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The meaning of transit times in NDT of reinforced concrete

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Abstract
Transit times of longitudinal pulses that travel near reinforcing steel bars are usually smaller than in plain concrete. Chung first, and then Bungey, demonstrated the importance of including bar diameters in the correction factors when pulses propagating parallel and near to the bars can not be avoided. Besides the influence of geometric dispersion and mode conversion on pulse propagation, there is an effect due to a specific pattern of radial variation of longitudinal wave velocities from the surface of the embedded steel bar. Wall effects produce a region (a sheath) of concrete surrounding the steel bar, with smaller longitudinal pulse velocities in comparison with the velocity farther away from the bar. Then the acoustic energy travels trapped in a kind of composite leaky waveguide with a velocity less than the compression (P) wave velocity in steel and in concrete but greater than the extensional wave velocity in steel. A mathematical model of the propagation of a longitudinal wave-packet is constructed, taking into account both attenuation and dispersion effects. An asymptotic analysis of pulse propagation and the introduction of a threshold of detection in the receiving transducer, allows the derivation of an approximate analytical formula for transit times. Then the meaning of transit times in NDT of concrete is discussed. The abovementioned formula is used to study the effects of the reinforcing bar radius, the propagation parameters in plain concrete and in the sheath surrounding the steel bar, the path length between the transmitting and the receiving transducers, and the energy and the spectral composition of the pulse injected by the emitter in the tested body. Chung's empirical correlation between the measured pulse velocity and bar radius is reviewed and a new correlation is proposed.

Key words: ultrasonic testing of reinforced concrete, transit times of longitudinal pulses, leaky waveguide, mathematical model

1. Introduction

Ultrasonic testing of concrete is currently used for quality assessment in large concrete structures cast on site and in mass-produced prefabricated units. The intrinsic heterogeneity of concrete limits the carrier frequencies that can be used to 250 kHz and less (being 54 kHz the most common frequency) when probing distances are greater than 1 m. It usually demands two ultrasonic transducers, one for pulse emission and the other for pulse reception. In practice, probes for ultrasonic testing of concrete are not much larger (50 mm) than the customary ones for metals. The contact face of a common cylindrical probe, whose diameter is of the order of magnitude of the wavelength, or even less, radiates besides longitudinal waves, also shear and surface waves of appreciable intensity. The radiation and reception beam patterns are much less collimated than in the case of metals, although still not entirely isotropic (1). In any case, the absence of any significant directional effect in the probes, as well as the multiple scattering of waves inside concrete, makes it possible, (in principle) to couple directly any two points on the surface of the specimen being tested. The fastest ultrasonic signal received in this way is then always a direct longitudinal wave-pulse. This is followed by shear and surface wave-pulses, and by reflected longitudinal waves. For quality tests on concrete the longitudinal acoustic velocity is determined measuring the distance between probes and the time that the direct longitudinal pulse takes to go from the emitting to the receiving probe. A higher velocity usually means a better concrete quality (strength, durability and dimensional stability). Dry cracks in concrete members may be detected and their depth and inclination assessed by careful measuring time-versus distance relationships for longitudinal pulses. Cavities and micro-crack fields can be detected under suitable conditions.

In all this measurements, reinforcement, if present, should be avoided. Indeed, considerably uncertainty is introduced by the higher velocity of pulses in steel and by possible compaction shortcomings in heavily reinforced regions. If the reinforcing bars run in a direction at right angles to the pulse path and the sum of bar diameters is small in relation to path length, the effect is generally not significant. But if the reinforcing bars lie along or parallel to the path of the pulse, and their diameter is greater than 10 mmm, the effect may be very significant. It is not only the measurements taken along the reinforcing bars that are affected, but also those in the neighbourhood of the bars. The first pulse taking an indirect path connected with the reinforcing bar may reach the receiving probe before that going along the direct path through bulk concrete, resulting in a shorter apparent transit time. Now, a relatively small difference in pulse velocity usually corresponds to a relatively large difference in the quality of concrete. So, if reinforcement cannot be avoided, it may be necessary to make some corrections in order to estimate the true longitudinal pulse velocity in bulk concrete. The purpose of the present work is to give a theoretical discussion of the relationship between the propagation time of longitudinal pulses and the parameters of reinforced concrete, of the ultrasonic transducers (emission and reception properties) and the detection threshold of the equipment. The intention is to improve and enlarge, with the help of fairly simple mathematical models, the discussion already begun in a previous paper (2). As was noted there, even if enough spatial symmetry is assumed, the problem of pulse propagation in steel bars embedded in concrete is fairly difficult to solve using directly an ab-initio numerical analysis of the equations of elastodynamics. So, a simplified analytical approach may be of interest, both for the design of realistic digital simulations and for the interpretation of the experimental results obtained with non-destructive testing (ultrasonic) equipments.

RILEM recommendations commonly used in ultrasonic testing of structures for making corrections in pulse velocity measurements, when the reinforcement influence can not be avoided, are briefly reviewed. An extension of this approach to estimate the transit times of longitudinal pulses in a reinforced concrete with two layers of different quality surrounding the steel bar is suggested.

Some significant limitations of the ray theory of wave propagation that serves as framework for the abovementioned recommendations are considered. The results of Chung's experiments are briefly summarized. Then, a simplified model called "almost-fluid leaky waveguide model", is used to discuss the dispersive propagation of longitudinal pulses in reinforced concrete. Powerful asymptotic methods are applied to give a better description of wave-packet propagation, beginning with an analysis of the elastic precursor. A new and theoretically grounded correlation is proposed between velocity, bar radius, and other parameters of practical interest in ultrasonic NDT of concrete in structures.

2. The standard corrections and some of their limitations

The following correction is recommended by RILEM and British Standards (3) for the situation depicted in fig 1:

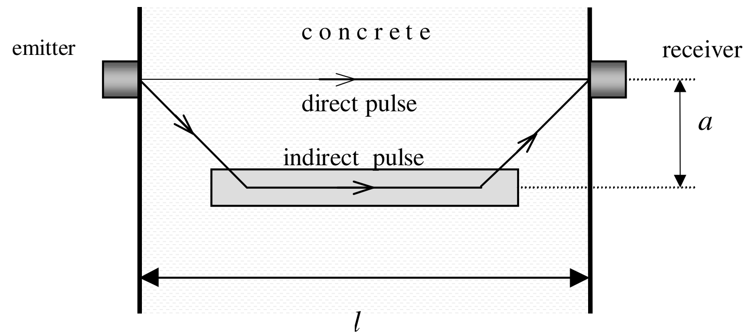

Figure 1: Direct and direct paths for ultrasonic pulses in concrete.

If t is the measured travel time of the pulse, l the apparent path length, V_S the pulse velocity in steel (which may be from 1.4 to 1.7 higher than the pulse velocity in plain concrete) and a is the distance shown in the figure, then the travel time of an indirect pulse, arriving first, and given by elementary acoustic ray theory is:

$$t = \frac{l}{V_s} + 2.a.\frac{\sqrt{V^2{}_s - V_c{}^2}}{V_s.V_c} \qquad [1]$$

According to BS 4408 pt.5 and from [1], it follows that the true pulse velocity in bulk concrete, V_c is given by $V_c = \dfrac{2aV_s}{\sqrt{4a^2 + (tV_s - l)^2}}$, while the apparent pulse velocity in bulk concrete would be $V_{c,app} \equiv \dfrac{l}{t}$. This correction should be applied if $V_c \le V_s$ and if a/l is less than the critical value:

$$\frac{1}{2}\sqrt{\frac{V_s - V_c}{V_s + V_c}} = \left(\frac{a}{l}\right)_{critical} \qquad [2]$$

The effect of the reinforcement disappears if a/l is greater than this critical value, because in this case the direct pulse arrives first. As a consequence, the "apparent velocity" l/t is now the true velocity in plain concrete.

The aforementioned correction formula is based in the acoustic ray theory of the old refraction method of the seismologists. In this case we have an elastic layer of thickness a over an elastic half-infinite bed. P waves are produced by an emitter at the upper face of the layer, and the waves are detected by a receiver, also located at the upper face. If the bulk P wave velocity in the bed is higher than the bulk P wave velocity in the layer (concrete), an indirect P wave arrives first after travelling totally reflected at the interface. All the other indirect P waves arrive later.

However, the concrete surrounding the bar is often of different quality that plain concrete (in the adjacent layer, P waves travel slower than in the bulk of the material).

By a direct extension of the acoustic ray theory to two layers in a half infinite bed (4) well known to geophysicists (Fig.2), the following generalization of the travel time of indirect ultrasonic pulses is obtained for reinforced concrete:

$$t = \frac{l}{V_s} + 2.a_1.\frac{\sqrt{V^2{}_s - V^2{}_{c1}}}{V_s.V_{c1}} + 2.a_2.\frac{\sqrt{V^2{}_s - V^2{}_{c2}}}{V_s.V_{c2}} \qquad [3]$$

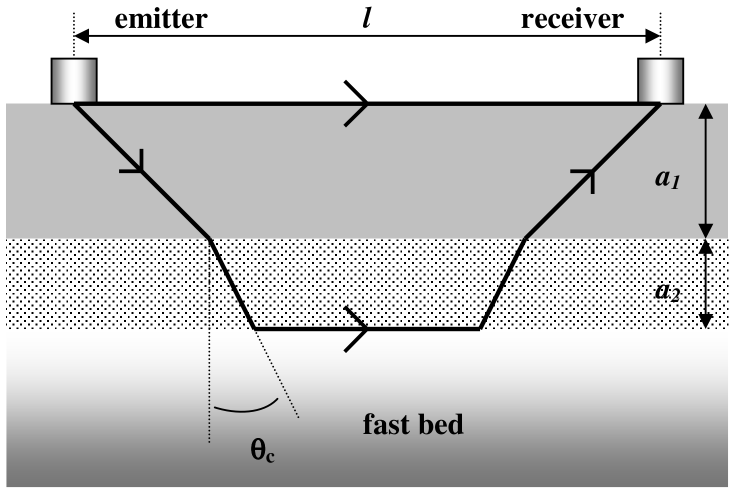

Figure 2: The refraction method for two layers on a fast half-infinite bed

Here a_1 is the depth of the upper layer with higher P wave velocity V_{c1}, while a_2 and V_{c2} are the corresponding parameters for the layer adjacent to the steel bed, with slower P velocity. The travel time of the direct pulse is: $t = l/V_{c1}$ The indirect pulse arrives first

if ($a = a_1 + a_2$): $$\frac{a}{l} \leq \frac{1}{2}\sqrt{\frac{V_s - V_{c1}}{V_s + V_{c1}}} - \frac{a_2}{l}\left(\sqrt{\frac{V^2{}_s - V^2{}_{c2}}{V^2{}_s + V^2{}_{c2}}} - 1\right) = (\frac{a}{l})_{critical} \qquad [4]$$

In this formula V_{c1} corresponds to V_c in formula [2]. So, the critical ratio $(a/l)_{critical}$ is less if there is a layer of relatively low velocity concrete between plain concrete and steel. Formula [4] could give a more realistic but less conservative estimation of the critical ratio of influence of steel on longitudinal pulse velocity measurements in reinforced concrete. Nevertheless, to represent reinforcement bars as equivalent to a half infinite steel bed, is indeed a very coarse simplification of the actual situation, because the radius of the bar is fairly small and the wavelengths are fairly large compared with the other lengths involved in pulse propagation.

Chung (5) has shown that for pulses travelling in the direction of the axis of the reinforcing bars through a steel-concrete medium, the effective pulse velocity V_e is less than the above mentioned value V_s for P waves in bulk steel and it is influenced by bar's diameter according to the approximate empirical formula:

$$V_c = 5.90 - \frac{5.2}{R}.(5.90 - V_c) \qquad [5]$$

Here R is bar radius (in mm and greater than 5.2) and V_c is the true velocity in bulk concrete (in km/sec.). Chung's experiments were done for six concretes mixes (of different proportions of water, cement, sand and coarse aggregates) but always with the same transducers of a carrier frequency of 54 kHz and for fixed path lengths (750 mm) and transverse dimensions (150 by 150 mm).

As a consequence of Chung's findings, it seems that some corrected V_e must be used instead of V_S in formula [1]. In BS 4408 pt.5 a fixed value of 5.5 km/sec is suggested for V_S. This figure is a fixed average between longitudinal bulk wave velocity (5.9 km/sec) and extensional (bar) wave velocity (5.2 km/sec) in steel. As such, it doesn't take into account neither bar diameter nor a possible effect due to the pulse spectrum and path length that may be reasonably expected due to geometric dispersion and spatial attenuation of the longitudinal wave-packet (6). Further experimental work about the influence of reinforcement and a critical discussion of the proposed corrections was given by Bungey (7). However, even now there is controversy about the meaning and the feasibility of these corrections. It is the pulse travelling in the sheath of concrete surrounding the bar which is being measured, rather than the pulse travelling through the bulk of the material (which is almost always the goal of the measurements). Besides, the degree of adherence between steel and concrete must be taken into account when considering the influence of reinforcement bars in ultrasonic measurements. Let us try another approach to these problems, leaving geometric acoustics aside and considering waveguide effects in reinforced concrete members.

3. The almost-fluid waveguide approach and a theoretical foundation to Chung´s empirical formula

In a steel bar in air, as the wave train progresses along the bar, disturbances along the boundary are produced by total reflection and mode conversion at the steel-air interface, so as to reduce there the stress to zero. At the receiving probe a complex time signal is obtained corresponding to a longitudinal wave packet arriving first followed by several trailing pulses. Experimental results obtained by Mc Skimin in the Bell Laboratories during a research in acoustic delay lines, suggest that, as a first approximation when the wavelength is small relative to bar radius, the dilatational component may be considered to be guided along the bar much as in the case of a fluid wave-guide that releases pressure at its boundary (8). Due to interference from waves produced at interfaces, both the axial and the radial displacement fields, as functions of the distance from bar axis, show small oscillations superposed on a fairly smooth trend. If we make an average of these fields over a suitable chosen (and movable) plane region, we obtain a smoothed axial field that seems to obey very approximately a leaky classical wave equation. In the solid wave guide case energy is continually being drained off through mode conversion processes at the boundary. Nevertheless, at any cross section of the bar, the leading pulse can be reconstructed from the Fourier spectrum of the longitudinal pulse injected by the emitter using suitably defined propagation parameters and working with a properly defined average dilatational wave field. The displacement field imposed by the emitting transducer excites mainly the first mode of the almost fluid waveguide, and well known formulae for phase and group velocity as functions of frequency can be used as a first approximation to the propagation process. (The fluid waveguide concept was further developed and used to study some problems related with the influence of geometric dispersion in ultrasonic testing with longitudinal pulses in (9) and (10)).

In a steel bar immersed in concrete and well bonded to it, the P wave velocity in concrete and the S (shear) wave velocity in concrete are less than the P and S velocities in steel. So, contrary to the bar in air case, now there is no possibility of total reflection when a wave comes to the steel-concrete interface from the steel side. Then a dilatational pulse that travels along an embedded reinforcement bar is continuously loosing energy towards the adjacent concrete. Once the waves go through the steel-concrete interface towards the concrete, they scatter, reflect and convert modes many times at the interfaces between the aggregates and the cement paste. Part of this energy comes back and now part of it can be trapped by total reflection in the concrete layer adjacent to the steel bar. This trapping is enhanced when a sheath of concrete surrounding the steel bar has lower propagation velocities than the bulk of the material, due to wall effects during filling and compaction. A certain dilatational wave field is thus produced, in the concrete surrounding the bar and in the steel, inside the bar, moving along the bar axis and tending fairly quickly to zero as the distance from the bar axis grows. A suggestion was made (2) to model the propagation of the leading longitudinal pulse (whose travel time is measured in non-destructive testing) using an equivalent leaky waveguide, coaxial with the steel bar, but with an effective radius R_e, greater than bar radius R. To simplify the analysis, let us suppose for a moment that the waveguide can be treated as almost-fluid. Then, for the first propagation mode we can apply the following formula for the wave number k as a function of frequency ω:

$$k(\omega) = \frac{\sqrt{\omega^2 - {\omega_{MS}}^2(\omega)}}{V_S} \qquad [6]$$

What may be called called Mc Skimin frequency function $\omega_{MS}(\omega)$ is given by the following asymptotic expression that was introduced in reference (9):

$$\omega_{MS}(\omega) \approx \omega_C \cdot \left(1 + \sum_{n=1}^{\infty} c_n \cdot \left(\frac{\omega_C}{\omega} \right)^n \right) \qquad [7]$$

The asymptotic cut-off frequency of the mode is given by: $\omega_C = \dfrac{a_0 \cdot V_S}{R_e}$ [8]

Here a_0 is the first zero of the Bessel function of the first kind and order zero $J_0(z)$, and may be approximated by 2.40.

The group velocity will be:

$$V_g = V_S \cdot \frac{\sqrt{1 - \dfrac{\omega^2{}_{MS}(\omega)}{\omega^2}}}{\left(1 - \dfrac{\partial^2 \omega_{MS}(\omega)}{\partial \omega^2} \right)} \qquad [9]$$

If the measured pulse velocity were the group velocity, formulae [7] to [9] taken together would be a theoretical counterpart to the empirical formula due to Chung, but with the equivalent bar radius R_e instead of the true bar radius R. Now, a relation between these two parameters must be found. In order to find it, let us work in the high frequency limit, where $\omega_{MS} \approx \omega_C$ is taken to be constant. The group velocity then simplifies to the well known waveguide formula:

$$V_g = V_S \cdot \sqrt{1 - \frac{{\omega_C}^2}{\omega^2}} \qquad [10]$$

It decreases when R_e decreases, being equal to the longitudinal velocity in the sheath of concrete V'_C for a critical value $\tilde{R}_e$ of the radius of the equivalent waveguide. From [8] and [10] it follows that:

$$\tilde{R}_e = \frac{\frac{a_0 \cdot V_S}{\omega}}{\sqrt{1 - \left(\frac{V'_C}{V_S}\right)^2}} \qquad [11]$$

If R_e is greater than $\tilde{R}_e$, the first pulse that arrives to the receiving transducer would be the leaky waveguide pulse. If R_e is smaller than $\tilde{R}_e$, the first pulse that arrives to the receiving probe would be the direct pulse that travels with velocity V'_C. Then, the effective velocity V_e of longitudinal pulses in reinforced concrete would be given by equations [8] and [10] if $R_e \geq \tilde{R}_e$, and by $V_e = V'_C$ if $R_e < \tilde{R}_e$.

But the formula for group velocity in the leaky waveguide can be approximated by the linear relationship: $V_g \approx a + \frac{b}{R_e}$ When R_e tends to ∞ then V_g must tend to V_S, so that $a = V_S$. When $R_e = \tilde{R}_e$ then V_g must be equal to V'_C, so that $b = -\tilde{R}_e \cdot (V_S - V'_C)$.

Thus the following Chung-like formula is derived:

$$V_e \approx V_g \approx V_S - (V_S - V'_C) \cdot \frac{\tilde{R}_e}{R_e} \qquad [12]$$

Chung's experimental formula can be re-written in this way:

$$V_e \approx V_S - (V_S - V_C) \cdot \frac{R_0}{R} \qquad [13]$$

Here V_C is the longitudinal wave velocity in the bulk of concrete, while $V'_C \approx \kappa \cdot V_C$ is the longitudinal wave velocity in the layer that surrounds the bar. The parameter κ, usually less than one, gives the ratio of velocities. If in a first approximation κ is taken equal to one, then a direct comparison between [12] and [13] gives:

$$\frac{R_0}{R} = \frac{\tilde{R}_e}{R_e} \qquad [14]$$

Following an argument already given in (2), it is possible to show that:

$$R_0 = \frac{a_0 \cdot V_S}{\omega} \qquad [15]$$

From this last equation and from [11] and [14] it follows that the radius of the equivalent waveguide is related with bar radius by:

$$R_e = \frac{R}{\sqrt{1 - \left(\frac{V'_C}{V_S}\right)^2}} \qquad [16]$$

The same equation relates $\tilde{R}_e$ with R_0. If the steel bar is in void ($V'_C = 0$), we obtain $R_e = R$ as it should be. If V'_C approaches to V_S, R_e tends to infinity so we will have homogeneous dilatation plane waves moving without geometric dispersion in an unbounded steel medium, also as it should be.

4. Asymptotic expansions for propagating pulses and elastic precursors

Now let us put an emitter to inject at a certain cross-section of the waveguide, of axial coordinate $z = 0$, a propagating wave packet. Let us suppose that the axial displacement of the injected longitudinal pulse may be represented as a damped sinusoid:

$$u(t, z = 0) = A_0 \cdot e^{-\mu_0 \cdot t} \sin(\omega_0 \cdot t) \cdot H(t) \qquad [17]$$

Here ω_0 is the carrier frequency and $H(t)$ is Heavside's unit step function (equal to 1 if $t > 0$ and to 0 if $t < 0$). If the pulse is a short one, not more than 5 or 6 cycles, as in ultrasonic non-destructive testing of concrete, the damping parameter μ_0 is of the order of $\omega_0/2\pi$. The amplitude spectrum $A(\omega)$ and phase $\phi(\omega)$ of this pulse are given by:

$$A(\omega) = \frac{1}{2\pi} \frac{\omega_0 . A_0}{\sqrt{\left(\mu_0{}^2 + \omega_0^2 - \omega^2\right)^2 + 4\mu_0{}^2 . \omega^2}} \qquad [18a]$$

$$\tan \phi(\omega) = \frac{-2 \cdot \mu_0 \cdot \omega}{\omega^2 - \left(\omega^2{}_0 + \mu^2{}_0\right)} \qquad [18b]$$

So, even if we have spectral components of every possible frequency, their amplitudes tend to zero as $1/\omega^2$ when ω grows to infinity.
A receiver is located in another cross-section. This probe produces a scalar (voltage) time signal after making a certain weighted average of the elastodynamic field. So, it will be enough to consider the propagation of a representative scalar field (for example, the average axial displacement) given by:

$$u(t, z) = \int_{-\infty}^{+\infty} A(\omega) . e^{i\phi(\omega)} . e^{-z\alpha(\omega)} . e^{i(\omega \cdot t - z \cdot (k(\omega) + \beta(\omega)))} d\omega \qquad [19]$$

Here $\alpha(\omega)$ is an attenuation coefficient related with the dilatational energy drainage off the equivalent waveguide, by mode conversion and by leakages towards the bulk of concrete. (The attenuation due to energy dissipation in concrete at testing frequencies, related with hereditary effects (11) or coupled elastic and thermal effects (12), is much less important than the attenuation due to heterogeneities in the material (6)). If there is energy dissipation at a given frequency, given by $\alpha(\omega)$, there will be accompanying dispersion given by the wave-number $\beta(\omega)$. These functions of frequency are related by the Hilbert Transform or, if $\alpha(\omega)$ is symmetric and $\beta(\omega)$ anti-symmetric, by the Kramers-Krönig relations (13). Nevertheless, in what follows, and as a first approximation to simplify the analysis, we will neglect the effect of $\beta(\omega)$ relative to $k(\omega)$ in the description of the propagated disturbance.

When t is big enough, the method of stationary phase (14) can be applied to the evaluation of the integral [19]. For a given t and z, the main contribution to the integral comes from the frequencies $\omega_e(t,z)$ that verify:

$$\frac{\partial k}{\partial \omega} = \frac{t}{z} \qquad [20]$$

If [20] does not have real roots, $u(t,z)$ is negligible. In our case, from [9] and [20] it follows that:

$$\frac{z}{V_S \cdot t} = \frac{\sqrt{1 - \frac{\omega^2{}_{MS}(\omega_e)}{\omega_e{}^2}}}{\left(1 - \frac{\partial^2 \omega_{MS}(\omega_e)}{\partial \omega^2}\right)} \qquad [21]$$

If there is only one solution to equation [21], and $\frac{\partial^2 k(\omega_e)}{\partial \omega^2} \neq 0$ is not too small, the scalar field [19] can be approximated (14), (15), when $\omega_C \cdot t$ is large enough, by

$$u(t,z) \approx \frac{2e^{-z.\alpha(\omega)}.A(\omega_c)}{\left[\frac{1}{2\pi}\cdot\left|\frac{\partial^2 k(\omega_c)}{\partial \omega^2}\right|.z\right]^{\frac{1}{2}}}.\cos\left(\omega_e.t - k(\omega_e).z + \phi(\omega_e) + s.\frac{\pi}{4}\right) \qquad [22]$$

Here s is the sign of $\frac{\partial^2 k(\omega_e)}{\partial \omega^2}$. Formula [22] is a local approximation by a harmonic wave of frequency $\omega_e(z/V_S \cdot t)$ and wave number $k(\omega_e(z/V_S \cdot t))$. But when $z/t \cdot V_S$ varies, ω_e changes as well (according to formula [21]), so that we obtain a wave modulated in amplitude, frequency and phase. When the dimensionless number $z/t \cdot V_S$ approaches to 1 from below, $\omega_e(z/V_S \cdot t)$ tends to infinity, ω_{MS} tends to ω_C and $\frac{\partial^2 k(\omega_e)}{\partial \omega^2}$ as well as all the higher order derivatives tends to zero. So, we can expect that at and near the wave-front of the propagated ultrasonic disturbance formula [22] is not valid. However, the relation between k and ω as well as the spectrum of the injected pulse, extended to the complex plane are such that the method of uniform asymptotic expansions developed in (16) can be applied. Neglecting the attenuation related with $\alpha(\omega)$ the following simplified asymptotic formula is obtained, as a coarse approximation for the wave-front of the pulse, when $\left(1 - z/t \cdot V_S\right)$ and $\omega_C \cdot t \cdot \left(1 - z/t \cdot V_S\right)$, are both small:

$$u(t,z) \approx A_0 \cdot \frac{\omega_0}{2\pi} \cdot \sqrt{\frac{t - \frac{z}{V_S}}{\left(\frac{z \cdot \omega^2{}_C}{2 \cdot V_S} + 2 \cdot \mu_0\right)}} \cdot J_1\left(2 \cdot \sqrt{\left(\frac{z \cdot \omega^2{}_C}{2 \cdot V_S} + 2 \cdot \mu_0\right)} \cdot \sqrt{t - \frac{z}{V_S}}\right) \qquad [23]$$

Here $J_1(x)$ is the Bessel function of first kind and order one, and $t \geq z/V_S$. When $t < z/V_S$, all the fields are zero, because the acceleration front has not arrived yet to the corresponding points z of the waveguide axis. Formula [23], albeit different, is closely related with Sommerfeld´s formula for the propagation of precursors in optical dispersive media and has a similar meaning: immediately upon its arrival at the cross-section located in z, the initial amplitude is very small compared with A_0, and the initial frequency of oscillation is very high compared with the emitter nominal frequency. But the amplitude (due to the factor $\sqrt{t - \frac{z}{V_S}}$) and the period of oscillation (due to the location of the zeroes of the Bessel function), increase with increasing $t - \frac{z}{V_S}$

5. Conclusions: detection thresholds and the meaning of the measured transit times of longitudinal pulses.

The leading edge of the first dilatational pulse that is measured by the receiving probe is occupied by the highest frequencies that give amplitudes above a certain measurement threshold (1). If these frequencies are big enough, we can expect to be able to discuss the measured transit time between emitter and receiver using the almost-fluid approach. Let $\omega_{e,u}$ be the biggest frequency whose amplitude is over the detection threshold. If the asymptotic approximation [22], that represents the propagated disturbance far enough from the acceleration front of the pulse, describes the local behaviour that corresponds to the threshold frequency, and assuming that the wave-number is given by [6], at least with enough approximation, then [21] is verified. It follows that for a certain distance l between the receiving and the emitter transducers, the first signal detected would be registered in an instant t such that:

$$t = V_S \cdot l \cdot \frac{\left(1 - \frac{\partial^2 \omega_{MS}(\omega_e)}{\partial \omega^2}\right)}{\sqrt{1 - \frac{\omega^2{}_{MS}(\omega_e)}{\omega_e{}^2}}} \qquad [24]$$

This would give us the apparent dilatational pulse velocity l/t measured with a propagating wave-packet of a given spectrum of Fourier components and with a receiver with a certain detection threshold. The effective longitudinal pulse velocity V_e would be given by formula [9], but for $\omega = \omega_{e,u}$ instead of the carrier frequency ω_0 of the emitted pulse. Furthermore, we must use $\omega_{e,u}$ instead of ω_0 in formula [15] for the radius R_0 of the bar such that for it $V_e = V_C$.

In principle, the spectrum of the incoming pulse has component frequencies as big as we want. As a consequence, a certain fraction of the pulse, represented by equation [23], will always travel with a velocity very near to V_S. But this fraction can be detected, and its corresponding travel time can be measured, only if their amplitudes are above the

detection threshold of the ultrasonic equipment. This is the case when the length l of the trajectory of the pulse is of the same order of magnitude or less than the radius R_e of the equivalent waveguide: here we do not have geometric dispersion. But when l increases, this high frequency part of the wave-packet will remain under the threshold of detection and the measured transit time will correspond to the travel time of the highest detectable frequency $\omega_{e,u}$. This threshold frequency will depend of l, of the amplitude spectrum of the emitted ultrasonic pulse, of the signal to noise characteristics of the time measuring equipment and of the propagation distortions from the emitter to the receiver described by formula [22]. The fluid waveguide model predicts that $\omega_{e,u}$ should grow if l decreases, if ω_0 (the carrier frequency) increases, and if the total energy of the incoming pulse increases (for the same relative amplitudes). Now, considering the generalization of Chung´s formula given by equation [13], from the above remarks and from equation [15] it follows that in this equation the parameter R_o should decrease if l decreases, or if ω_0 increases, or if the total energy of the pulse increases. As was already noticed in (2), from Chung´s experimental value for R_o (0.0052m) and using [15], it results $\omega_{e,u} \approx 14.160/0.0052 = 2.723$ MHz. The carrier frequency is $\omega_0 = 6.28*540$ kHz = 0.3391 MHz so that in this case $\omega_{e,u}$ would be nearly eight times ω_0 .
Using the concept of an equivalent almost-fluid waveguide we have thus explained some known aspects of the geometric dispersion of longitudinal pulses in reinforcement bars.
This almost-fluid approximation is very useful to study the effects of the geometric dispersion when the wavelengths are small relative to the smallest dimension of the solid body perpendicular to the direction of pulse propagation. But for the low frequencies used in ultrasonic testing of concrete the wavelengths λ are not small in comparison with bar radius R (for 54 kHz in steel, λ= 0.11 m). So, as was also stressed in reference (2), even if the spectra of short ultrasonic pulses of 54 kHz of carrier frequency have significant components at higher frequencies (well above 150 kHz), a less simplified approach must be used to study the travel time of the leading dilatational pulse.
As shown in (17) and (18), a model with two or three coupled propagation modes may be enough to describe the main features of mechanical vibrations and pulse propagation in a bar of any uniform cross-section, immersed in air. If $R_e \cdot \omega / V_S$ is not large enough, the almost-fluid model cannot be applied to reinforced concrete and a more realistic model of an equivalent solid waveguide must be constructed. The simplest solid waveguide model can be obtained coupling a purely extensional mode with a suitable shear mode. This, as will be developed elsewhere, gives two emerging modes. When the dimensionless number $R_e \cdot \omega / V_S$ increases, the upper mode should approach the first dilatational mode of the almost fluid waveguide. At its turn, it seems that the lower mode approaches a kind of interface concrete-steel wave propagation mode, which is very slow and it is less and less excited by the emitting transducer as $R_e \cdot \omega / V_S$ increases. All this deserve further experimental and theoretical research, including digital simulations using suitable numerical codes. Several qualitative and quantitative predictions that stem from the results obtained in sections 3 and 4 are also amenable to experimental or digital simulation testing.

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