ISOLATION OF THE RELATIVE DECREASE IN MAGNETIC PERMEABILITY CONTRIBUTION TO RELUCTANCE OF AN OPEN MAGNETIC CIRCUIT BASED ON STRAY FIELD MEASUREMENTS

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ABSTRACT

The goal of the study was to evaluate a relative decrease in magnetic permeability in open magnetic circuits based on stray magnetic field measurements. The boundary element method was used to simulate magnetic circuits with variable geometry and variable magnetic permeability. The simulated magnetic circuits were rectangular or bone-shaped flat samples. Reluctance was increased in the middle of the samples resulting in leakage of magnetic flux. An analysis was performed to determine relations between the leakage and factors that increase the reluctance of the samples. These factors were the cross-sectional area and magnetic permeability. In case of strong demagnetization of a ferromagnetic sample, an approach to quantitative evaluation of a stray magnetic field distribution was proposed. Results of the analysis indicate that the amplitude of the tangential component of the stray magnetic field increases with the increasing absolute value of the relative change of the reluctance. The same tendency was observed in the case of the gradient of the normal component. The decrease in the cross-sectional area of the sample (and more precisely its width) has similar influence on the stray magnetic field as the decrease in the magnetic permeability. These two factors were also studied in the coincidence and the possibility of isolation of information about a relative decrease in the magnetic permeability was verified. The isolation scheme was formulated and validated based on experimental as well as numerical data. Results of the validation indicate that it is possible to make a rough estimation of a local reduction in the magnetic permeability based on stray field measurements.

KEYWORDS: Magnetic Permeability, Magnetic Reluctance, Stray Magnetic Field, Boundary Element Method

1. INTRODUCTION

The classical concept of magnetic reluctance refers to closed or quasi-closed magnetic circuits with small air gaps. The fraction of magnetic flux that diffuses into an ambient space is negligible in such cases. Nevertheless, a local increase in the reluctance of a magnetic circuit results in leakage of magnetic flux, which can be detected via stray magnetic field (SMF) measurements. A local reduction of magnetic permeability caused by plastic deformation is one of sources of the increased reluctance. Therefore, measurements of SMF can be potentially used to solve an inverse problem, namely to determine a value of plastic strain. Regions of magnetic permeability decreased by plastic strain can be accompanied by geometric discontinuities such as changes in a cross-sectional area [1]. A relative decrease in magnetic permeability and a local change of a cross-sectional area in very similar way contribute to creation of the leakage. The motivation of this study was to verify the possibility of isolation of these two contributions, and thus to evaluate the relative decrease in magnetic permeability of the investigated sample.

2. NUMERICAL ANALYSIS

Parametric simulations using the boundary element method (BEM) were performed. The object under consideration was a sample made of a structural steel placed in a weak external magnetic field. The simulated sample, presented in Fig. 1, was a projection of the sample used in previous experiments [2,3]. Measurements of initial magnetic permeability were performed for the material of the sample. The relative value of the initial magnetic permeability μ1 was equal to 240. A magnetic field of 50 μT was applied coaxially to the sample longest dimension. First series of simulations was made for the variable width w₂ of the middle part of the bone-shaped sample presented in Fig. 1. The length of the middle part has a fixed value of 40 mm. The radius of curvature was also fixed at 40 mm. The width w₂ was changed in the range of 0-52 mm. The width w₁ of the sample endings was constant and equal to 52 mm. The
relative change of the sample width, $\delta w = (w_2 - w_1)/w_1$, was used to assign each simulation in this series. The numerical model shown in Fig. 1 had three symmetry planes. Therefore, a number of boundary elements could be reduced eight times. Mesh adaptation was performed using the h-refinement strategy to minimize the discretization error. The mesh was refined by increasing the number of all elements in the model (by ~1000 of elements for each adaptive step). The supremum of a difference between two consecutive solutions, $\sup\{0 < x < 120 \mid |B_i(x) - B_i(x-1)|\} \leq 0.5 \mu T$, was adopted as the convergence criterion (where $B_i$ is one of components of the magnetic induction determined 2 mm above the top surface of the BEM model).

![Fig. 1 Dimensions of the sample analyzed in first series of simulations and the corresponding geometry used for the BEM analysis.](image)

Results of the series of simulations performed for the variable $\delta w$ are presented in Fig. 2. The component of the magnetic induction being tangential to the longest dimension of the sample depicted in Fig. 1 was assigned as $B_x$. The component of the magnetic induction being normal to the top surface of the sample was assigned as $B_z$. However, in this study, the distributions of the gradient $\partial B_z/\partial x$ were analyzed as they are very similar to those of $B_x$. It can be seen that, when $\delta w$ changes from 0 to -100%, amplitudes of SMF distributions increase both for $B_z$ and for $\partial B_z/\partial x$. The main difference between them is opposite polarity of their distributions. Therefore, the same approach to quantitative analysis of these two kinds of distributions was applied.
Fig. 2 SMF distributions: a) $B_x$ and b) $\partial B_z/\partial x$; for different values of $\delta w$.

Second series of simulations was made for the variable magnetic permeability $\mu_2$ of the middle part of the rectangular sample depicted in Fig. 3. The same mesh adaptation method and boundary conditions were used as in first series of simulations. The influence of cross-section changes in the middle part of the sample was eliminated in this series of simulations. The value of $\mu_2$ was changed in the range of 0-240 to simulate a magnetic permeability change caused by plastic deformation. This approach to modelling of magnetic behaviour of plastically deformed samples was applied in the study of Yao et al. [4]. The relative change of the permeability, $\delta \mu = (\mu_2 - \mu_1)/\mu_1$, was used to assign each simulation in this series.

Fig. 3 Dimensions of the sample analyzed in second series of simulations and the corresponding geometry used for the BEM analysis.
SMF distributions determined for different values of $\delta\mu$ are presented in Fig. 4. These distributions exhibit higher slope in a region of reluctance transition than those presented in Fig. 2. It is a consequence of a step change of the magnetic permeability of the sample, while the change of the width was gradual in first series of simulations.

![Fig. 4 SMF distributions: a) $B_x$ and b) $\partial B_x/\partial x$; for different values of $\delta\mu$.](image)

Quantitative analysis of the results presented in Fig. 2 and Fig. 4 is impeded by the demagnetization effect. This effect is significant for open magnetic circuits, which the considered samples belong to. Consequently, direct determination of a peak-to-peak value of an SMF distribution is possible only in few cases. Another quantity was defined, i.e. the amplitude $\Delta B_x$, as an alternative for the peak-to-peak value. It was the difference between an extremum of the middle part of an SMF distribution and a reference value at an inflection point of a side part of the distribution. First step to determine the reference value is to find an inflection point, where the second derivative of a distribution curve changes its sign. The distribution of $B_x$ for $\delta\mu=-25\%$ with its second derivative is presented in Fig. 5a. The distribution of $B_x$ was computed numerically, and thus the second derivative $B_x''$, presented in Fig. 5a, was determined using the symmetric finite difference formula. Numerical differentiation magnified numerical noise included in the $B_x$ distribution. As a result, uncertainty of inflection point determination was increased. Alternatively, the right part of the $B_x$ distribution was firstly approximated with the use of a fifth order polynomial $W_5$ and then differentiated analytically. This operation enabled to find the exact zero of the polynomial $W_5''$, and thus also the reference value for $\Delta B_x$ determination. The same approach was applied to analyze $\partial B_x/\partial x$ distributions.
Fig. 5 (a) $B_x(x)$ with its second derivative $B_x''(x)$ for $\delta \mu = -25\%$. (b) Their zoom with a smoothing polynomial $W_5$ and its second derivative $W_5''$.

Fig. 6 shows that both amplitudes $\Delta B_x$ and $\Delta (\partial B_z/\partial x)$ have qualitatively the same relation with $\delta w$ and $\delta \mu$. This results confirmed that the concept of the reluctance formulated for a closed magnetic circuit is also applicable to an open magnetic circuit, even though the magnetic flux is not conserved in the latter case. Both relations presented in Fig. 6b can be interchangeably used as calibration data for solving the inverse problem, i.e. to find an unknown value of $\delta \mu$.

Third series of simulations included simultaneous changes of $\delta w$ and $\delta \mu$. All three series of simulations refer to samples used in the previous study [2]. These samples were characterized by a specific combination of $\delta w$ and $\delta \mu$. Values of these two parameters for each of modelled samples were included in Table 1.
Table 1. Values of $\delta w$ and $\delta \mu$ characterizing each of modelled samples (denotation of the samples used in this study differs from that used in [2]).

<table>
<thead>
<tr>
<th>Sample</th>
<th>$\delta w$ [%]</th>
<th>$\delta \mu$ [%]</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>-77</td>
<td>0</td>
</tr>
<tr>
<td>B</td>
<td>0</td>
<td>-50</td>
</tr>
<tr>
<td>C</td>
<td>-77</td>
<td>-50</td>
</tr>
</tbody>
</table>

3. ISOLATION SCHEME

Results of $B_x$ experimental measurements and results of BEM simulations are presented side by side in Fig. 7. Despite the fact that numerically derived distributions are ideally symmetrical, they are very similar to experimental distributions. Both graphs included in Fig. 7 show that $B_x$ distribution for sample C has the highest amplitude, what is a result of coincidence of two factors increasing the reluctance, i.e. $\delta w$ and $\delta \mu$.

![Fig. 7](image_url) Comparison of $B_x$ distributions obtained (a) experimentally and (b) with the use of BEM for three investigated samples.

Fig. 8 also shows both experimental and numerical results, but they concern $\partial B_z/\partial x$. Experimental values of $\partial B_z/\partial x$ were derived indirectly based on $B_z$ measurements and then interpolated using polynomial splines. Therefore, distributions presented in Fig. 8 are characterized by significant fluctuations. Similar to results depicted in Fig. 7, the amplitude of $\partial B_z/\partial x$ for sample C also is the highest one.
Fig. 8 Comparison of $\frac{\partial B_z}{\partial x}$ distributions obtained (a) experimentally and (b) with the use of BEM for three investigated samples.

Table 2 comprises data about amplitudes of $B_x$ and $\frac{\partial B_z}{\partial x}$ for experimentally investigated samples and their numerical equivalents. Only sample A reveals discrepancy, which is a consequence of determining the values of permeability $\mu_1$ and $\mu_2$, used in simulations, based on sample B (due to technical limitations).

<table>
<thead>
<tr>
<th>Sample</th>
<th>$\Delta B_x$ [µT]</th>
<th>$\Delta(\frac{\partial B_z}{\partial x})$ [mT/m]</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Experiment</td>
<td>Simulation</td>
</tr>
<tr>
<td>A</td>
<td>32.2±2.3</td>
<td>41.0±5.4</td>
</tr>
<tr>
<td>B</td>
<td>19.6±3.1</td>
<td>19.2±2.4</td>
</tr>
<tr>
<td>C</td>
<td>49.6±5.7</td>
<td>58.5±7.6</td>
</tr>
</tbody>
</table>

The idea of the isolation scheme is based on the assumption that experimental data for two objects of similar geometry but different SMF distributions, like samples A and C, are available. Moreover, the calibration curves of $\delta \mu(\Delta B_x)$ or $\delta \mu(\Delta(\partial B_z/\partial x))$ should be derived numerically. Data presented in Table 2 and calibration curves depicted in Fig. 6 were used to estimate the permeability decrease $\delta \mu$ in the middle of sample C. Firstly $\Delta B_x$ for sample A was subtracted from $\Delta B_x$ for sample C. Secondly the value of $\delta \mu$ for the result of the subtraction was read from Fig. 6b. The same procedure was applied with the use of $\Delta(\partial B_z/\partial x)$ data. Due to lower uncertainties of $\Delta B_x$, an estimation of $\delta \mu$ based on them is more reliable than an estimation based on $\Delta(\partial B_z/\partial x)$. The value of $\delta \mu$ determined this way was equal to -43% and was close to experimental value $\delta \mu=-50±13%$ measured for sample B.

4. CONCLUSION

The estimated value of the reduction in magnetic permeability $\delta \mu$ in the case of sample C is in the uncertainty range of the actual value of the permeability reduction experimentally determined for sample B. It can be concluded that results of SMF measurements can be used for rough estimation of a local reduction in magnetic permeability. However, it is necessary to meet two prerequisites. First of them requires availability of a ferromagnetic object with similar geometry to the investigated one but without a local change in magnetic permeability. Such an object plays a role of the reference. Second prerequisite is availability of numerically determined relations between $\delta \mu$ and $\Delta B_x$, or between $\delta \mu$ and $\Delta(\partial B_z/\partial x)$. Future studies will be devoted to similar quantitative analysis but performed for the gradient measured directly with the use of two adjacent sensors.
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