Sound insulation of the plane end walls of a semicylindrical housing

D. Gužas

Šiauliai University, Vilniaus St. 141, LT-5419, Šiauliai, Lithuania

E-mail: danielius.guzas@fondai.com

Abstract

In our previous work [1] a problem is solved for estimation of sound insulation R_c – the lateral wall of the housing on condition that edges are rigid and do not transmit the sound. This is not always accomplished in practice, therefore when designing a question concerning the choice of the parameters of the ends so that they would not worsen R_c comes into forth. In this paper a problem will be considered on the sound insulation of an end wall by making a assumption that an end wall is rigid. In this work also a frequency case will be considered when a point source is located on the axis of a semicylinder.

The problem is solved by series expansion of the sound pressure p_1 inside the housing and the flexural displacement of the wall u in eigenfunctions of the boundary value problem.

Keywords: sound insulation of the plates and shells, sound insulation of a housing.

Introduction

When considering the sound insulation of different machines and mechanisms (compressors, electric engines, ventilators, ducts, etc.), it is recommended to use semicylindrical housings [2].

Previously in our work [3] a problem was considered on the sound insulation of walls of a semicylindrical housing of the limited length. In it, however, the end walls were taken as rigid.

It is of interest to define the sound insulation of end walls and to compare it with the sound insulation of the lateral ones. A problem is not simple, since it is difficult to calculate the insulation of the end outside. As a result, we shall consider a simplified model: a rigid semicylindrical shell is placed on the rigid base so that its radial displacement at $r = a$ and azimuthal displacement at $\varphi=0$ and $\varphi=\pi$ equal zero.

In the problem of a particular case, we shall solve the sound insulation by means of expansion of the sound pressure P_1 inside the housing and the flexure of the wall u in eigenfunctions of the boundary value problem.

By satisfying the boundary functions a displacement of the neutral surface or flexure of the wall u was defined at the end.

General solution of a problem about the sound insulation of end walls of a semicylindrical housing

We shall consider the simplified model: a rigid semicylindrical shell lies on the rigid base so that its radial displacement at $r = a$ and azimuthal displacement at $\varphi = 0$ and $\varphi=\pi$ equal zero (see Fig. 1).

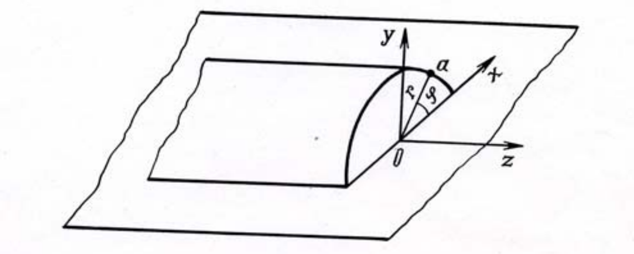

Fig. 1. **Diagram of the arrangement of the coordinate axes**

In this case the sound pressure inside the shell will be constituted of the waves of type

$$F(r,\varphi,z) = P_{nk} \cos n\varphi J_n\left(\alpha_{nk}\frac{r}{a}\right)e^{\pm i\sqrt{k_b^2 - \frac{\alpha_{nk}^2}{a^2}}\cdot z}, \quad (1)$$

where J_n is the Bessel's function of n series, α_{nk} is the root of a dispersion equation

$$J_n'(\alpha_{nk}) = \left[\frac{dJ_n}{dr}\right]_{r=a} = 0,$$

$k_b = \omega/c_b$ is the wave number in air, signs + and – in the exponential index denote waves travelling in the positive and negative directions along the axis z. For simplicity, we shall omit the time factor $\exp(-i\omega t)$ here and further on.

This follows from the fact that $F(r, \varphi, z)$ satisfies Helmholtz's equation

$$\Delta F(r,\varphi,z) + k_b^2 F(r,\varphi,z) = 0, \quad (2)$$

(it is possible to make sure of this by simple substitution) and the above-given boundary conditions.

We shall represent the sound pressure under the housing in the form of three components: $P_n (r, \varphi, z)$ is the primary incident wave on the end wall; $P_n (r, \varphi, z)$ is the reflected wave from the end wall, if it is treated as absolutely rigid and P_{ref} is the wave, reflected from the end wall, which is related to its radiation inside the housing. At $z = 0$, the first two components equal each other, i.e. $P_n(r, \varphi, 0) = P_n(r, \varphi, 0)$. Thus the sound pressure under the housing will be

$$Pb(r,\varphi,z) = P_n(r,\varphi,z) + P_n(r,\varphi,-z) - P_{ref}(r,\varphi,z). \quad (3)$$

An equation of flexural vibrations of the end wall of the housing we shall write in the form of

$$B\Delta\Delta U(r,\varphi) - \omega^2 mU(r,\varphi) = P_b(r,\varphi,0) - P_c(r,\varphi,0). \quad (4)$$

Here $B = Eh^3/12(1-\sigma^2)$ is the flexural rigidity of the wall; $m=\rho h$ is its linear density; E, σ and ρ – Young's module, Poisson's ratio and density of material; h is the wall thickness; u is the displacement of the neutral surface of the wall; P_c is the sound pressure, related to the wall radiation outside. If the conditions of radiation of the end wall inside and outside of the housing are the same, then

$P_{ref}(r,\ \varphi,\ 0)$ and $P_c(r,\ \varphi,\ 0)$ would differ only by the sign, and the right part of Eq.4 will be written as

$$P = 2P_n(r,\varphi,0) - 2P_{ref}(r,\varphi,0). \qquad (5)$$

The solution of the problem in this case will be simplified considerably. Correlation Eq.5 will take place at the frequencies higher than the critical f_k, at which the airborne wavelength λ_B equals the length of the flexural wave λ_n [2] at $2a \gg \lambda_B$. Let us consider a problem in this approximation.

Let us select the conditions for fastening of the end wall so as its flexure $U(r,\ \varphi)$ is expressed in the form of

$$U(r,\varphi) = \sum_{n,k} U^u_{nk} \cos n\varphi I_n\left(\alpha_{nk}\frac{r}{a}\right). \qquad (6)$$

It is necessary to note that in our case it is possible to use Eq.6 practically at any method of fastening of the wall edges, since it has an effect on the form $U(r,\ \varphi)$ only in the region of the first resonances, and they are placed significantly lower of the frequency range under study. In accordance with Eq.1 the sound field of radiation of wall P_{ref} shall be written as

$$P_{ref}(r,\varphi,z) = \sum_{n,k} P_{nk}\cos n\varphi I_n\left(\alpha_{nk}\frac{r}{a}\right) e^{i\sqrt{k_b^2 - \frac{\alpha_{nk}^2}{a^2}}\cdot z}. \qquad (7)$$

At $z = 0$. the boundary condition must be fulfilled

$$U(r,\varphi) = \frac{1}{\rho b\omega^2}\frac{\partial P_{ref}(r,\varphi,z=0)}{\partial z}, \qquad (8)$$

expressing the equation of displacements of particles and the plate. Here ρ_b is the air density. Substituting Eq.6, 7 in Eq.5, 4 and 8, we shall get

$$\left.\begin{aligned} &\sum_{n,k}\left[B\frac{\alpha_{nk}^4}{a^4} - \omega^2 m\right] U_{nk}\cos n\varphi J_n\left(\alpha_{nk}\frac{r}{a}\right) = \\ &= 2P_n(r,\varphi,0) - 2\sum_{n,k} P_{nk}\cos n\varphi J_n\left(\alpha_{nk}\frac{r}{a}\right), \\ &P_{nk} = \frac{\rho_B\omega^2}{i\sqrt{k_B^2 - \frac{\alpha_{nk}^2}{a^2}}}\cdot U_{nk}. \end{aligned}\right\} \qquad (9)$$

Making use of the second expression Eq. 9, we shall transform the first one to the form

$$\sum_{n,k}\left[B\frac{\alpha_{nk}^4}{a^4} - \omega^2 m + 2\frac{\rho b\omega^2}{i\sqrt{k_b^2 - \frac{\alpha_{nk}^2}{a^2}}}\right]\cdot \qquad (10)$$

$$\cdot U_{nk}\cos n\varphi I_n\left(\alpha_{nk}\frac{r}{a}\right) = 2P_n(r,\varphi,0).$$

In order to specify the unknown amplitude of vibrations U_{nk}, it is necessary to represent the wave of excitation P_n in terms of series according to eigenfunctions:

$$P_n(r,\varphi,0) = \sum_{n,k} q_{nk}\cos n\varphi J_n\left(\alpha_{nk}\frac{r}{a}\right), \qquad (11)$$

where

$$q_{nk} = \frac{4}{\pi a^2 J_n^2(\alpha_{nk})\left[1 - \frac{n^2}{\alpha_{nk}^2}\right]}$$

$$\int_0^{\pi}\cos n\varphi d\varphi\int_0^{a} P_n(r,\varphi,0)\cdot I_n\left(\alpha_{nk}\frac{r}{a}\right)\cdot rdr.$$

Then, substituting Eq. 11 to Eq. 10, we shall get:

$$U_{nk} = \frac{2q_{nk}}{\left[B\frac{\alpha_{nk}^4}{a^4} - \omega^2 m + 2\frac{\rho_b\omega^2}{i\sqrt{k_b^2 - \frac{\alpha_{nk}^2}{a^2}}}\right]}, \qquad (12)$$

$$P_{nk} = \frac{2\rho_b\omega^2 q_{nk}}{i\sqrt{k_b^2 - \frac{\alpha_{nk}^2}{a^2}}\left[B\frac{\alpha_{nk}^4}{a^4} - \omega^2 m + \frac{2\rho_b\omega^2}{i\sqrt{k_b^2 - \frac{\alpha_{nk}^2}{a^2}}}\right]}.$$

The sound pressure beyond the edge wall is given by

$$P_c|r,\varphi,0| = \sum_{n,k}\frac{2\rho_b q_{nk}\cos n\varphi J_n\left(\alpha_{nk}\frac{r}{a}\right)}{im\sqrt{k_b^2 - \frac{\alpha_{nk}^4}{a^2}}\left[\frac{\alpha_{nk}^4}{k_n^4 a^4} - 1 + \frac{2\rho_b}{im\sqrt{k_b^2 - \frac{\alpha_{nk}^2}{a^2}}}\right]}. \qquad (13)$$

Sound insulation is defined as

$$R = 10\lg\left|\frac{P_n(r,\varphi,0)}{P_c(r,\varphi,0)}\right|^2.$$

Making use of Eq. 11 and 13, we shall obtain a formula for calculation of sound insulation:

$$R = 10\lg\left|\frac{\sum_{n,k} q_{nk}\cos n\varphi J_n\left(\alpha_{nk}\cdot\frac{r}{a}\right)}{im\sqrt{k_b^2 - \frac{\alpha_{nk}^2}{a^2}}\left[\frac{\alpha_{nk}^4}{a^2} - 1 + \frac{2\rho_b}{im\sqrt{k_b^2 - \frac{\alpha_{nk}^2}{a^2}}}\right]}\right|. \qquad (14)$$

As it is seen from the formula, the sound insulation depends on the conditions of excitation, i.e. on the amplitude q_{nk} in the series expansion of an incident sound wave in eigenfunctions.

Practical special case of sound insulation of the end wall

In this section we shall consider one more case that is found to approach practice more closely: the source is situated on the axis of a semicylinder. In this case the sound field is not dependent on the angle φ. This statement is also valid for any axially symmetric excitation.

Then Eq. 14 will be written as

$$R = 10\lg \left| \frac{\sum_{k=0}^{\infty} J_0\left(\alpha_k \cdot \frac{r}{a}\right)}{\sum_{k=0}^{\infty} \frac{2 q_b J_0\left(\alpha_k \cdot \frac{r}{a}\right)}{im\sqrt{k_b^2 - \frac{\alpha_k^2}{a^2}} \left[\frac{\alpha_k^4}{k_u^4 a^4} - 1 + \frac{2\rho_b}{im\sqrt{k_b^2 - \frac{\alpha_k^2}{a^2}}}\right]}} \right|. \tag{15}$$

The point source is represented mathematically by the Dirac's functions δ. In its series expansion in eigenfunctions of the boundary problem excitation amplitudes will be constant and equal each other. The sound insulation R in this case is equal to

$$R = 10\lg \left| \frac{\sum_{k=0}^{\infty} J_0\left(\alpha_k \cdot \frac{r}{a}\right)}{\sum_{k=0}^{\infty} \frac{2 \rho_b J_0\left(\alpha_k \cdot \frac{r}{a}\right)}{im\sqrt{k_b^2 - \frac{\alpha_k^2}{a^2}} \left[\frac{\alpha_k^4}{k_u^4 a^4} - 1 + \frac{2\rho_b}{im\sqrt{k_b^2 - \frac{\alpha_k^2}{a^2}}}\right]}} \right| \tag{16}$$

Inconvenience of calculation according to Eq. 14, 15 and 16 lies in the dependence of the magnitude of sound insulation on the point coordinates, which the sound field is emitted. This difficulty can be avoided if to define the sound insulation of the end part by means of the relation of the energy of sound waves that passed through the end to the energy of an incident wave.

The excitation wave $P_n(r, \varphi, z)$ is written as

$$p_n(r,\varphi,z) = \sum_n \sum_k q_{nk} \cos n\varphi J_n\left(\alpha_{nk} \frac{r}{a}\right) e^{i\sqrt{k_b^2 - \frac{\alpha_{nk}^2}{a^2}} \cdot z}.$$

At axially symmetric excitation

$$p_n(r,z) = \sum_k q_k J_0\left(\alpha_k \frac{r}{a}\right) e^{i\sqrt{k_b^2 - \frac{\alpha_k^2}{a^2}} \cdot z}. \tag{17}$$

The particle velocity component along the axis z

$$v_z = \frac{1}{i\rho_b\omega}\left(\frac{\partial p_n}{\partial z}\right) = \frac{1}{\rho_b\omega}\sum_k q_k J_0\left(\alpha_k \frac{r}{a}\right) \sqrt{k_b^2 - \frac{\alpha_k^2}{a^2}}\, e^{i\sqrt{k_b^2 - \frac{\alpha_k^2}{a^2}} \cdot z}. \tag{18}$$

The flux of energy $q_z(r)$ along the axis z may be written as

$$q_z(r) = \frac{1}{2}\mathrm{Re}\left(p_n v_z^*\right) = \frac{1}{2}\mathrm{Re}\left[\sum_k q_k J_0\left(\alpha_k \frac{r}{a}\right) e^{i\sqrt{k_b^2 - \frac{\alpha_k^2}{a^2}} \cdot z} \cdot \frac{1}{\rho_b\omega}\sum_n q_n^* J_0^*\left(\alpha_k \frac{r}{a}\right)\left(\sqrt{k_b^2 - \frac{\alpha_k^2}{a^2}}\right)^* \cdot e^{-i\left(k_b^2 - \frac{\alpha_n^2}{a^2}\right)^* \cdot z}\right] \tag{19}$$

Here * denotes a complex-conjugate value.

The total energy of a sound wave which is incident on the end is equal to:

$$Q_z = \int_0^a q_z(r) \cdot \pi r dr.$$

Substituting here $q_z(r)$ from Eq. 19, we obtain:

$$Q_z = \frac{1}{2}\mathrm{Re}\left[\cdot \frac{1}{\rho_b\omega}\sum_k \sum_n q_k q_n^* e^{i\sqrt{k_b^2 - \frac{\alpha_k^2}{a^2}} \cdot z} \cdot e^{-i\left(\sqrt{k_b^2 - \frac{\alpha_k^2}{a^2}}\right)^*} \cdot \left(\sqrt{k_b^2 - \frac{\alpha_k^2}{a^2}}\right)^* \cdot \pi \int_0^a J_0\left(\alpha_k \frac{r}{a}\right) J_0^*\left(\alpha_n \frac{r}{a}\right) r dr\right]. \tag{20}$$

Due to the orthogonality of the Bessel's function the integral in Eq. 20 is equal to zero at $k \neq n$. With $k=n$ it equals to [4]:

$$\int_0^a = \frac{a^2}{2}\left[J_0^2(\alpha_k) + J_1^2(\alpha_k)\right].$$

From the boundary conditions $J_0' - J_1 = 0$ and the second member on the right side is equal to zero.

The integral

$$\int_0^a = \frac{a^2}{2} J_0^2(\alpha_k).$$

By making use of this formula we shall write Eq. 20 in the form of

$$Q_z = \frac{a^2\pi}{4\rho_b\omega}\mathrm{Re}\left[\sum_k |q_k|^2 \left(\sqrt{k_b^2 - \frac{\alpha_k^2}{a^2}}\right)^* J_0^2(\alpha_k)\right]. \quad (21)$$

The summing up here is carried out by these values k, in which $\sqrt{k_b^2 - \frac{\alpha_k^2}{a^2}}$ will be valid, since at an imaginary root the real part of the expression Eq. 21 is equal to zero. At the selected frequency f in this case the following condition should be satisfied

$$k_b a > \alpha k \quad \text{or} \quad \frac{2\pi f a}{c_b} > \alpha_k \quad (22)$$

The last value k, at which Eq. 22 is still valid, we shall denote through k_r.

In an analogous way the energy of the transmitted wave Q_{znp} will also be written, only $|q_k|$ will be substituted by $\left|p_k^2\right|$:

$$Q_{znp} = \frac{\pi a^2}{4\rho_b\omega}\mathrm{Re}\left[\sum_{k=0}^{k_r}\left|p_k^2\right|\left(\sqrt{k_b^2 - \frac{\alpha_k^2}{a^2}}\right)J_0^2(\alpha_k)\right],$$

where

$$\left|p_k^2\right| = \frac{4\rho_b^2|q_k|^2}{m^2\left(k_b^2 - \frac{\alpha_k^2}{a^2}\right)\left|\frac{\alpha_k^4}{k_u^4a^4} - 1 + \frac{2\rho_b}{im\sqrt{k_b^2 - \frac{\alpha_k^2}{a^2}}}\right|^2}. \quad (23)$$

Let us define the sound insulation of the end wall of a semicylinder as

$$R_э = 10\lg\left|\frac{Q_z}{Q_{znp}}\right|.$$

$$R_э = 10\lg\left[\frac{m^2}{4\rho_b^2}\frac{\sum_{k=0}^{k_r}|q_4|^2\sqrt{k_b^2 - \frac{\alpha_k^2}{a^2}}J_0^2(\alpha_k)}{\sum_{k=0}^{k_r}\frac{|q_k|^2J_0^2(\alpha_k)}{k_b^2 - \frac{\alpha_k^2}{a^2}\left[\frac{\alpha_k^4}{k_u^4a^4} - 1 + \frac{2\rho_b}{im\sqrt{k_b^2 - \frac{\alpha_k^2}{a^2}}}\right]^2}}\right]. \quad (24)$$

With the point excitation of the amplitude $|q_4|^2$ =const Eq. 24 transforms into:

$$R_э = 10\lg\left[\frac{m^2}{4\rho_b^2}\frac{\sum_{k=0}^{k_r}\sqrt{k_b^2 - \frac{\alpha_k^2}{a^2}}J_0^2(\alpha_k)}{\sum_{k=0}^{k_r}\frac{J_0^2(\alpha_k)}{\sqrt{k_b^2 - \frac{\alpha_k^2}{a^2}}\left[\frac{\alpha_k^4}{k_u^4a^4} - 1 + \frac{2\rho_b}{im\sqrt{k_b^2 - \frac{\alpha_k^2}{a^2}}}\right]^2}}\right]. \quad (25)$$

If to take into account that $\sqrt{k_b^2 - \frac{\alpha_k^2}{a^2}}$ in Eq. 25 has the real value, then the square of the modulus of an expression in the denominator is equal to:

$$\left[\left(\frac{\alpha_k^4}{k_u^4a^4} - 1\right)^2 + \frac{4\rho_b^2}{m^2\left(k_b^2 - \frac{\alpha_k^2}{a^2}\right)}\right]$$

and Eq. 25 will be equal to

$$R_э = 10\lg\left\{\frac{m^2}{4\rho_b^2 a^2}\times \frac{\sum_{k=0}^{k_r} J_0^2(\alpha_k)\sqrt{k_b^2 a^2-\alpha_k^2}}{\sum_{k=0}^{k_r}\frac{J_0^2(\alpha_k)}{\sqrt{k_b^2a^2-\alpha_k^2}\left[\left(\frac{\alpha_k^4}{k_u^4a^4}-1\right)^2+\frac{4\rho_b^2a^2}{m^2\left(k_b^2a^2-\alpha_k^2\right)}\right]}\right\}. \quad (26)$$

The values of the roots α_k are given in [4] (page 232). The first four roots equal to $\alpha_0=0$; $\alpha_1=3,8317$; $\alpha_2=7,0156$; $\alpha_3=10,1735$. It is possible to define the remaining roots with a precision sufficient for practice from the asymptotic Bessel's function $J_1(-\alpha)=0=\cos(\alpha-3\pi/4)$ from where $\alpha-\frac{3\pi}{4}=\left(-\frac{\pi}{2}+k\pi\right)$ and

$$\alpha_k \approx \frac{\pi}{4}(4k+1),$$

where k is the integer number.

Thus, we have come to the formula convenient for calculation Eq. 13, which does not depend on the coordinate of the point of observation and amplitudes of excitation. Moreover, $R_э$ determines that part of energy, which passes through the edge. This characterizes in the best way its sound insulation in practice.

Conclusions

1. In solving this problem, certain difficulties have been encountered.

2. To make its solution easier two assumptions were made:

- boundary conditions of fastening of the end are chosen such that its eigenfunctions $\psi_{nm}(r)$ according to the radius coincide with $\psi_{nm}(r)$ of the housing;

- the reaction of the outside air to the oscillations of the end wall is neglected. Both these assumptions may lead to the errors only in the area of the first resonances of the end.

3. When considering particular cases, it was taken that under the housing the mode with amplitude q_{nk} is excited, and the remaining are equal to zero. Then

$$R_{nk} = 10\lg\left|1+\frac{im\sqrt{k_b^2-\frac{\alpha_{nk}^2}{a^2}}}{2\rho_b}\left(\frac{\alpha_{nk}^4}{k_n^4a^4}-1\right)\right|^2. \quad (27)$$

This is a standard expression for the sound insulation of the plate. Let us transform it by introducing the designations: $\sin\Theta=\alpha_{nk}/k_b a$, where Θ is the angle between the direction of the propagation of a wave with the normal to the end of the housing (Fig. 2).

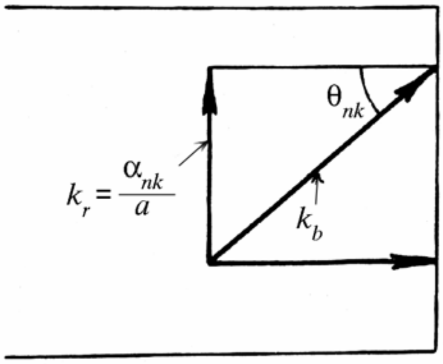

Fig. 2. **Calculation scheme**

4. The radial wave number in our case is equal to $k_r=\alpha_{nk}/a$, the axial to $k_z=\sqrt{k_b^2-k_r^2}=k_b\cos\Theta$. With the account of this, Eq. 27 takes the form:

$$R_{nk} = 10\lg\left|1-\frac{i\omega m}{2\rho_b c_b}\cos\Theta\left(1-\frac{f^2}{f_k^2}\sin^4\Theta\right)\right|^2, \quad (28)$$

where f_k is the frequency of coincidence (critical). Like in the plate of minimum sound insulation modes (n, k) will be at the angles of coincidence, when $\sin\Theta_k=\sqrt{f_k/f}$.

5. As a result Eq. 26, convenient for practical computations, was obtained. This formula does not depend on the coordinates of the point of observations and excitation amplitudes. In addition, the value $R_э$ defines that part of energy, which passes through the edge and characterizes in the best way its sound insulation in practice.

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D. Gužas

Pusiau cilindrinio gaubto galinių plokščių sienelių garso izoliacija

Reziumė

Praktikoje triukšmui mažinti taikomi pusiau cilindriniai gaubtai, kurių galuose įmontuojamos plokščios sienelės. Tirta, kokių savybių įgyja plokščia sienelė, įmontuota į pusiau cilindrinį gaubtą. Tyrinėjant sienelės garso izoliaciją priklausomai nuo triukšmo šaltinio padėties po gaubtu, buvo gautos praktinės reikšmės formulės, kurias galima naudoti projektuojamų gaubtų triukšmo mažinimo efektyvumui nustatyti.

Pateikta spaudai 2007 06 18