ⁻¹]	0,0165					
Stationary method						
$\bar{\lambda}$ [W.m ⁻¹ .K ⁻¹]	0,4246	0,4084	0,4144	0,4166	0,3942	0,4116
s [W.m ⁻¹ .K ⁻¹]	0,0005	0,0005	0,0005	0,0005	0,0004	0,0005
$s_{\bar{x}}$ [W.m ⁻¹ .K ⁻¹]	0,0101					

Table 3. The mean values of heat conduction coefficients for samples from brick plant no. 3

Quantity	Sample 1	Sample 2	Sample 3	Sample 4	Sample 5	Average
Hot wire method						
$\bar{\lambda}$ [W.m ⁻¹ .K ⁻¹]	0,4267	0,3970	0,4251	0,4278	0,4427	0,4238
s [W.m ⁻¹ .K ⁻¹]	0,0208	0,0274	0,0296	0,0185	0,0201	0,0233
$s_{\bar{x}}$ [W.m ⁻¹ .K ⁻¹]	0,0149					
Stationary method						
$\bar{\lambda}$ [W.m ⁻¹ .K ⁻¹]	0,4248	0,4298	0,4356	0,4259	0,4196	0,4271
s [W.m ⁻¹ .K ⁻¹]	0,0005	0,0008	0,0006	0,0003	0,0010	0,0006
$s_{\bar{x}}$ [W.m ⁻¹ .K ⁻¹]	0,0054					

Table 4. The mean values of heat conduction coefficients for samples from brick plant no.4

Quantity	Sample 1	Sample 2	Sample 3	Sample 4	Sample 5	Average
Hot wire method						
$\bar{\lambda}$ [W.m ⁻¹ .K ⁻¹]	0,4016	0,3994	0,4261	0,3878	0,3936	0,4017
s [W.m ⁻¹ .K ⁻¹]	0,0330	0,0283	0,0238	0,0114	0,0187	0,0230
$S_{\bar{x}}$ [W.m ⁻¹ .K ⁻¹]	0,0131					
Stationary method						
$\bar{\lambda}$ [W.m ⁻¹ .K ⁻¹]	0,3836	0,4120	0,3926	0,4144	0,4138	0,4033
s [W.m ⁻¹ .K ⁻¹]	0,0017	0,0006	0,0005	0,0005	0,0010	0,0009
$S_{\bar{x}}$ [W.m ⁻¹ .K ⁻¹]	0,0127					

It is obvious, as follows from the above mentioned results that the mean values of standard deviations measured by the hot wire method are higher, than the mean values of results obtained by the stationary plate method. Nevertheless the standard deviations of mean values in the case of individual test samples show, that this deviation is lower than the standard deviation when the measurement is realized by the hot wire method, and in the same time it is considerable higher as the standard deviation in the case of measurement with the stationary plate method.

The next step was the determination of the percentage deviation of measured values (general average) by the hot wire method and by the plate method for individual brick plants.

Table 5: Percentage deviations of measured values by the hot wire and by the stationary plate method

Brick plant	1	2	3	4
Deviation	0,9%	-1,7%	0,8%	0,4%

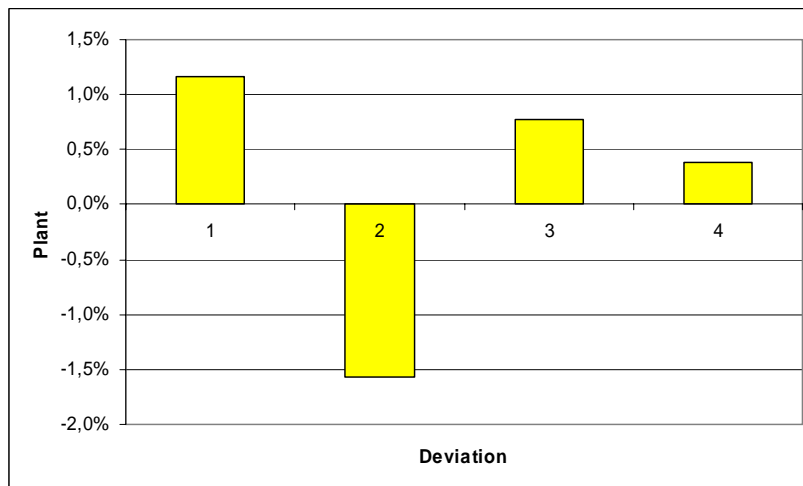


Figure 3. Deviations of measured values by the hot wire and by the stationary plate method

It is obvious from the above mentioned measured values that the measured values in one brick plant do not differ more than $\pm 1.7\%$.

Selected files were made of measured data from individual brick plants by random selection. These files contained always 50, 40, 30, 20, and 10 elements. The measurement error (the error of the arithmetic mean) was determined for individual selections. The probability fractile of 95 % was considered in the calculation. These errors include the error of measurement and in the same time the error caused by the variability of individual samples.

Table 6: The calculated errors of measured values in the hot wire method

Brick plant	Number of measurements				
	50	40	30	20	10
1	1,60%	1,75%	2,02%	2,87%	3,98%
2	1,99%	2,36%	2,82%	3,94%	6,16%
3	1,93%	2,17%	2,66%	3,64%	6,00%
4	2,00%	2,25%	2,42%	3,21%	4,27%

The error of measurement decreases by the number of measurements as obvious from the obtained results in the case of all brick plants. The error decreased with 30 measurements in all brick plants under 3 % and after the total number of 50 measurements it was not higher than 2 %. The error of measurement determined on the case of the stationary plate method was in the range 0.5 – 1.4 %.

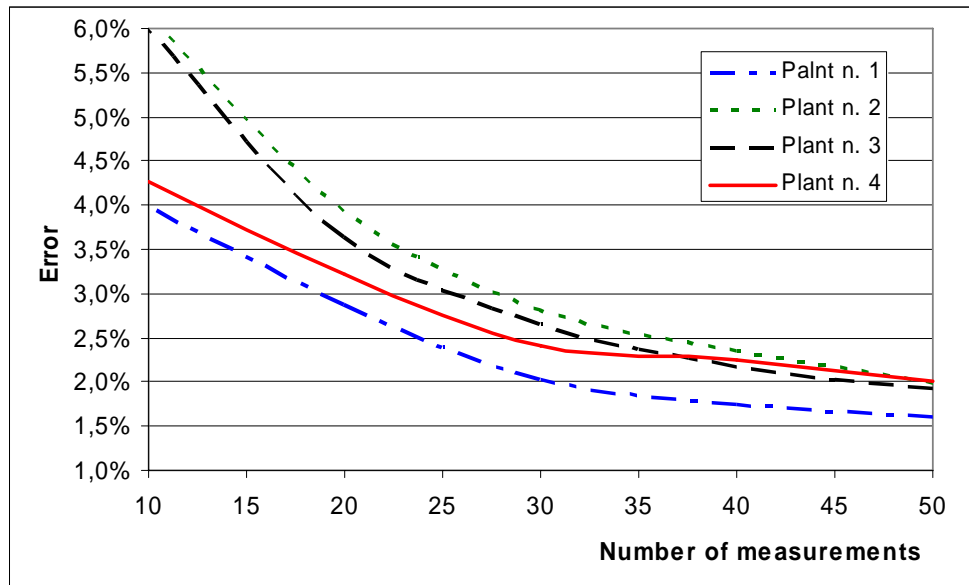


Figure 4. The dependence of the measured values error on the number of measurements by hot wire method

3. Determination of thermal conductivity coefficient of building materials

The non-stationary hot wire method measuring equipment has thanks to its design many advantageous properties. It is possible to measure by this apparatus easily and rapidly the value of thermal conductivity coefficient in the case of any building material:

- 1) **Measurement velocity.** In contrary to classical methods is this method incomparably quicker. The measurement itself lasts only some seconds and therefore it is possible (in comparison with stationary methods) to determine the value of thermal conduction coefficient in dependence on moisture of the tested sample,
- 2) **Measurement flexibility.** This measurement method is useful for many types of build materials,
- 3) **Measurement accuracy.** As with every measuring method even in the case of non-stationary measuring instrument the greatest error comes from the test sample, if the surface of the test piece uneven. If the test piece surface is sufficiently even the method provides relatively very exact results.
- 4) **Small dimensions of samples.** Considering the small dimensions of the test pieces and the fact that the testing probe is placed on the surface of the test piece, this method can be used for the determination of heat conduction coefficient of readymade products (for instance of form pieces for bricklaying etc.)
- 5) **Simplification of the measuring method.** The whole measuring method is controlled by a software (including evaluation), and the output quantity is the heat conduction coefficient value (λ).
- 6) **The low price of the apparatus**

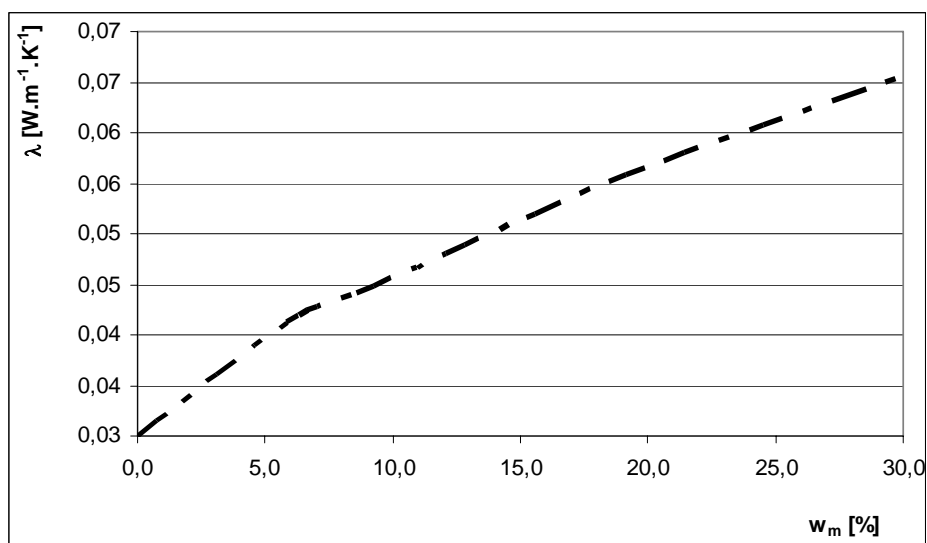


Figure 5: Dependence of the heat conduction coefficient of a wood based heat insulating back fill on moisture content.

The measuring apparatus can thanks to its favorable properties be applied for the determination of the thermal conductivity coefficient measurement in a great diversity of materials and products as for instance:

- Cellular concrete,
- Plastic insulation materials,
- Materials from mineral fibers,
- Insulation materials on natural base,
- Fireproof heat insulating ceramics,
- Insulation materials on cellular gypsum an anhydrite base,

- Building ceramics,
- Heat insulating mortars and plaster mixtures, etc.

Most important is the use of the measuring method in special applications, where it is possible to apply to the full extent its specific favorable properties, for example:

- determination of thermal conductivity in dependence on moisture,
- determination of thermal conductivity of cement binders during hydration

4. Conclusion

A set of measurements was realized on the test pieces brick bodies from four brick plants. The aim of the measurement was the comparison of measured values by means of the non-stationary hot wire method and by means of the stationary plate method. Following the measured results and the calculated statistical values we can conclude:

- the non-stationary hot wire method is relatively less exact in the comparison with the stationary plate method,
- the standard deviations of values measured by means of the hot wire method are comparable with the standard deviation determined on mean measured values of individual test samples in the production sphere of one plant and they characterize the properties variability of individual test samples in one tested set,
- the resulting measured values determined by the hot wire method and by the stationary plate method don't differ significantly. The highest determined standard deviation between the results in one and the same brick plant was 1.7 %,
- the calculated errors in dependence on the number of measurements by the method of hot wire show that in the case of sufficient number of measurements we can significantly minimize the measurement error, which was with 50 measurements not higher than 2 %.

In conclusion we can state that the hot wire method can be used in practice when we use sufficient number of measurements. The increase of measurements decreases the degree of the measuring method error on the total measurement error and in the case of 50 measurements the measurement accuracy in the case of the hot wire method is near to the measurement accuracy in the case of the measurement by the plate method where the total error in one and the same testing set was in the range 0.5 – 1.4 %.

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References

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