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Optimization of the Inspection Schedule for Surface-Breaking Cracks under Fatigue Loading

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Abstract

Inspections play an essential role in the damage tolerance approach to control fatigue damage. In this paper a method is presented to optimize the cost-effectiveness of inspection scheduling for single and multiple inspections during the expected lifetime. Detailed results are given for a surface-breaking crack of depth a in a cyclic tension field. The original crack depth is treated as a random variable with a known probability density function. The crack growth is governed by Paris law, and the inspection technique has a known probability of detection. The component fails when the crack depth reaches a critical value a_{cr}. As a part of the damage tolerant approach the component is replaced only when the depth of the detected crack is larger than a pre-defined value a_r.

The analysis is based on theoretical considerations, which take into account three aspects of fatigue damage using a statistical approach.

- Stage 1: Development of pre-crack fatigue damage leading to the formation of a very small macrocrack
- Stage 2: Growth of the macrocrack
- Stage 3: Determination of the probability of a detected crack with crack length $a > a_r$

The authors have discussed Stage 1 in considerable detail elsewhere. Here, the emphasis will be on Stages 2 and 3.

The point of departure is a macrocrack, represented by a probability distribution $f(a;N_0)$ of surface breaking macrocracks that Stage 1 has yielded, with known statistics. By the use of an evolution law (for example, Paris law) we can then compute the probability distribution $f(a;N)$ after N cycles. We take into account that cracks can only be detected with a known probability of detection (POD).

For a pre-determined number of cycles that defines the expected lifetime, we obtain the

optimal number of cycles for inspection scheduling, by minimizing a total cost function. The total cost function is the sum of the cost functions for failure, component replacement and inspection. These functions have been expressed is terms of the relevant probabilities. Analytical and numerical results are given for one, two and multiple inspections. These results show the effects of the magnitude of a_r, and the POD of the inspection technique on the optimum schedule of the inspection process.

1. Introduction

In recent years, the damage-tolerance design approach to fatigue has supplanted more traditional approaches like infinite-life design and safe-life design. The damage-tolerance design assumes that the component has some initial damage and deals with the ability of the component to resist a specific amount of damage for a given period of service. In a generalized sense, the successful implementation of the damage-tolerance approach depends on the following factors: (i) predicting macrocrack initiation, (ii) modeling of the macrocrack growth, and (iii) component inspection. In current applications, the predictions of macrocrack initiation are generally left out of consideration. However, a detailed discussion of measuring and quantifying pre-crack fatigue damage and predicting time for macrocrack initiation has been given by Kulkarni et. al [1]. Modeling of the crack growth is based on the determination of the stress-intensity factor and on establishing a suitable crack growth law. Component inspection deals with nondestructive techniques to detect cracks and with scheduling of inspections. A probabilistic approach to quantifying imperfect inspections, which is relevant to the current paper, has been discussed by Kulkarni and Achenbach [2]. In scheduling inspections, one needs to account for the fact that early in the life cycle, the crack growth rates and the crack sizes are small and hence the chances of cracks being detected by the inspection technique are small. To derive maximum benefit from the damage-tolerance approach, a sufficient but not an excessive number of inspections must be scheduled based on their POD (probability of detection) and the expected crack growth characteristics. This problem of scheduling inspections in the context of implementing the damage-tolerance approach has attracted the attention of researchers and a brief literature review is presented in Ref. [3].

For safety critical components, the present practice is to replace a component once a crack has been detected. This paper presents, however, a framework for scheduling inspections, which assumes an understanding of crack growth, and accounts for the case that a component may be allowed to continue in service when a crack is detected. Replacement or repair is implemented only if the detected crack exceeds a certain pre-defined size. We also allow for the fact that the crack length density in a replaced component need not be the same as the crack length density in a virgin component. In section 2, we begin our discussion by first describing the problem under consideration along with the growth law used to model the crack growth. In the next section, we briefly discuss the model used to describe the inherent uncertainty of an inspection process. The main contribution of the paper is presented in section 4 where we discuss a framework to optimize the inspection schedule of a component or structure. Expressions for the efficient implementation of the proposed strategy are presented. Numerical results obtained for a sample problem are shown and discussed in section 5. Finally, conclusions based on the numerical results and general remarks regarding the framework presented are given in sections 6 and 7.

2. Modeling of Crack Growth

We restrict our attention to the problem of finding the optimum inspection schedule for a component having a two-dimensional surface-breaking crack with depth ‘ a ’, located in a tensile field. We assume that the crack depth is much smaller than the component thickness and hence this case is equivalent to a surface breaking crack in a half space (see Figure 1) with stress intensity factor

$$K = 1.12\sigma\sqrt{\pi a} \quad (1)$$

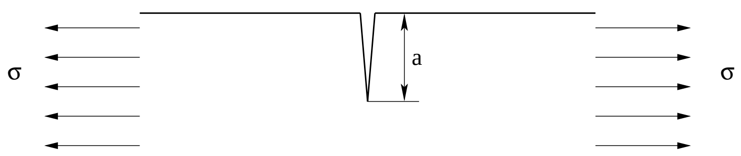

Figure 1: Surface-breaking crack in a tensile field

For cyclic loading, a commonly used crack growth law is Paris law, which is written as

$$\frac{da}{dN} = A\left(\Delta K\right)^m , \quad (2)$$

where N is the number of cycles, da/dN is the rate of crack growth, A and m are material parameters and ΔK is the amplitude of the stress intensity factor. For constant amplitude loading and by substituting Eq. (1), we can integrate Eq. (2) ($m \neq 2$) to get the crack depth after N cycles as

$$a^{1-m/2} = a_0^{1-m/2} + N\,A\left(1-\frac{m}{2}\right)\left(1.12\Delta\sigma\sqrt{\pi}\right)^m . \quad (3)$$

Here a_0 is the crack depth at $N = 0$. As mentioned earlier, we account for the inherent uncertainty in the fatigue process by treating the initial crack length a_0 as random variables with a known distribution.

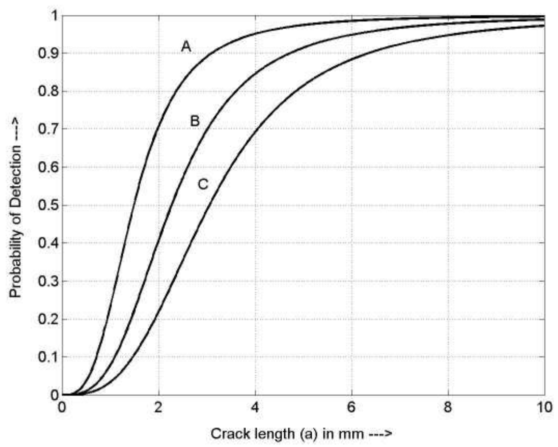

Figure 2. POD curves. A: $\alpha = 0.3$ mm$^{-\gamma}$, $\gamma = 3.0$
B: $\alpha = 0.085$ mm$^{-\gamma}$, $\gamma = 3.0$; C: $\alpha = 0.035$ mm$^{-\gamma}$, $\gamma = 3.00$

3. Characterization of the Nondestructive Inspection Technique

A common approach to model the inherent uncertainty in a nondestructive inspection technique to monitor crack size is to specify the probability of detection (POD) as a function of crack size. In the present paper the POD curves are assumed to be represented by the Log-Odds-Log scale model

$$POD(a) = \frac{\alpha a^{\gamma}}{1+\alpha a^{\gamma}} \tag{4}$$

where a is the crack depth and α and γ are regression parameters. Typical POD curves for three different inspection techniques labeled 'A', 'B' and 'C' are shown in Figure 2.

4. Optimization of the Inspection Schedule

We now present the framework for finding the optimum inspection schedule for a component subjected to fatigue loading. In the present case, the optimization is carried out by minimizing a predefined cost function which accounts for the cost due to failure, the cost of replacement and also the cost associated with each individual inspection. To mathematically formulate the cost function we make the following assumptions:

- The expected lifetime, N_{LT}, of the component has been estimated.
- The component fails if the crack depth is greater than a predefined critical size, i.e. $a > a_{cr}$.
- The initial density of crack depth in the virgin component is known.
- At each inspection the component is inspected with a technique whose probability of detection (POD) curve is known.
- If during an inspection a crack with depth $a > a_r \left(a_r < a_{cr}\right)$ is detected, the component is replaced. The crack depth density of the replaced component is also known.
- If during an inspection a crack with depth $a < a_r \left(a_r < a_{cr}\right)$ is detected, no action is taken.
- The cost due to failure (c_F), the cost of replacement (c_R) and the cost associated with each individual inspection (c_I) are known.

We formulate a cost function taking into account the assumptions listed above. The optimum inspection schedule is then obtained by minimizing a the cost function using a constrained optimization routine.

5. Numerical Examples

We consider the example of a surface-breaking crack whose crack growth is modeled by Paris law given by Eq. (2). The model parameters are taken as $m = 2.67,\ A = 5.069\times10^{-12},\ \Delta\sigma = 280\ \text{MPa}$ and $R = 0$. We apply the methodology presented in the paper to this problem, to obtain an optimum inspection schedule. We consider an expected lifetime of the component, N_{LT}, of 200,000 cycles. The initial crack depth, a_0, distribution is represented by a lognormal distribution with density

$$f_0(a_0) = \frac{1}{a_0\sigma\sqrt{2\pi}} \exp\left[-\frac{1}{2\sigma^2}\left(\ln(a_0)-\mu\right)^2\right]. \quad (5)$$

Here μ and σ are the mean and the standard deviation of the random variable $y = \ln(a_0)$. The mean and the standard deviation of the initial crack depth a_0 are taken to be 0.250 mm and 0.1 mm, respectively. For simplicity we also assume that the crack depth distribution in a replaced component is the same as the initial crack depth distribution, i.e.,

$$f_R\left(a_{N_i}\right) = f_0(a_0), i = 1, 2, \ldots.$$

For most of the simulations, the maximum tolerable crack depth (crack depth which does not require replacement), a_r, and the critical crack depth, a_{cr}, are taken to be 0.3 mm and 3.0 mm, respectively. The value of $a_r = 0.3$ mm is chosen because a crack with an initial depth of 0.3 mm will not reach the critical size during the expected lifetime under the assumed parameters of the crack growth law. We assume that we have three inspections techniques available for inspection with POD's given in Figure 2. As seen from the figure, the technique with POD A is the best, while the technique with POD C is the worst technique. To avoid assuming absolute values for the failure cost c_F, replacement cost c_R and inspection cost c_I, the replacement cost and the inspection cost are expressed in terms of the failure cost. We assume $c_R / c_F = 0.01$, $c_I^A / c_F = 0.001$, $c_I^B / c_F = 0.0001$, $c_I^C / c_F = 0.00001$. Here c_I^A, c_I^B and c_I^C are the inspection costs associated with techniques having POD A, POD B and POD C, respectively. In assuming these particular ratios for the inspection costs, we have accounted for the fact that based on the POD curves we expect $c_I^A > c_I^B > c_I^C$.

To calculate the probabilities required to evaluate the cost function we need to carry out numerical integration of integrands over infinite intervals of the type $\left(a^*, \infty\right)$ where $a^* = a_r$ or $a^* = a_{cr}$. We first transform such integrals to integrals defined over a finite interval $(0,1)$ using the transformation $x = e^{-\left(a-a^*\right)}$, and then use the adaptive quadrature routines to evaluate the integrals.

The minimization of the cost function is carried out using the *fmincon* function available in the Optimization Toolbox of MATLAB®. For the problem under consideration, *fmincon* attempts to find the solution to a constrained minimization problem with the Sequential Quadratic Programming (SQP) algorithm. For more details, we refer to the user guide of the Optimization Toolbox for use with MATLAB®.

We now present the results obtained for the case of a single inspection, two inspections and multiple(four) inspections. Note that the computed costs are rounded to five decimal places and the cycle numbers are rounded to the nearest hundredth cycle.

6.1 Case of a Single Inspection

Following the assumptions presented earlier, a decision tree for the case of a single inspection in the interval $[0, N_{LT}]$ is shown in Figure 3.

For the purpose of describing the decision tree, we assume that the virgin component has a single crack with known initial depth. The crack grows due to fatigue up to cycle N_1 at which point the component is subjected to an inspection. At the inspection we either detect or do not detect the crack. There are probabilities associated with each of these two events which depend on the inspection technique through the probability of detection curves. If a crack is detected

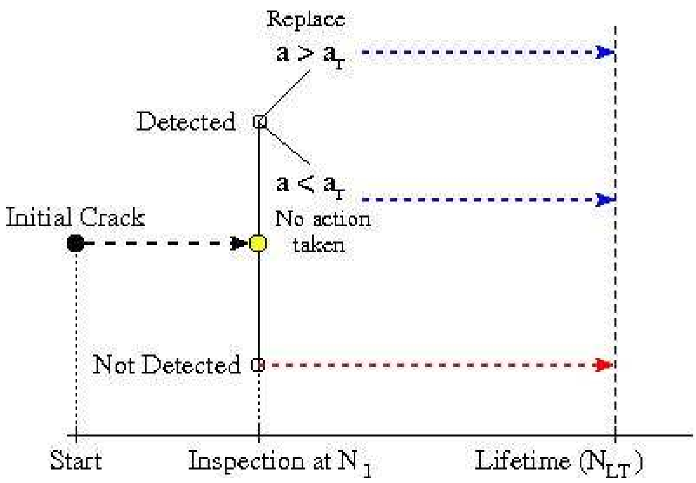

Figure 3: Decision Tree for the Case of Single Inspection

then depending on the depth of the detected crack, we decide whether to replacement the component or not. After the inspection, the undetected crack or the detected crack $a < a_r$, or a crack in the replaced component continues to evolve up to cycle N_{LT}. We can account for the stochastic nature of the fatigue process by treating the crack growth process (see Eq. (3)) in a probabilistic sense.

Based on the decision tree, we can now identify three different costs with the fatigue process and the inspections in the interval $[0, N_{LT}]$ - the expected cost of failure in the interval $[0, N_{LT}]$, $C_F(N_1)$, the expected cost that the component is replaced at the inspection, $C_R(N_1)$, and the cost of the inspection, C_I. For the case that the inspection is after N_1 cycles, we define a total cost function C_T in terms of the relevant probabilities, as follows

$$\begin{aligned} C_T(N_1) &= c_F \times \Pr\left(a > a_{cr} \text{ in the interval } [0, N_{LT}]\right) + c_R \times \Pr\left(a > a_r \text{ and detected at } N_1\right) + c_I, \\ &\equiv C_F(N_1) + C_R(N_1) + C_I. \end{aligned} \tag{6}$$

Here the first term is the expected cost of failure in the interval $[0, N_{LT}]$, the second term is the expected cost that the component is replaced after the inspection and the last term is the cost of the inspection itself. Observing that the probability that the component fails in the interval $[0, N_{LT}]$ is given by the sum of the probabilities that the component fails in the interval $[0, N_1]$ and the interval $[N_1, N_{LT}]$, we rewrite Eq. (6) as

$$C_T(N_1) = c_F \times \left[\Pr\left(a > a_{cr} \text{ in the interval } [0, N_1]\right) + \Pr\left(a > a_{cr} \text{ in the interval } [N_1, N_{LT}]\right)\right] + c_R \times \Pr\left(a > a_r \text{ and detected at } N_1\right) + c_I. \quad (7)$$

Note that the term $\Pr\left(a > a_{cr} \text{ in the interval } [N_1, N_{LT}]\right)$ is given by the sum of the probabilities that $a > a_{cr}$ along each of the three branches arising after the inspection as shown in Figure 3. To find the optimum schedule for the inspection, Eq. (7) is then minimized subject to the constraint $0 \le N_1 \le N_{LT}$.

To efficiently carry out the minimization we need to develop efficient means to evaluate the cost function. One way to evaluate the cost function would be to use Monte Carlo simulation to calculate the probabilities appearing in Eq. (7). But this is a computationally intensive task. For the case in which the only random variable in Eq. (3) is the initial crack size, a_0, we have developed expressions for the crack depth density which can be efficiently integrated using numerical integration to give the required probabilities. Details are given in Ref. [3].

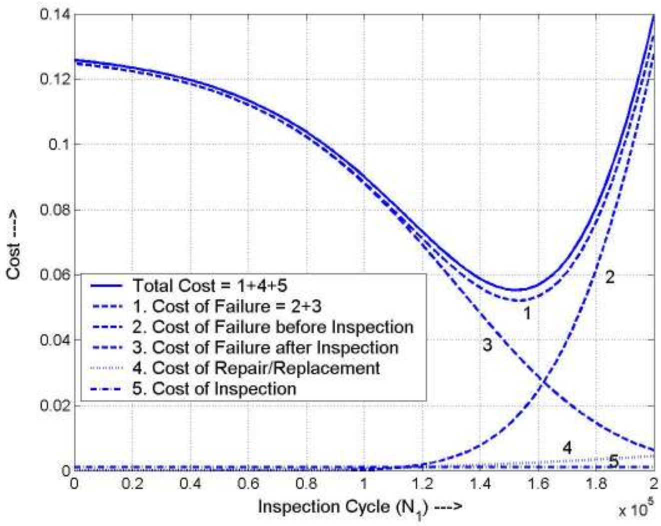

Figure 4: Variation of Cost versus Inspection Cycle (POD A)

Before finding the optimal schedule, we first graphically illustrate the variation of the cost function (see Eq.5) as a function of the inspection cycle number (see Figure 4). These results have been obtained when the inspection technique has POD A, but they are qualitatively similar to the ones obtained for POD B or POD C. It is seen from the figure that we do have a minimum in the variation of the total cost function and this minimum defines the optimum number of cycles for an inspection. It is also seen from the figure that the cost of failure (curve 1) is the dominant contributor to the total cost function. The variation of the total cost function can therefore be explained by focusing on the variation of the cost of failure. The cost of failure as a function of inspection cycle N_1 consists of two contributions :– a) the cost of failure in the interval $\left[0, N_1\right]$ (curve 2), and b) the cost of failure in the interval $\left[N_1, N_{LT}\right]$ (curve 3).

We now present the individual costs and the number of cycles for the optimum inspection schedule by minimizing the cost function given by Eq. (5). We have used the three POD's (see Figure 2) to compare the effect of inspection quality on the inspection schedule. The results appear in Table 1. Here C_T, C_F, C_R and C_I denote the expected total cost, the expected cost due to failure, the expected cost of replacement and the total cost of the inspection, respectively (see section 4.3). The percentage contributions of the expected cost due to failure, expected replacement cost and the inspection cost to the expected total cost are also indicated in Table 1.

Table 1: Inspection Schedule for the Case of Single Inspection

POD	C_T	C_F	C_R	C_I	N_1
A	0.05533	0.05218 (~ 94%)	0.00215 (~ 4%)	0.00100 (~ 2%)	152500
B	0.08660	0.08533 (~ 99%)	0.00117 (~ 1%)	0.00010 (~ 0%)	157600
C	0.10880	0.10823 (~ 100%)	0.00056 (~ 0%)	0.00001 (~ 0%)	152400

We can make the following observations from Table 1:

1. Though the number of cycles for the inspections is nearly the same for the three POD's for the problem under consideration, the expected total cost is significantly different for the three POD's.
2. The expected cost due to failure is the major contributor towards the expected total cost for all the three cases.
3. As expected, the cost of replacement increases as the quality of inspections improves.

6.2 Case of Two Inspections

We now present the formulation of the cost function and discuss methods for its efficient calculation for the case of two inspections. The decision tree for the case of two inspections is shown in Figure 5.

Similar to the case of a single inspection, we associate three different costs – the expected cost of failure in the interval $[0, N_{LT}]$, the expected cost of replacement at the inspections and the cost of inspections, and we define the total cost function as follows:

$$
\begin{aligned}
C_T(N_1, N_2) &= c_F \times \Pr\left(a > a_{cr} \text{ in the interval } [0, N_{LT}]\right) \\
&\quad + c_R \times \left[\Pr\left(a > a_r \text{ and detected at } N_1\right) + \Pr\left(a > a_r \text{ and detected at } N_2\right)\right] + 2c_I \\
&\equiv C_F(N_1, N_2) + C_R(N_1, N_2) + C_I
\end{aligned}
\tag{8}
$$

The first term in Eq. (8) is the expected cost of failure in the interval $[0, N_{LT}]$, the second term is the expected cost if the component is replaced at the inspections and the last term is the cost of the inspections. For simplicity we have assumed that both the inspections are of the same type

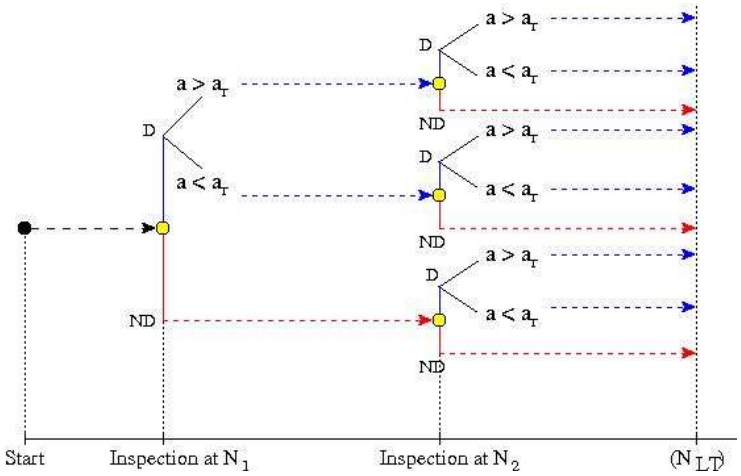

Figure 5: Decision Tree for the Case of Two Inspections
(D: Detected, ND: Not Detected)

and hence the cost of the inspections is twice the cost of a single inspection. Equation (8) can be rewritten as

$$
\begin{aligned}
C_T(N_1, N_2) &= c_F \times \begin{bmatrix} \Pr\left(a > a_{cr} \text{ in the interval } [0, N_1]\right) + \Pr\left(a > a_{cr} \text{ in the interval } [N_1, N_2]\right) + \\ \Pr\left(a > a_{cr} \text{ in the interval } [N_2, N_{LT}]\right) \end{bmatrix} \\
&\quad + c_R \times \left[\Pr\left(a > a_r \text{ and detected at } N_1\right) + \Pr\left(a > a_r \text{ and detected at } N_2\right)\right] + 2c_I
\end{aligned}
$$

(9)

where we have made use of the fact that the probability that the component fails in the interval $[0, N_{LT}]$ is given by the sum of the probabilities that the component fails in the interval $[0, N_1]$, $[N_1, N_2]$ and the interval $[N_2, N_{LT}]$. The optimum schedule is then obtained by minimizing the cost function given by Eq. (9) subject to the constraint $0 \le N_1 \le N_2 \le N_{LT}$.

We now discuss efficient ways to evaluate the cost function. We observe that all the probabilities in Eq. (9), except $\Pr\left(a > a_{cr}\right.$ in the interval $\left[N_2, N_{LT}\right]\big)$, which is the probability that the component fails in the interval $\left[N_2, N_{LT}\right]$, can be calculated by using the expressions of the crack depth densities derived in Ref. [3]. The expression for the probability of failure in the interval $\left[0, N_1\right]$ is given by Eq. (31) of Ref. [3], and the expression for the probability that the component is replaced at the first inspection is given by Eq. (33) of Ref. [3].

For the case of two inspections. Figure 6 graphically illustrates the variation of the expected total cost as a function of the first inspection at N_1 followed by a second inspection at N_2. We have considered the case that the quality of both the inspections can be represented by POD A. It is seen from the figure that a minimum does exists for the expected total cost and the values of N_1 and N_2 corresponding to the minimum are the cycles at which the inspections should be scheduled.

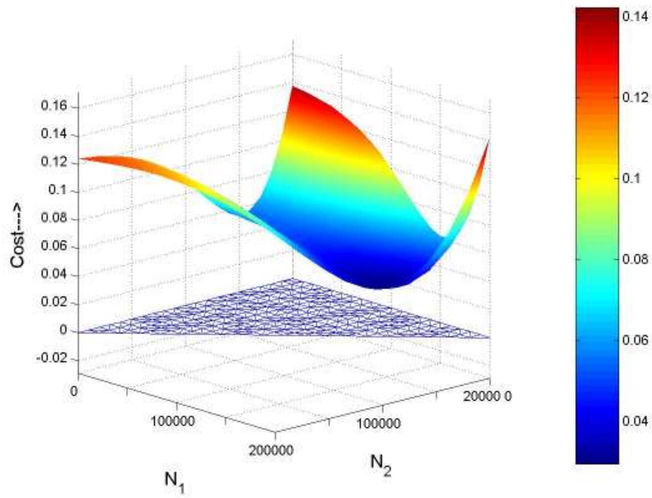

Figure 6: Variation of Total Cost versus Inspection Cycles (POD A)

The optimum schedule and the individual contributions of the different costs (failure, replacement and inspections) toward the total cost are listed in Table 2.

Table 2: Inspection Schedule for the Case of Two Inspections

POD	C_T	C_F	C_R	C_I	N_1	N_2
A	0.02922	0.02393 (~ 82%)	0.00329 (~ 11%)	0.00200 (~ 7%)	135300	161300
B	0.06082	0.05873 (~ 97%)	0.00189 (~ 3%)	0.00020 (~ 0%)	147400	164100
C	0.09234	0.09132 (~ 99%)	0.00100 (~ 1%)	0.00002 (~ 0%)	148400	158000

We can make the following observations which from Table 2:

1. The optimum schedule obtained is significantly different for the three POD's.
2. Comparison with Table 1 shows that as the number of inspections increases, the contribution of the cost of failure to the total cost decreases, while the contributions of the cost of replacement and the cost of inspections increase.
3. The expected cost of failure is again the major contributor to the expected total cost but in a significantly different proportions for the three cases.
4. The expected total cost when compared to the case of the single inspection does not decrease uniformly in the three cases. The total cost decreases by about 1.9 times, 1.4 times and 1.2 times for POD A, POD B and POD C, respectively.

6.3 Case of Multiple Inspections

The cost function is now formulated for the case of case of scheduling n inspections at cycles $N_1,\ldots,N_n$ in the interval $[0, N_{LT}]$. As before, we define a cost function, which we wish to minimize, as follows

$$C_T(N_1,\ldots,N_n) = C_F(N_1,\ldots,N_n) + C_R(N_1,\ldots,N_n) + C_I. \tag{10}$$

Here $C_T(N_1,\ldots,N_n)$ is the expected total cost in the interval $[0, N_{LT}]$, $C_F(N_1,\ldots,N_n)$ is the expected cost due to failure in the interval $[0, N_{LT}]$, $C_R(N_1,\ldots,N_n)$ is the expected cost of replacement in the interval $[0, N_{LT}]$ and C_I is the cost of the inspections. The expected cost due to failure $C_F(N_1,\ldots,N_n)$, the expected replacement cost $C_R(N_1,\ldots,N_n)$ and the cost of the inspection C_I now written as:

$$C_F\left(N_1,\ldots,N_n\right)=c_F\times\Pr\left(a>a_{cr}\text{ in the interval }\left[0,N_{LT}\right]\right)$$

$$=c_F\times\begin{bmatrix}\Pr\left(a>a_{cr}\text{ in the interval }\left[0,N_1\right]\right)+\\ \Pr\left(a>a_{cr}\text{ in the interval }\left[N_1,N_2\right]\right)+\ldots+\\ \Pr\left(a>a_{cr}\text{ in the interval }\left[N_n,N_{LT}\right]\right)\end{bmatrix}, \tag{11}$$

$$C_R\left(N_1,\ldots,N_n\right)=c_R\times\left[\Pr\left(a>a_r\text{ and detected at }N_1\right)+\ldots+\Pr\left(a>a_r\text{ and detected at }N_n\right)\right], \tag{12}$$

and

$$C_I=n\,c_I, \tag{13}$$

respectively. Note that for simplicity we have assumed that all the inspections are of the same type. It is clear that we need expressions for the crack depth density in the intervals $\left[0,N_1\right],\left[N_1,N_2\right],\left[N_1,N_3\right],\ldots$ and $\left[N_n,N_{LT}\right]$ to compute the probabilities necessary to evaluate the cost function. This implies that we need expressions for the crack depth density after one inspection, two inspections, … and n inspections. Proceeding in a manner similar to the one described for the case of a single inspection and two inspections, we can derive an expression for the crack depth density after an arbitrary number of inspections. Ref. [3] gives an expression for the crack depth density after $p\left(p=1,2,\ldots\right)$ inspections at cycles $N_1,\ldots,N_p$, with $0\le N_1\le\ldots\le N_p$.

The optimum schedules obtained for the case of four inspections are shown in Table 3. In addition to the optimum schedule, the tables also list the individual contributions of the different costs (failure, replacement and inspection) towards the total cost.

Table 3: Inspection Schedule for the Case of Four Inspections

POD	C_T	C_F	C_R	C_I	N_1	N_2	N_3	N_4
A	0.01460	0.00610 (~ 42%)	0.00450 (~ 31%)	0.00400 (~ 27%)	111600	132900	151600	168400
B	0.03277	0.02956 (~ 90%)	0.00281 (~ 9%)	0.00040 (~ 1%)	132200	146600	159300	170800
C	0.06711	0.06517 (~ 97%)	0.00190 (~ 3%)	0.00004 (~ 0%)	140500	149400	157400	165000

Most of the observations which can be made from Table 3 are similar to ones made from Table 2 which lists the results for the case of two inspections. Hence to avoid repetition they are not repeated here. We can make the following observations which are specific to

Table 3:

1. The intervals between consecutive inspections gets progressively smaller for all three cases, i.e. $\left(N_{i+1}-N_i\right)<\left(N_i-N_{i-1}\right), i=2,3,\ldots$. This is to be expected due to the nonlinear nature of the crack growth law.
2. The expected total cost when compared to the case of the single inspection does not decrease uniformly in the three cases. The total cost decreases by about 3.8 times, 2.6 times and 1.6 times for POD A, POD B and POD C, respectively.

Next we summarize the different inspection schedules obtained when using a single technique but with different number of inspections for the case that the quality of all the inspections can be represented by POD A.

Table 4: Inspection Schedule for Different Number of Inspections

No. of Inspections	N_1	N_2	N_3	N_4
1	152500	-	-	-
2	135300	161300	-	-
3	122200	145600	165800	-
4	111600	132900	151600	168400

We can make the following observations from Table 4:

1. The number of inspections have a significant effect on the cycle numbers at which the inspections are carried out, e.g. the cycle at which the first inspection is carried out is significantly different for the cases of one, two, three and four inspections.
2. There is a pattern to the inspection cycles selected based on the optimum inspection schedule. If we increase the number of inspections from n to $n+1$, we find that the new inspection schedule, i.e. cycles $N_1,\ldots,N_{n+1}$ are distributed in the $n+1$ intervals formed by the inspection schedule obtained for n inspections. For example, when we increase the number of inspections from 2 to 3, we observe that $122200\in[0,135300]$, $145600\in[135300,161300]$ and $165800\in[161300,200000]$. This observation implies that as the number of inspections increases, the first and the last inspections are scheduled progressively earlier and later, respectively.

Finally we investigate the influence of a_r on the optimum schedule and the associated costs. Recall that if during an inspection a crack with depth $a>a_r$ is detected, the component is replaced. We will present the case four inspections that are represented by POD A. The results are illustrated in Table 5

Table 5: Dependence of the schedule on a_r

a_r (mm)	C_T	C_F	C_R	C_I	N_1	N_2	N_3	N_4
0	0.01460	0.00609	0.00451	0.00400	111600	132900	151600	168400
0.3	0.01460	0.00610	0.00450	0.00400	111600	132900	151600	168400
0.6	0.01418	0.00610	0.00408	0.00400	111300	132700	151500	168400
0.9	0.01375	0.00610	0.00365	0.00400	110900	132800	151500	168100
1.2	0.01350	0.00693	0.00257	0.00400	108600	133900	153500	170500
1.5	0.01504	0.00915	0.00189	0.00400	110500	139300	155900	170400
1.8	0.02498	0.02498	0.01950	0.00400	72400	125600	159400	173200

We can make the following observations from Table 5:

1. The inspection schedule and the associated costs are not overly sensitive to the different values of a_r (except $a_r = 1.8$ mm).
2. The total cost decreases with increasing a_r, reaches a minimum ($a_r = 1.2$ mm) and then increases.
3. The variation in the total cost as function of a_r can be understood in terms of the variation in the cost of failure and the cost of replacement. The initial decrease in the total cost is due to a substantial reduction in the cost of replacement accompanied by a slight increase in the cost of failure. The increase in the total cost is due to a significant increase in the cost of failure which cannot be compensated for by the reduced cost of replacement.

We have presented the results for the case of four inspections with POD A, but similar trends were observed for the cases of one, two and three inspections with POD's A, B and C.

7. Concluding Remarks:

An analytical method has been presented to optimize the inspection schedule for monitoring the growth of fatigue cracks. Explicit results are given for a two-dimensional surface-breaking crack. The point of departure of the analysis is a crack defined by a probability density of crack depths whose growth is governed by Paris law, and an inspection technique with a known probability of detection. Two critical crack lengths are defined, a crack length a_r at which a component will be replaced, and a crack length a_{cr}, at which the component fails. A total cost function is defined as a sum of cost functions for failure, component replacement and inspection. For the general case of n inspections the cost functions are expressions in terms of probabilities that $a > a_r$ and $a > a_{cr}$. For a pre-determined number of inspections during the expected lifetime, the total cost function has been minimized to obtain the optimal number of cycles for each of the inspections. Analytical details and numerical results are given for the case of one, two and multiple inspections. The main conclusions which can be drawn from the numerical simulations are as follows:

1. For a pre-defined number of inspections, the optimum schedule strongly depends on the POD of the inspection technique.
2. Increasing the number of inspections does not necessarily imply a proportional decrease in the total cost. The reduction in the total cost which can be expected depends on the POD of the inspection technique.
3. The value $a_r = 0$ corresponds to the case in which the component is replaced as soon as a crack is detected. The size of the detected crack is not taken into account when making this decision. So far, only this case has been considered in the literature. The results in the current paper indicate there exists an optimum value of a_r which reduces the total cost and does not significantly affect the optimum schedule.

The main utility of the framework presented here, along with the expressions to evaluate the cost functions, lies in the fact that the use of computational intensive techniques to find the optimum schedule can be avoided completely at least for a simple crack growth law. Use of commonly available tools like MATLAB® can be made, to explore the effects of the number of inspections, and the type of inspections on the inspection schedule.

It is important to point out that the derivation of expressions of the probabilities which can be easily computed was facilitated by the following assumptions:

1. Crack growth can be represented by Paris law.
2. The initial crack length is the only random variable.

For more general cases, alternative methods like the Monte Carlo integration may have to be used.

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References:

1. S S Kulkarni, L Sun, B Moran, S Krishnaswamy, and J D Achenbach, ‘Probabilistic Method to Predict Fatigue Crack Initiation’, International Journal of Fracture, 137:9-17, 2006.

2. S S Kulkarni and J D Achenbach, ‘Structural Health Monitoring and Damage Prognosis in Fatigue’, Structural Health Monitoring, in press.

3. S S Kulkarni and J D Achenbach, ‘Optimization of Inspection Schedule for a Surface-breaking Crack Subject to Fatigue Loading’, Probabilistic Engineering Mechanics, in press.