3rd European-American Workshop on NDE Reliability
September 10-13, 2002 in Berlin

On the Relation Between Inspection Quantity and Quality

Daniel Straub[1] and Michael Havbro Faber[2]
Swiss Federal Institute of Technology (ETH)

Abstract

The quality of inspection techniques is generally expressed in terms of Probability of Detection (PoD) curves and sometimes in addition by the Probability of False Indication (PFI). Such a format facilitates a quantitative description of the inspection quality and allows for risk based inspection planning (RBI). Existing PoD curves describe the inspection quality implicitly assuming that the entire area or volume subject to possible defects is inspected. Such an assumption stands in gross contrast to most practical applications where full inspection coverage is hardly possible nor relevant and consequently only a percentage of the considered area is inspected.

In the present paper an extension of the classical PoD / PFI concept to systems with only partial inspection is introduced based on theoretical considerations. The proposed approach takes into account the aspects of the interrelation of the spatial variability of the deterioration processes and the coverage of the performed inspections. It is illustrated on an example highlighting how this may be applied for the identification of the optimal inspection coverage in risk based inspection and maintenance planning.

Keywords: Probability of Detection, Inspection coverage, Reliability, Risk Based Maintenance Planning, NDE

1 Introduction

The quantitative modeling of Non-Destructive Evaluation (NDE) quality and its use in risk based maintenance optimization has a long tradition. Most models implicitly assume a 100% inspection, i.e. with full inspection coverage of the area subject to deterioration and/or initial defects. The inspection quality is then expressed by means of a Probability of Detection (PoD) function for the largest defect in that area, e.g a fatigue crack or a material loss due to corrosion. In practice full inspection coverage of the deterioration sensitive parts is not a realistic assumption as noted in e.g. Banon et al. (1994) considering offshore structures. In general for large engineering structures, such as ship hulls or pipelines, only a smaller fraction of the total structure is inspected using NDE, since a full coverage would not be economical. But what is the optimal inspection coverage and how can it be assessed?

---

[1] Research Assistant, Institute of Structural Eng. (IBK), straub@ibk.baug.ethz.ch, Tel. +41 1 633 36 97

[2] Professor, Dr., Institute of Structural Eng. (IBK), faber@ibk.baug.ethz.ch, Tel. +41 1 633 31 17

By taking into account the stochastic and functional dependency between the predicted deterioration in different locations, it has been shown how inspection of one location influences the nominal reliability of excessive deterioration at other locations, see also Straub and Faber (2002). As discussed in Faber and Sorensen (1999) and in Straub and Faber (2002) a simple optimization of the inspection effort according to decision theory as done for 100% inspection coverage, e.g. Faber et al. (2000), is in general not feasible due to the required numerical efforts. Straub and Faber (2002) propose a methodology for the inspection planning of systems based on a generic approach to Risk Based Inspection Planning (RBI). This approach is however still numerically demanding and the present paper therefore aims at developing a simpler methodology on how the inspection quantity can be related to inspection quality.

For systems consisting of elements subject to identical conditions and for which preventive and corrective maintenance actions are the same for all elements, i.e. when all elements are replaced or repaired given that a certain percentage of the elements have reached an unacceptable state, the inspection and maintenance planning problem reduces to a classical quality control problem. This approach is used by Faber and Sorensen (2002) for inspection planning of concrete structures based on indicators. Also Wall and Wedgwood (1998) use this approach in a simplified way. The statistical uncertainty associated with the estimation of the failure rate of the individual elements of the system is reduced with increasing number of inspections N_{insp} according to sample theory with the square root of N_{insp}. This is exact as long as the individual inspections are perfect and uncorrelated.

In the present paper an approach is followed where only those elements of the system for which the inspection yield a positive indication of the presence of critical defects are repaired or replaced. An adaptive inspection strategy is then introduced so that the number of inspections is varied according to the results of already performed inspections. This allows for expressing the fraction of detected critical defects as a function of the inspection coverage and in this way relating the inspection quantity to the inspection quality.

2 Scope & Limitations

In practical applications most inspections are performed on deteriorating structures i.e. systems with explicit time dependent behavior. However, for the sake of simplicity the paper considers only the point-in-time modeling of defects. The extension to a formulation including time dependent aspects (deterioration modeling and discounting) will increase the complexity of the problem but does not change the general nature of the problem.

3 Definitions

3.1 System

The systems considered (structures, pipelines etc.) are modeled as consisting of N "finite elements", for which the largest defect is described by the marginal distribution functions $F_{S_i}(s_i), i = 1,2,.., N$. The stochastic dependency between the largest defects of the individual elements is assumed represented by the covariance matrix, COV_{SS}, modeling the correlation between the single elements. Typical elements in real systems are e.g. hot spots in a steel structure subject to fatigue failures or finite elements of a structure subject to corrosion.

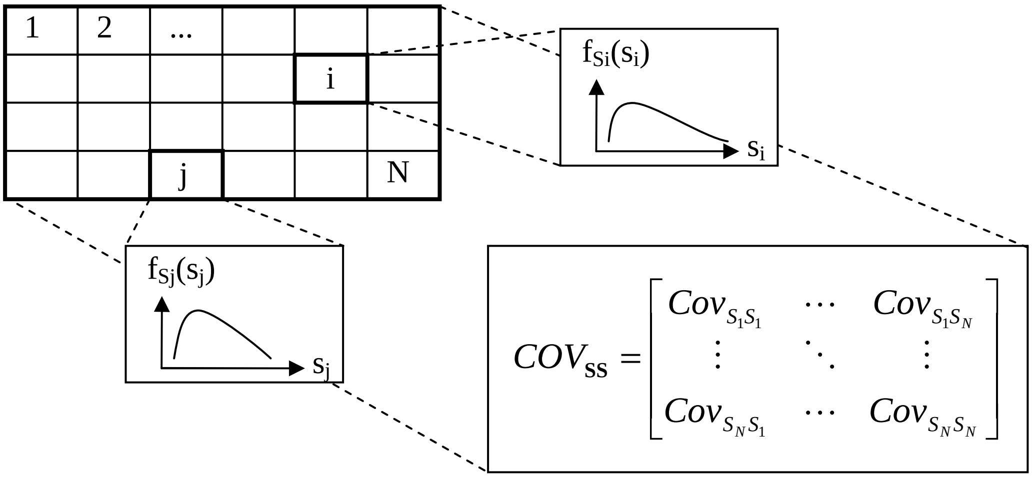


Figure 1 - Illustration of the system model

In principle the individual $F_{S_i}(s_i)$ can be different for all elements. For many systems the model of the deterioration will however be identical for a large number of elements. For a straight pipeline with constant temperature the prior model of corrosion depth will be the same for all elements along the bottom.

3.2 Defect modeling

An important aspect when considering the structure as a system of finite elements is the coherence between element size and defect distribution. Most often in practical applications the largest defect in an element is of interest, in which case the interesting matter is the extreme value distribution of the largest defect in an element. The extreme value distribution is obviously a function of the element size such that e.g. the mean value of the distribution will increase with increasing element size.

3.3 Failure

For the purpose of the present paper it suffices to differentiate defects into critical and non-critical. Here criticality is not necessarily associated with a state of failure but rather any condition associated with marginal consequences. The defect in an element is considered critical when its dimension exceeds the critical defect dimension s_{cr}. For structural reliability calculations the corresponding limit state function is expressed as

$$g_{cr} = s_{cr} - s \tag{1}$$

The probability of having a critical defect, $P(F)$ is then evaluated by integration over the domain where $g_{cr} \leq 0$, Equation (2) in accordance with the classical formulation in structural reliability, see Madsen et al. (1986).

$$P(F) = \int_{g_{cr} \leq 0} f_S(s) ds \tag{2}$$

Generally s_{cr} is associated with uncertainty but for simplicity it is modeled by a deterministic value in the following.

4 Bayesian Decision Theory

When the inspection planning and the modeling of inspection quality for systems is considered, it is essential to have a model of how inspection results are used and how they influence the risk associated with the defects in the structure. Bayesian decision theory, as described in e.g. Raiffa and Schlaifer (1961) and Benjamin and Cornell (1970), provides a consistent framework for such modeling. The following section summarizes the pre-posterior analysis from Bayesian decision theory aimed at identifying the optimal decision on inspections (or experiments).

4.1 Classical Bayesian pre-posterior theory

Generally, the inspection and maintenance planning decision problem can be described in terms of the following sets of decision and events:

$E = \{e\}$, the family of possible inspections or experiments, i.e. number, time, location and type of inspections

$Z = \{z\}$, the sample space, i.e. inspection outcomes such as no-detections, detections, observed crack length, observed crack depth.

$A = \{a\}$, the space of terminal acts, i.e. the possible actions after the inspection such as do nothing, repairs, change of inspection and maintenance strategy.

$\Theta = \{\theta\}$, the state space, i.e. the true but unknown “state of the nature” (non-failure, failure, degree of deterioration).

$u(e, z, a, \theta)$, the utility assigned, by consideration of the preferences of the decision maker, to any combination of the above given decisions and events. Utilities are often, but not necessarily, expressed in monetary terms.

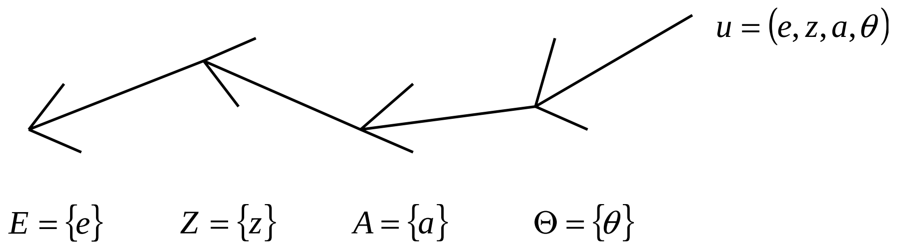


Figure 2 - The general decision tree

By means of pre-posterior analysis it is possible to determine the expected utility resulting from the inspection decision e by assigning to the possible inspection results z a decision rule d on the actions in regard to repairs and/or changes in future inspection and maintenance plans, such that

$$a = d(e, z) \tag{3}$$

The expected utility corresponding to a particular inspection strategy (e, d) then is:

$$\mathrm{E}[u(e, d)] = \mathrm{E}_{\Theta, Z|e,d}[u(e, Z, d(Z), \Theta)] \tag{4}$$

where $\mathrm{E}[\cdot]$ is the expectation. Upper case letters are used for random variables and the corresponding lower case letters for their realizations.

With the aid of structural reliability analysis and model updating in particular, as described in Madsen (1987), it is possible to evaluate the optimal inspection strategy by maximizing Equation (4). Thereby allowances can be made for constraints on the maximal accepted failure probability as specified by codes, authorities or operators themselves. In Faber et al. (2000) a practical approach to the solution of the optimization problem for deteriorating elements is presented.

4.2 Extended Bayesian pre-posterior analysis (adaptive inspection strategy)

The decision tree as shown in Fig. 2 is applicable directly to systems. It however misses one important aspect: After the analysis of the inspection results further inspections can be performed, i.e. that based on "sample" inspections, decisions on further inspections are made. The decision tree is therefore extended as shown in Figure 3.

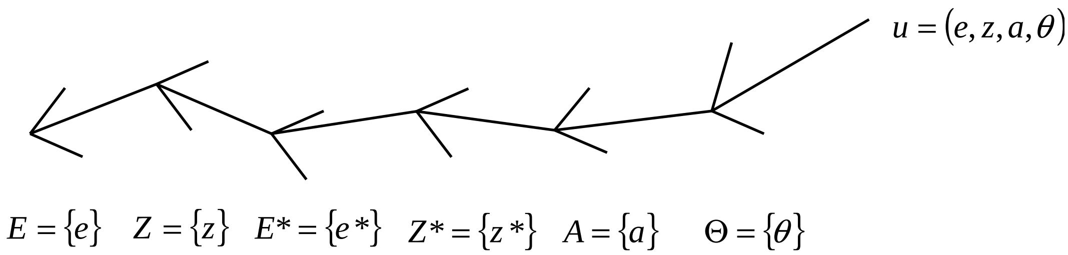


Figure 3 - The extended decision tree

E^* and Z^* are the decisions on the second inspections and their outcome, respectively. They are related to the first inspections by a new decision rule d^*:

$$e^* = d^*(e,z) \tag{5}$$

whereas the original decision rule is now based on the outcome of all inspections:

$$a = d(e,z,e^*,z^*) \tag{6}$$

If the decision rule d^* is to not perform any additional inspection, no matter what the results of the first inspection are, then the analysis reduces to normal pre-posterior analysis (section 4.1). If no adaptive inspection strategy is considered, then the inspection quality model for systems is a simple linear function of the one for individual elements, because the inspection effort is then independent of the outcome of correlated inspections and the expected outcome therefore is also independent of the other elements. This is shown in the following:

The quality of an inspection with outcome z of the binary type, detection ($z=1$) or non-detection ($z=0$), can be described by the probability of detection (PoD). The PoD is then defined as the expected inspection outcome, see e.g. Berens and Hovey (1983). Generally but not necessarily the PoD is a function of one or more defect properties. For a single element it is therefore written as

$$PoD = \mathrm{E}_{Z,\Theta|e}[Z] = \int_Z \int_\Theta z \cdot f_Z(z|e,\theta) \cdot f_\Theta(\theta) \cdot d\theta \cdot dz \tag{7}$$

For illustration a system with two elements is considered in the following. Extension to a system with N elements is accordingly. The two elements are identified by the indices 1 and 2. Due to the correlation in the system, the performance of the second element θ_2 is dependent on the performance of the first θ_1. If no adaptive strategy is applied, then the total inspection effort on the second element, denoted e_2, is independent of z_1, the outcome of the inspection on the first element. The PoD for the second element is then written as

$$PoD_2 = \mathrm{E}_{Z,\Theta|e_2}[Z_2] = \int_{Z_2}\int_{\Theta_2}\int_{\Theta_1} z \cdot f_{Z_2}(z|e_2,\theta_2)\cdot f_{\Theta_2}(\theta_2|\theta_1)\cdot f_{\Theta_1}(\theta_1)\cdot d\theta_1 \cdot d\theta_2 \cdot dz \tag{8}$$

Because

$$\int_{\Theta_1} f_{\Theta_2}(\theta_2|\theta_1)\cdot f_{\Theta_1}(\theta_1)\cdot d\theta_1 = f_{\Theta_2}(\theta_2) \tag{9}$$

Equation (8) reduces to Equation (7), as Equation (9) is simply the marginal probability density of θ_2. The PoD of the second element is shown to be independent of the first element.

If an adaptive strategy is applied, e_2 becomes a function of z_1 and d^* and the PoD is then

$$PoD_2 = \mathrm{E}_{Z,\Theta|d^*,e_1}[Z_2] = \int_{Z_2}\int_{Z_1}\int_{\Theta_2}\int_{\Theta_1} z_2 \cdot f_{Z_2}(z_2|d^*(e_1,z_1),\theta_2)\cdot f_{Z_1}(z_1|e_1,\theta_1)\cdot f_{\Theta_2}(\theta_2|\theta_1)\cdot f_{\Theta_1}(\theta_1)\cdot d\theta_1 d\theta_2 dz_1 dz_2 \tag{10}$$

The PoD of the second element is now dependent on the inspection decision and outcome of the first element. Using an adaptive strategy therefore changes the PoD for the individual element and therefore also for the overall system.

4.2.1 Decision rule d^*

As a decision rule for updating the inspection plan after a first campaign of partly inspection of the system it is proposed to change the coverage of the inspections of the second campaign (i.e. increase or reduce the percentage of inspected elements) if, more (or equivalently less) than the q^{th} fraction of the inspections of the first campaign resulted in detection. The parameter q should then be subject to optimization. The application of other decision rules is straightforward.

5 Modeling inspection quality as a function of inspection coverage

5.1 PoD formulation for systems

Because in systems there is not just one but a multitude of possible defect locations, the PoD_S cannot be expressed as a simple function of the defect dimensions as normally done for single elements. Different alternative formulations are therefore considered:

a) Probability of detecting the largest defect in the system

b) Probability of detecting all critical defects

c) Expected fraction of identified critical defects (relative to the expect total number of critical defects)

The shortcoming of the formulations a) and b) is that they are highly dependent not only on the coverage (in percent) and the defect distribution, but also on the number of elements in the system. Moreover a) is not very expressive in most applications. Formulation c) is favored in the following as it is can directly be utilized for solving many inspection optimization problems.

It is important to realize that all possible formulations, including c), will be dependent on the defect distribution in the system and the decision rule d^*, and no general formulation only being dependent on the inspection method and the coverage is possible.

5.2 Evaluating the system PoD_S

Applying the decision rule given in section 4.2.1 with parameter q, the system PoD_S as defined above can be readily evaluated by simulation. Given the system model (section 3), the expected fraction of identified critical defects, $\mathrm{E}[\delta]$, is calculated as a function of the initial inspection coverage, γ.

For a single element we have the two states; critical (F) and non-critical ($\overline{F}$), and the inspection results in either; indication (I) or no-indication ($\bar{I}$). The probabilities of having these can be evaluated given a PoD function as described in the literature, Hong (1997).

5.3 Probability of false indications

It has been recognized since long that the PoD is not the only important characteristic of an inspection method, but also the probability of false indication and subsequent unnecessary rejection and corrective actions has to be considered, Johnson (1977). In the context of inspection of systems the Probability of False Indications (PFI) can be expressed as the fraction of unnecessarily repaired elements. This combines the probability of making a false decision on additional inspections and the probability of a false indication at the individual element, i.e. it includes errors in the first decision d^*, and the second decision, d.

6 Risk based inspection optimization

For demonstrating the application of the above introduced concept in a risk based framework for inspection and maintenance planning the following cost consequences are considered

- Expected cost of having a critical defect at one element, C_F including all possible effects of that defect on the overall system
- Cost of repairing one element, C_R
- Cost of inspecting one element, C_I

Generally these costs will be a function of the number of elements, see Straub and Faber (2002). This is, however, neglected in the following for simplicity, but implementation in the analysis is in principle straightforward.

Based on the cost model, the expected total cost of an inspection strategy, C_T, is evaluated as the sum of the expected inspection cost, the expected repair cost and the expected cost due to critical defects:

$$C_T = \left(C_I \cdot \mathrm{E}[\gamma_{tot}] + C_R \cdot \mathrm{E}[\zeta] + C_F \cdot (1 - \mathrm{E}[\delta]) \cdot P(F)\right) \cdot N \tag{11}$$

The expected fraction of repairs, $\mathrm{E}[\zeta]$, as well as the expected fraction of inspected elements, $\mathrm{E}[\gamma_{tot}]$, are determined numerically.

7 Example

7.1 Defect model

In the following example the largest defect in the individual elements of the considered system is modeled by identical lognormal distributions with mean $\mu_S = 0.15mm$ and standard deviation $\sigma_S = 0.5mm$. The lognormal model has the advantage that the multivariate distribution of all defects is exactly described by the marginal distributions and the covariance matrix. The correlation between the largest defect sizes of the individual elements is modeled by a correlation coefficient ρ_S which is assumed identical between all elements. A defect is considered critical when it exceeds the (deterministic) critical size $s_{cr} = 1.5mm$.

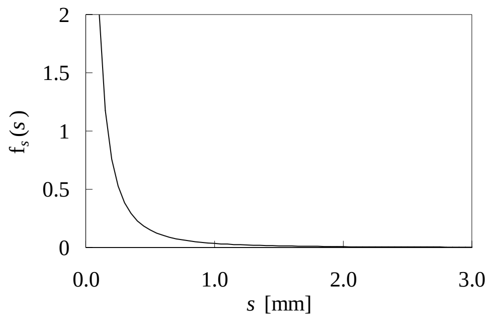


Figure 4 – Marginal probability density function for the largest defect in the individual elements.

7.2 Inspection model

The inspection quality for the individual elements is modeled by a PoD curve. The PoD is represented by a logLogistics model with parameters $\alpha = 1.02$ and $\beta = 1.52$:

$$PoD(s) = \frac{\exp(\alpha + \beta \ln(s))}{1 + \exp(\alpha + \beta \ln(s))} \tag{12}$$

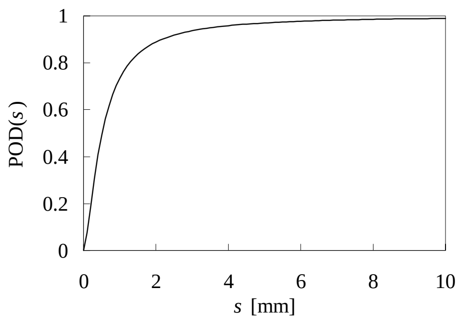


Figure 5 – Probability of detection curve for the inspections of the individual elements of the system

The considered case is representative for systems of elements subject to cracks. Inspection on systems subject to corrosion can be modeled by defining the element size as the inspected areas. When the entire element is inspected, the individual inspection can be modeled by $P(I|F)$ and $P(I|\overline{F})$ which are then functions of the measurement error.

7.3 Cost model

The following model is assumed for the investigations:

$C_F = 1$, $C_R = 0.05$, $C_I = 0.005$

*7.4 Decision on additional inspections, d**

In the present example it is assumed that all elements of the system will be inspected if during the first inspection campaign more than the q^{th} fraction of the inspected elements exhibit defects.

7.5 Calculations

A system with 400 elements was considered, however, the number of elements only has an influence for low inspection coverage. The probability calculations were performed using crude Monte Carlo simulations with 20’000 simulations.

7.6 Results

For the individual elements the probability of having a critical defect is $P(F) = 0.013$. Inspection of the individual elements may result in the following different indications with corresponding probabilities

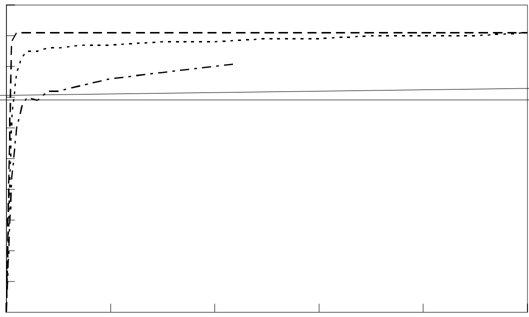
- True indication $P(I|F) = 0.91$
- False non-indication $P(\bar{I}|F) = 0.09$
- False indication (false alarm) $P(I|\overline{F}) = 0.09$

- True non-indication $P(\bar{I}|\bar{F}) = 0.91$

The total probability of having an indication at an inspection is $P(I) = 0.104$.

7.6.1 Influence of correlation on the system POD_S

Figure 6 shows that already for low correlation factors the system PoD_S is substantially higher than for independent elements. For independent elements ($\rho_S = 0$) the system PoD_S is as for the non-adaptive strategy. The maximum value of the PoD_S is given by the reliability of the individual inspection, $P(I|F)$.



7.6.3 Expected number of inspections $\mathrm{E}[\gamma_{tot}]$

The above mentioned effects are indicated in figure 8. A larger correlation has only little influence on the expected number of inspected elements as the higher PoD_S is mainly due to more targeted inspections. In contrast, a decreasing q leads to a higher inspection effort due to the larger probability of fulfilling the decision criteria.

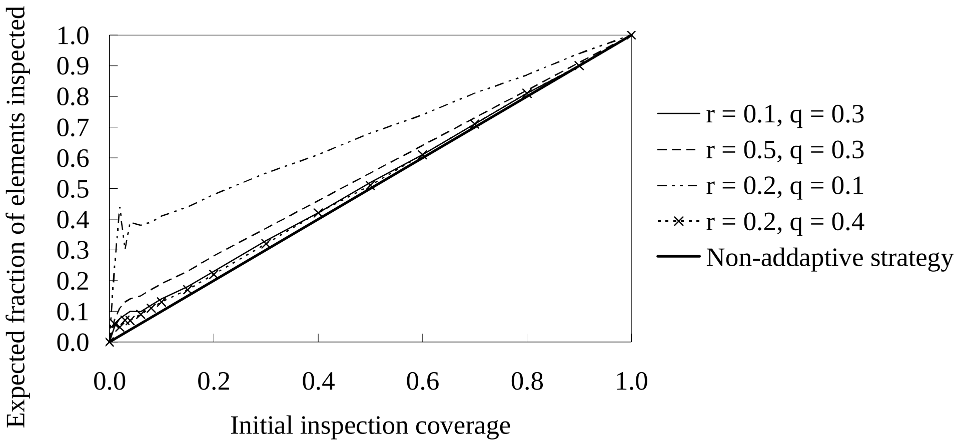


Figure 8 – Expected total number of inspections

The number of false indications is approximately proportional to the number of inspections as illustrated in Figure 8.

7.6.4 Optimal inspection effort

Combining the expected number of inspections, indications and unidentified critical defects, the total expected cost is evaluated according to Equation (11) and as illustrated in Figure 9.

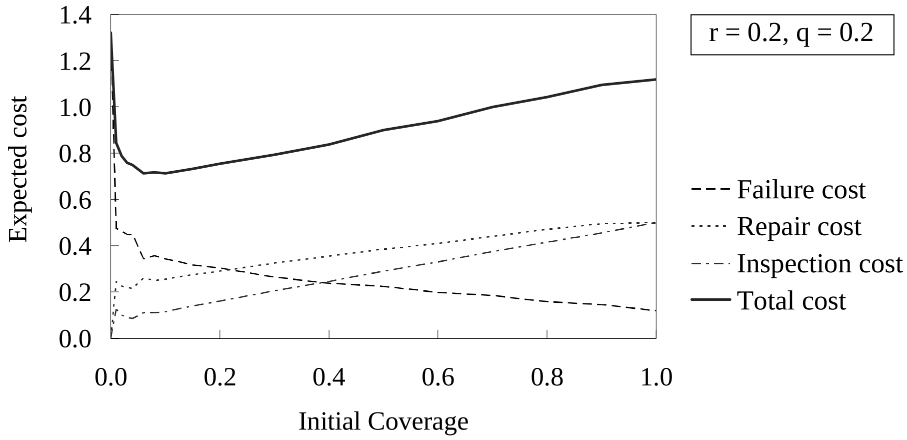


Figure 9 – Expected cost as a function of the initial inspection coverage

In Figure 10 the influence of the decision rule parameter q on the total expected cost is illustrated. For a correlation of $\rho_S = 0.2$ it is found that an initial coverage of 10% and a $q = 0.2$ is optimal with regard to the total expected costs.

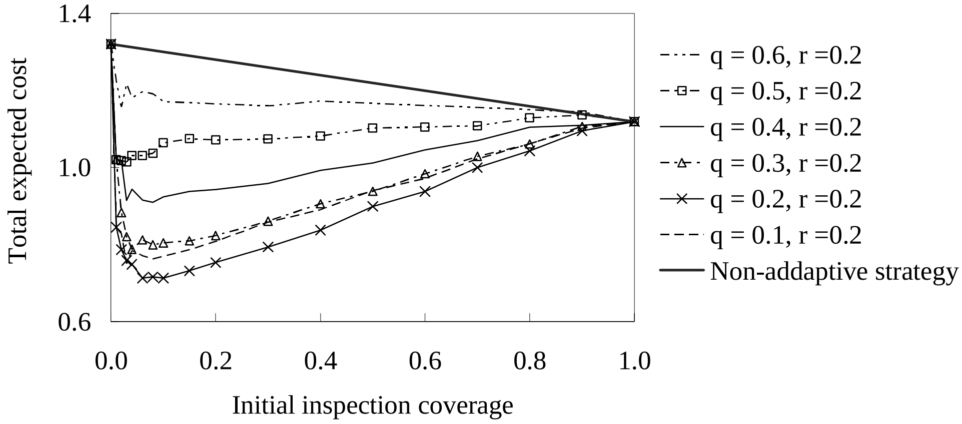


Figure 10 – Expected total cost for different q *given that* $\rho_S = 0.2$

In Figure 11 the optimal initial inspection coverage and the optimal decision rule parameter q is illustrated for different correlation in the system. The optimal q depends on the applied initial coverage. The values given in Figure 11 assume that the optimal coverage was applied.

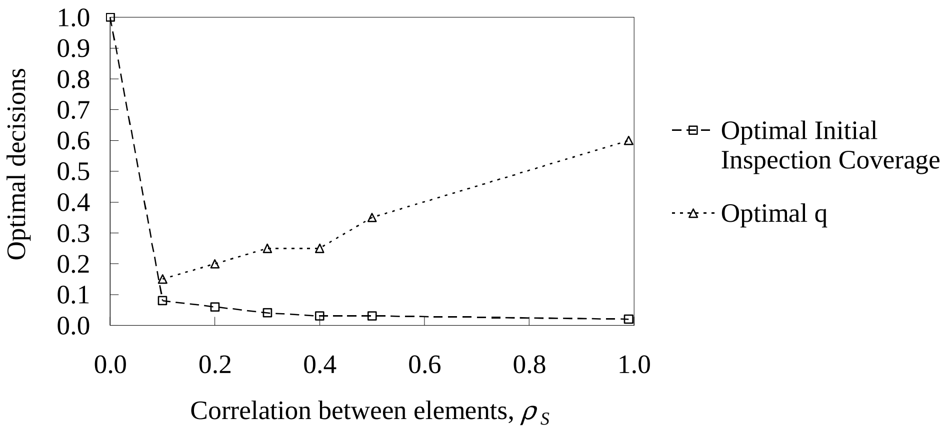


Figure 11 – Optimal inspection coverage and optimal q

8 Discussion and conclusion

The present paper introduces a probabilistic approach for the quantification of the effect of the coverage of inspections of systems consisting of several elements subject to the influence of defects. Thereby the inspection quality for systems may be related to the size of the system and the characteristics of the spatial variability (or correlation structure) of the defects of the system. It is shown how the inspection quality can be related to the quantity (expressed in terms of inspection coverage), if an adaptive inspection strategy is considered.

Explicit allowance can be made for the prior model of the defect as well as the correlation between the individual elements. Inspection is thereby characterized by its PoD and the probability of false indications. Cost optimal inspection coverage can be found. The model presented in this paper is a general model. For specific applications specific models should be established based on the specific practically relevant inspection and maintenance strategy as well as the deterioration mechanisms.

It has been observed that for all investigated cases (except for the extremes with no or full correlation) the optimal initial inspection coverage was between 3 – 10%, also if the costs (C_F, C_R, C_I) are varied. This result is, however, restricted to the presented model.

An important aspect that has not been treated in this paper is the correlation between the individual inspections. As discussed by Berens and Hovey (1983), the inspection quality estimations are uncertain themselves. This in fact results in PoD models for the description of the quality of individual elements which are correlated due to the effect of statistical and model uncertainties. How much this inspection performance dependency influences the inspection quality achieved for the entire system is an issue for future investigations.

9 References

Banon H, Bea RG, Bruen FJ, Cornell CA, Krieger WF, Stewart DA (1994). Assessing Fitness for Purpose of Offshore Platforms. I: Analytical Methods and Inspections. Journal of Structural Engineering, **120**(12), pp. 3595 - 3612

Benjamin JR, Cornell CA (1970). Probability, statistics and decision for civil engineers. McGraw-Hill, NY.

Berens AP, Hovey PW (1983). Statistical Methods for Estimating Crack Detection Probabilities. in Probabilistic Fracture Mechanics and Fatigue Methods: Applications for Structural Design and Maintenance, ASTM STP 798, J.M.Bloom and J.C.Ekevall (Eds.), pp. 79 - 94

Faber MH, Engelund S, Sorensen JD, Bloch A (2000). Simplified and Generic Risk Based Inspection Planning. Proc. 19th OMAE, ASME, New Orleans.

Faber MH, Sorensen JD (1999). Aspects of Inspection Planning – Quality and Quantity. Proc. ICASP8, Sydney.

Faber MH, Sorensen JD (2002). Indicators for inspection and maintenance planning of concrete structures. Structural Safety, **24**(4), pp. 377 - 396

Hong HP (1997). Reliability analysis with nondestructive inspection. Structural Safety, **19**(4), pp. 383 – 395

Johnson DP (1977). Inspection Uncertainty: The Key Element in Nondestructive Inspection. Nuclear Engineering and Design, **43**, pp. 219-226

Madsen HO (1987). Model Updating in Reliability Theory. Proc. ICASP5, Vancouver, Canada, pp. 565-577

Madsen HO, Krenk, S, Lind NC (1986). Methods of Structural Safety. Prentice Hall, New Jersey.

Raiffa H, Schlaifer R (1961). Applied Statistical Decision Theory. Cambridge University Press, Cambridge, Mass.

Straub D, Faber MH (2002). System Effects in Generic Risk Based Inspection Planning. 21st Offshore Mechanics and Arctic Engineering Conference, Oslo, paper S&R-28426.

Wall M, Wedgwood FA (1998), Economic Assessment of Inspection – the Inspection Value Method. The e-Journal of Nondestructive Testing, **3**(12), available under www.ndt.net/journal/archive.htm

10 Nomenclature

a	Mitigation action
C_F, C_I, C_R, C_T	Costs, see section 6
d, d^*	Decision rules
e	Inspection decision
F	Event of a critical defect
I	Indication event
N	Number of elements in the system
q	Decision rule parameter, see section 4.2.1 and 7.2
PFI	Probability of false indication
PoD	Probability of detection
R	Repair event
s	Defect size
s_{cr}	Critical defect size
u	Utilities
z	Inspection outcome
γ	Initial inspection coverage
γ_{tot}	Total inspection coverage
δ	Fraction of identified critical defects (relative to the total number of expected critical defects)
ζ	Fraction of elements repaired
θ	State of the considered system
ρ_S	Correlation between defect size in the individual elements