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The Application of EM Algorithm to Thickness Estimation of Thin Layer Materials

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1. Introduction

An important application of ultrasonic nondestructive testing (NDT) techniques is to acquire the quantitative material properties, such as the velocity of ultrasound in unknown material, the thickness of the material, and the attenuation of the material, etc. Most of the formulas describing the material properties in ultrasonic NDT technique include time of arrival (TOA) and time-difference of arrival (TDOA) parameters. TOA parameter can give us the information of the location of the target (flaw, layer … etc.). The thickness of the inspected specimen, the velocity of ultrasound in the unknown material, and some other information about the material require the estimation of TDOA between corresponding layer reflective echoes.

The cross-correlation method is widely used to obtain the high resolution TOA and TDOA [1]. This method uses a linear time invariant filter, which can be viewed as a “matched filter,” to produce maximum value at the TOA of the received echo.

However, to estimate the thickness of thin-layered materials is a challenging task in ultrasonic nondestructive testing. This is because when ultrasonic testing is applied to a thin-layered material, the echoes that come from the front surface and back wall of the layer will be overlapped in the domain. We cannot use the traditional cross-correlation method in this case to find out the high resolution TOA and TDOA.

C. Martin, et al. [2] have advanced homomorphic deconvolution method to obtain the thickness of thin-layered material. This method assumes there to be only two reflections from the specimen. In other words, it only gives the time delay of echoes from two layers. This method can only be used to calculate the TDOA of thin-layered material whose attenuation is large, such as thin-layered fiber. For the thin-layered material, which has multiple reflections, such as thin metal material, we need to use other methods to calculate the TDOA between different layers.

Recently, R. Demirli, et al. [3] presented the model-based method to obtain the high resolution TOA of echoes from thin-layered material that has multiple reflections. In this paper, we introduce the Gaussian model based space alternating generalized EM (SAGE) algorithm to estimate the parameters in the Gaussian model and system model to get the high-resolution estimate of the system response in thin-layered material. By combining the system model based SAGE algorithm with the Gaussian model base SAGE algorithm, the TDOA of thin-layered material is estimated.

The organization of the paper is as follows. Ultrasonic pulse-echo immersion testing scheme and the formula to get the material thickness in NDT are introduced in section 2. In section 3 and 4, we present the backscatter echo model and pulse-echo system model. We introduce the EM algorithms to estimate the backscatter echo model and pulse-echo system model. In section 6 and 7, we use the EM algorithms to estimate the thickness of thin-layered titanium and draw a conclusion.

2. Ultrasonic pulse-echo immersion testing

To obtain a precise result of the material ultrasonic characterization, we need to study the waveforms in the A-scan signal and their corresponding echoes. Figure 1 is the scheme of ultrasonic pulse-echo immersion testing. And the corresponding waveforms are shown in Figure 2 [4].

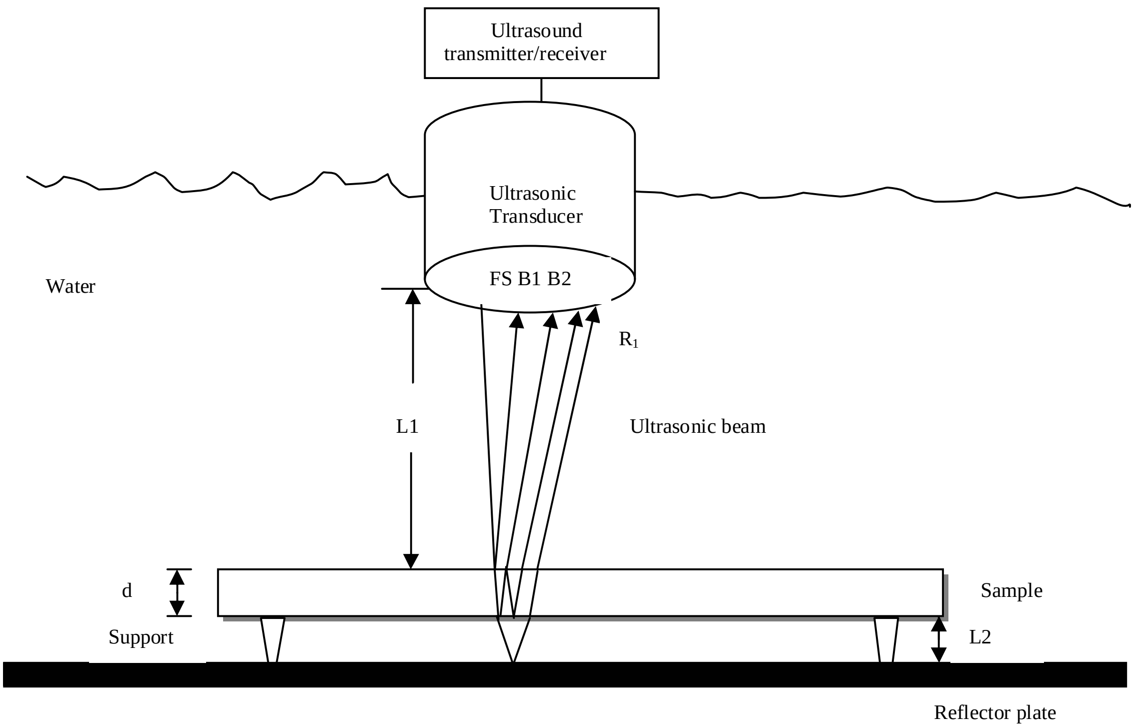

Figure 1 Ultrasonic pulse-echo immersion testing system

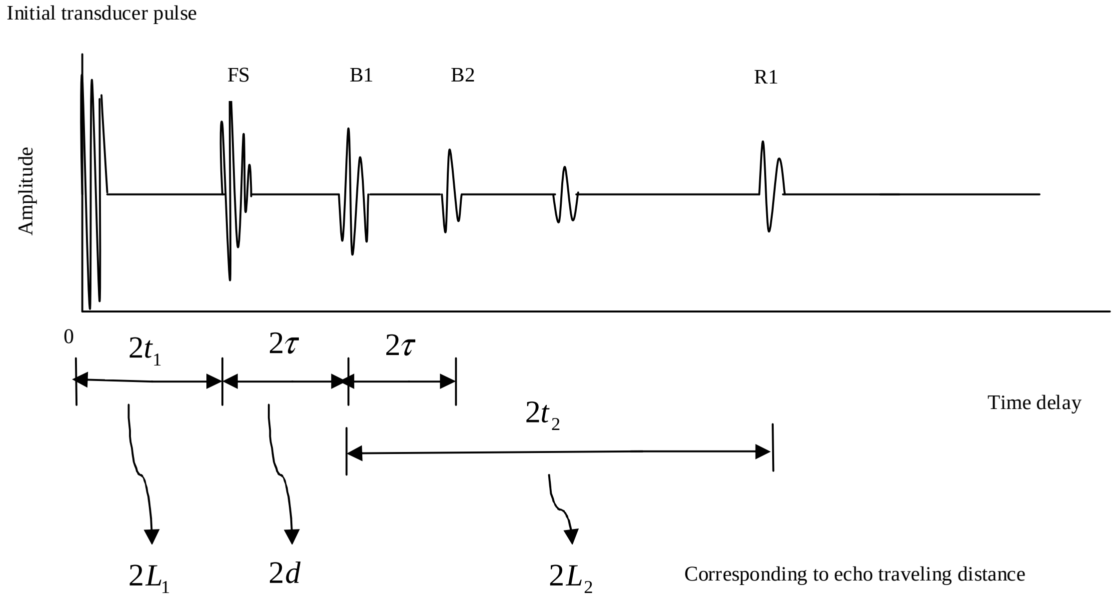

Figure 2 Corresponding waveforms of Figure 1

Here, L_1 is the distance between ultrasonic transducer and the sample; L_2 is the height of the support; d is the thickness of the sample; FS represents the ultrasonic front surface echo of the sample; B_1 represents ultrasonic first back wall echo of the sample; B_2 is the ultrasonic second back wall echo of the sample; R_1 is ultrasonic front surface echo of the reflector plate with the sample present; $2t_1$ represents the pulse-echo time delay between the front surface of the sample and the transducer; 2τ represents the pulse-echo time delay between a front surface and back wall echoes or two successive back wall echoes of the sample; $2t_2$ represents the pulse-echo time delay between the back wall of the sample and the front surface of the reflector plate.

The relationships between the above parameters are:

$$2t_1 = \frac{2L_1}{V_{water}} \tag{1}$$

$$2\tau = \frac{2d}{V_{sample}} \tag{2}$$

$$2t_2 = \frac{2L_2}{V_{water}} \tag{3}$$

We will notice that, 2τ is pulse-echo TDOA between a front surface and back wall echoes or two successive back wall echoes of the specimen, that is

$$2\tau = TOA(B_1) - TOA(FS) \tag{4}$$

or

$$2\tau = TOA(B_2) - TOA(B_1) \tag{5}$$

The thickness of the material can be determined by using equation (6),

$$d = \frac{2\tau \times V}{2} \tag{6}$$

Here, d is the thickness of the specimen. 2τ is the pulse-echo TDOA between a front surface and back wall echoes or two successive back wall echoes of the specimen, which is given in equation (4) and (5). V is the velocity of ultrasound in the material.

3. Ultrasonic backscattered echo model

The ultrasonic backscattered echo from a flat surface reflector can be written as in equation (7) [3],

$$y(t) = s(\theta;t) + v(t) \tag{7}$$

Here, $s(\theta;t)$ is a Gaussian echo model, and $s(\theta;t) = \beta e^{-\alpha(t-\tau)^2}\cos(2\pi f_c(t-\tau)+\phi)$,

$\theta = [\alpha \quad \tau \quad f_c \quad \phi \quad \beta]$. α is bandwidth factor, determines the bandwidth of the echo or the pulse duration in the time domain; τ is arrival time, indicates the location of the reflector; f_c is center frequency, governed by the transducer center frequency; ϕ is phase; β is amplitude, both ϕ and β are related to the impedance, size and orientation of the reflector. $v(t)$ is the additive white Gaussian noise process.

Equation (8) is an altered version to describe the multiple echoes from the reflector,

$$y(t) = \sum_{m=1}^{M} s(\theta_m;t) + v(t) \tag{8}$$

Here, M denotes the number of superimposed Gaussian echoes; echo vector θ_m indicates the shape and position of each echo. The summation term represents a signal model of multiple; the number of reflections is unknown.

4. Ultrasonic pulse-echo system model

The transducer pulse-echo wavelet can be represented by alter equation (7) with a number of M superimposed Gaussian echo wavelets [5],

$$h(t) = \sum_{m=1}^{M} c_m e^{-\alpha_m(t-\lambda_m)^2}\cos(2\pi f_m(t-\lambda_m)+\phi_m) \tag{9}$$

$$\theta_m = [\alpha_m \quad \lambda_m \quad f_m \quad \phi_m \quad c_m]$$

Here, α_m is bandwidth factor; λ_m is arrival time; f_m is center frequency; ϕ_m is phase; c_m is amplitude.

An ultrasonic echo reflected from a flat surface can be represented as the model:

$$s(t) = \beta h(t-\tau) \tag{10}$$

Where, $h(t)$ is the transducer pulse-echo wavelet defined in equation (9); τ corresponds to the arrival time of the ultrasonic echo traveling through the different reflective layers, and it can indicate the position of the reflector; β is the amplitude of the reflective echo, and is affected by the impedance, size, inner structures, and orientation of the reflector.

The system model (10) can be generalized to the M-echoes with noise effect:

$$y(t) = h(t) * \{\sum_{n=1}^{N} \beta_n \delta(t-\tau_n)\} + v(t) \tag{11}$$

$$\psi_n = [\tau_n \quad \beta_n]$$

Where, $\{\sum_{n=1}^{N} \beta_n \delta(t-\tau_n)\}$ is the impulse train, which denotes the unknown system response.

5. EM Algorithm

Expectation Maximization (EM) algorithm is an iterative optimization method to estimate a set of unknown parameters Θ of given measurement data [6, 7]. It is an algorithm for finding maximum likelihood estimates of parameters in probabilistic models, where the model depends on a set of unknown parameters. It includes two steps; one is expectation step (E-step), and the other is maximization step (M-step). In the E-step, we compute the expected value from the model using current parameters. In the M-step, we compute the maximum likelihood estimation of the parameters from the given data and the model, and set the resulting parameters as the current parameters for the E-step.

Now, we introduce the EM algorithms to estimate the parameters in ultrasonic backscatter echo model and pulse-echo system model, which we present in section 3 and 4.

For computational purpose, equation (8) can be rewritten as in equation (12):

$$y = \sum_{m=1}^{M} s(\theta_m) + v \tag{12}$$

We need to estimate parameters θ_1, θ_2,…, θ_M to construct the estimated M-superimposed echoes.

We first define x_m as in equation (13):

$$x_m = s(\theta_m) + v_m \tag{13}$$

Here, $s(\theta_m)$ is the m th Gaussian echo; v_m is the WGN sequence.

Then equation (12) can be rewritten as

$$y = \sum_{m=1}^{M} x_m \tag{14}$$

Where x_m and y are Gaussian random sequences.

In the E-step, we can use equation (15) to get the expected value $\hat{x}_m$ [3]:

$$\hat{x}_m^{(k)} = s(\theta_m^{(k)}) + \beta_m (y - \sum_{l=1}^{M} s(\theta_l^{(k)})), \quad \text{where } \sum_{m=1}^{M} \beta_m = 1 \tag{15}$$

In the M-step, we use equation (16) to get the next parameter:

$$\theta_m^{(k+1)} = \theta_m^{(k)} + (H^t(\theta_m^{(k)}) H(\theta_m^{(k)}))^{-1} H^t(\theta_m^{(k)})(\hat{x}_m^{(k)} - s(\theta_m^{(k)})) \tag{16}$$

Where, parameter $\theta = [\alpha \ \ \tau \ \ f_c \ \ \phi \ \ \beta]$; $H(\theta)$ is the gradient of the model with respect to parameters in θ.

Then we can use the EM algorithm to estimate the M-superimposed Gaussian echoes with WGN. However, E-step and M-step are computed in parallel in EM algorithm, that is, we obtain all the required current parameters $\Theta^{(k)}$ and the estimated data $\hat{X}^{(k)}$ in the E-step; then we use them to calculate the $\Theta^{(k+1)}$ in the M-step. One alternative to this parallel method is to update the $\Theta^{(k+1)}$ in the M-step right after estimating $\hat{x}_m^{(k)}$ in the E-step, without waiting for the other parameter vector to be estimated. This method is known as the generalized EM. It can help the computation to be faster. In the case of WGN, the generalized EM is also known as space alternating generalized EM (SAGE) algorithm [8]. The use of SAGE algorithm to estimate the M-superimposed Gaussian echoes with WGN is shown in below.

Algorithm 1 (Space Alternating Generalized EM to estimate Θ)

Step 1. Initialize $\Theta^{(0)} = [\theta_1^{(0)}; \theta_2^{(0)}; ...; \theta_M^{(0)}]$, Norm
Step 2. Set $k = -1$ and tolerance.
Step 3. Check the norm, while the norm greater than the tolerance, go to step 4, otherwise stop.
Step 4. Set $k = k + 1$, $m = 1$.
Step 5 Check m, while $m \le M$, go to Step 6, otherwise go to Step 9.
Step 6. E-step: compute the expected echoes

$$\hat{x}_m^{(k)} = s(\theta_m^{(k)}) + \frac{1}{M}(y - \sum_{l=1}^{M} s(\theta_l^{(k)}))$$

Step 7. M-step: using the corresponding parameters to calculate equation (16):

$$\theta_m^{(k+1)} = \theta_m^{(k)} + (H^t(\theta_m^{(k)}) H(\theta_m^{(k)}))^{-1} H^t(\theta_m^{(k)})(\hat{x}_m^{(k)} - s(\theta_m^{(k)}))$$

Step 8 Set $m = m + 1$, and then go to Step 5.
Step 9 Compute $\| \Theta^{(k+1)} - \Theta^{(k)} \|$ to get norm, set $\Theta^{(k)} = \Theta^{(k+1)}$, then go to Step 3.

The flow chart for SAGE algorithm is shown in Figure 3.

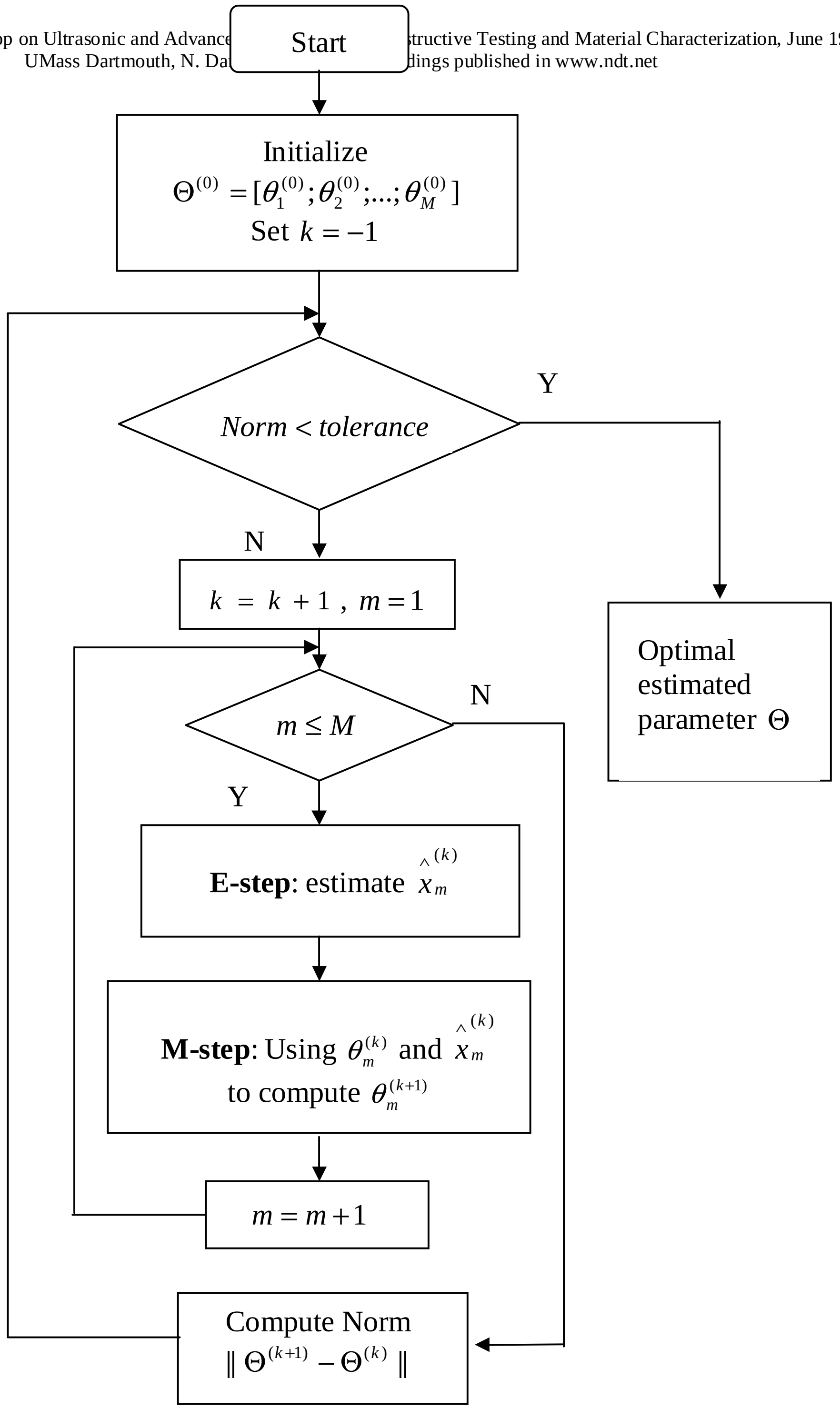

Figure 3 Flow chart for SAGE algorithm

We can also use SAGE algorithm to estimate the parameter $\psi_n = [\tau_n \quad \beta_n]$ from give data y in ultrasonic pulse-echo system model shown in equation (11).

Algorithm 2 (Space Alternating Generalized EM to estimate Ψ)

Step 1. Initialize $\Psi^{(0)} = [\psi_1^{(0)}; \psi_2^{(0)}; ...; \psi_N^{(0)}]$, Norm
Step 2. Set $k = -1$ and tolerance.
Step 3. Check the norm, while the norm greater than the tolerance, go to step 4, otherwise stop.
Step 4. Set $k = k + 1$, $n = 1$.
Step 5 Check n, while $n \le N$, go to Step 6, otherwise go to Step 9.
Step 6. E-step: compute the expected echoes

$$\hat{x}_n^{(k)} = s(\psi_n^{(k)}) + \frac{1}{N}(y - \sum_{l=1}^{N} s(\psi_l^{(k)})),$$

where $\psi_n = [\tau_n \quad \beta_n]$, $s(\psi_n) = \beta_n h(t - \tau_n)$, and $h(\cdot)$ is given by equation (9)

Step 7. M-step: using the corresponding parameters to calculate equation (17):

$$\psi_n^{(k+1)} = \psi_n^{(k)} + (H^t(\psi_n^{(k)}) H(\psi_n^{(k)}))^{-1} H^t(\psi_n^{(k)}) (\hat{x}_m^{(k)} - s(\psi_n^{(k)})) \quad (17)$$

$H(\psi)$ is the gradient of the model with respect to the parameters in $\psi = [\tau \quad \beta]$. The process to get $H(\psi)$, and $H^t(\psi) H(\psi)$ is given in [9].

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Step 8 Set $n = n + 1$, then go to Step 5.
Step 9 Compute $\| \Psi^{(k+1)} - \Psi^{(k)} \|$ to get norm, set $\Psi^{(k)} = \Psi^{(k+1)}$, then go to Step 3.

6. The application of EM algorithm to estimate the thickness of thin layer materials

We can use the ultrasonic pulse-echo system model to get the accurate TOA of different layer reflections of the thin layer. After getting the TOA, we can calculate the TDOA between the front surface and the back wall of the thin layer. For a given pulse-echo signal generated from the thin layer, first we estimate M sets of the five parameters $\theta_m = [\alpha_m \quad \lambda_m \quad f_m \quad \phi_m \quad c_m]$ using Algorithm 1, and substitute them into equation (9) to get $h(t)$; then assume $h(t)$ is known and unchanged to estimate N sets of the system parameters $\psi_n = [\tau_n \quad \beta_n]$ using Algorithm 2.

We use the above two algorithms to get the thickness of thin-layered titanium. Figure 4 (a) and (b) depict the A-scan signal of the thin titanium (using a 5-MHz transducer) and its magnitude spectrum:

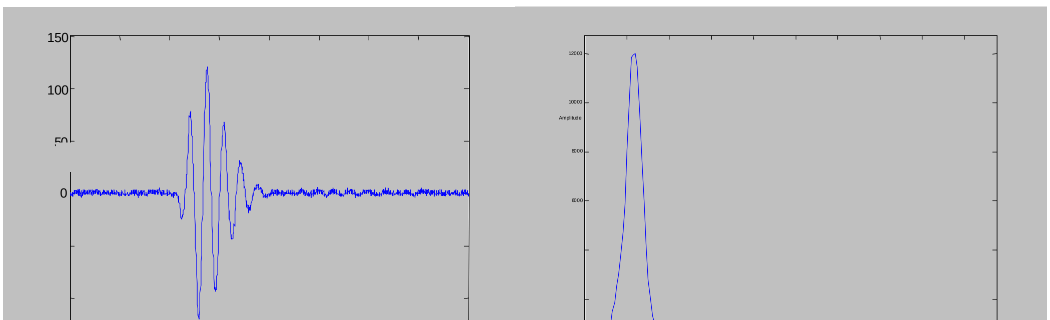

Figure 4 (a) A-scan signal of the thin-layered titanium and (b) its magnitude spectrum

We use the model order M=2 in Algorithm 1 to estimate the parameters. Figure 5 (a) is the estimated echo for Figure 4 (a). And Figure 5 (b) is the magnitude spectrum of Figure 4 (b).

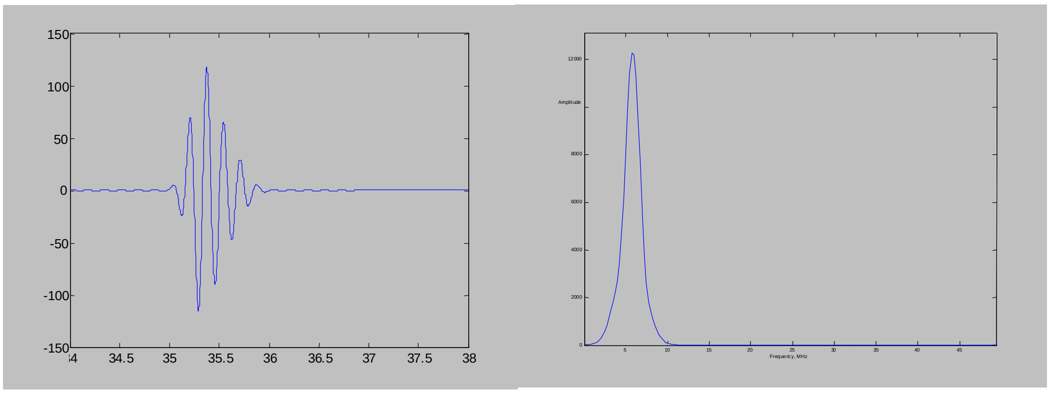

Figure 5 (a) Estimated echo of the thin titanium and (b) its magnitude spectrum

Then we use Algorithm 2 to estimate the time delay between the front surface and back wall of the thin titanium. Figure 6 shows the system response of the thin layer titanium using Algorithm 2.

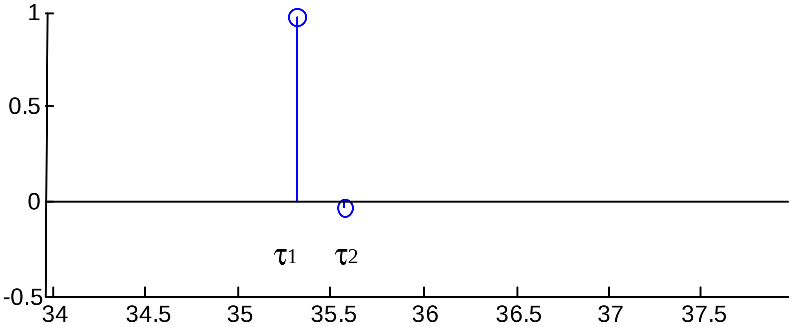

Figure 6 the system response of the thin-layered titanium

The difference between the two estimated parameters arrival time is

$$\text{TDOA} = \text{TOA (Back wall)} - \text{TOA (Surface)} = \tau_2 - \tau_1 = 0.265\mu s$$

And the thickness of the thin-layered titanium calculated by the estimated TDOA fits the actual thickness well.

7. Conclusion

In this paper, we introduced a model-based method to estimate a thin layer material, which may exhibit many more reflections. This model-based method includes two parts, the Gaussian echo model and the system model. We estimated the parameters of the Gaussian echo model, and then use the estimated result and assume they are invariant to estimate the parameters in the system model. Then we use these methods to estimate the TOA and TDOA of the thin titanium layer.

Acknowledgement

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