

# Wavelet Transform and Its Applications to Acoustic Emission Analysis of Asphalt Cold Cracking

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## Abstract:

Wavelet transform, a contemporary technique for data and signal analysis, has already found its distinguished place in nondestructive testing. Acoustic emission method could benefit greatly adopting this technique for better interpretation and control of data flow for materials testing and long term construction monitoring. Wavelet transform has recently been considerably developed adding new instruments and special functions. The art of their right choice and application for a practical task is a part of this work. Experimental data collected during acoustic emission testing of cold cracking asphalt was treated and analyzed by wavelet transform for noise elimination and better imaging. Further investigation and applications of this new technique to other practical tasks of acoustic emission testing are presented in the work.

**Keywords:** wavelet transform, acoustic emission, cold cracking, asphalt pavement.

## Introduction

Evaluation of strength and stress in pavement materials such as asphalt and concrete is a very challenging task for nondestructive testing (NDT) of these materials. Acoustic emission (AE) keeps one of leading place among others NDT methods, which may be applied to stress measurements in construction materials and pavements. Recently, wavelet transforms were widely used for analysis of AE data. These applications have been addressed in a number of recently published papers. This paper is discussing wavelet analysis applied to pavement materials evaluation and mapping in time – frequency domain, which provides useful information regarding strength and stress characterization of materials.

## 1. Wavelet Transform Versus Spectral Analysis of Acoustic Emission

Traditionally, analysis of AE from materials is utilizing Fourier transformation technique for consideration of signal properties in frequency domain [1]. Signal processing of AE is based on spectral analysis of events and AE process as average characteristics of different parameters, which were obtained through statistical analysis of these events.

The discrete Fourier transform, or DFT, is the primary tool of digital acoustic signal processing. The foundation of the DFT is the fast Fourier transform (FFT), a method for computing the DFT with reduced execution time. Many of the spectral analysis functions (including  $z$ -domain frequency response, spectrum and spectrum analysis, and some filter design and implementation functions) incorporate the FFT. MATLAB provides the functions of direct spectral transformations *fft* and reversed spectral transformations *ifft* to compute the DFT and return these data, respectively. For the input sequence of acoustic signal  $U(t)$  and its transformed version as spectrum  $F(\omega)$  the two functions implement the following relations [2]

$$F(k+1) = \sum_{n=0}^{N-1} U(n+1)W_N^{kn} \quad (1)$$

$$U(n+1) = \frac{1}{N} \sum_{k=0}^{N-1} F(k+1)W_N^{-kn}$$

where

$$W_N = \exp[-j \cdot (\frac{2\pi}{N})] \quad (2)$$

and  $k = 0, 1, 2, \dots, N-1, n = 0, 1, 2, \dots, N-1$  - numbers of spectral and timing data in analysis where  $\omega = 2\pi f$  and  $f$  - frequency of AE transformation.

Generally, FFT brings complex variant of spectral data, which contains as real part of acoustic signal, as imagery part of signal spectrum  $F(\omega)$ . The real part of signal spectrum is presented by amplitude spectrum  $A(\omega)$  and imagery part is presented by phase spectrum  $\Phi(\omega)$  that could be described as follows:

$$A(\omega) = \text{Re}(F(\omega)) \quad (3)$$

$$\Phi(\omega) = \text{Im}(F(\omega))$$

and their relation is presented by angle spectrum  $\Theta(\omega)$

$$\Theta(\omega) = \frac{A(\omega)}{\Phi(\omega)} \quad (4)$$

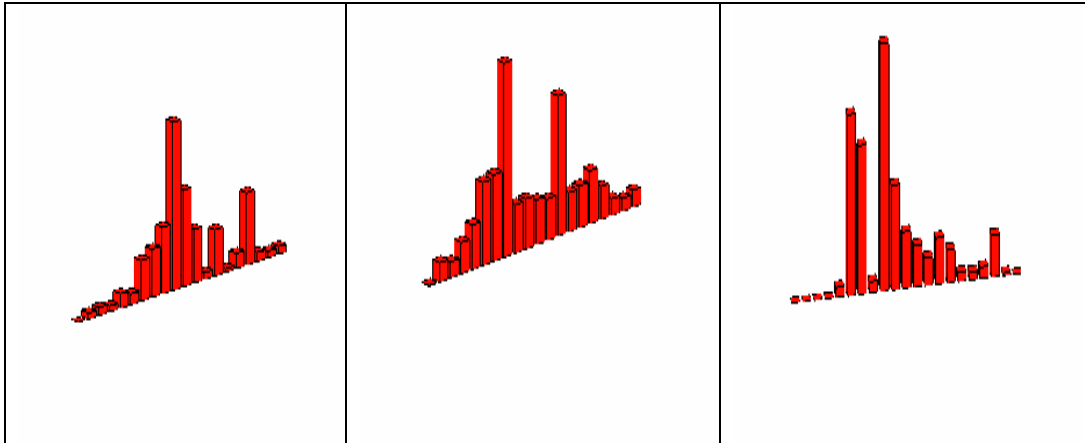
Spectral analysis of acoustic signals allows to present mixture of noise and AE event in frequency domain as well as to build a specific imaging for energy-pattern characteristics of AE.

Some examples of maximum amplitude spectra  $A_{MAX}(\omega)$  of AE events versus loading history are given below in the Table 1:

Table 1

Maximum amplitude spectrums  $A_{MAX}(\omega)$  of AE pulses with background noise versus loading history for different compositions of asphalt specimens

SPECIMEN 33-7-B1	SPECIMEN 33-7-T1	SPECIMEN 35-17-T1
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There are two important characteristics of AE patterns: phase angle  $\Theta(\omega)$  and phase spectrum  $\Phi(\omega)$ , which indicate inner relations in signal. They are very helpful for wavelets imaging and other types of visualization.

It is necessary to note that AE in construction materials is a stochastic process that depends on physical nature and random character of inner stress in local points of materials. As a result, AE energy may be presented as a sum of different sources of AE containing a background noise. So, AE spectra are more complicated characteristics comprising reflections from local and distant sources of energy as well as different mechanisms of acoustic noise formations. This assumption allows to develop a differential approach to the AE analysis using identification and recognition algorithms for separation of accumulating energy from different sources of acoustic signals.

One of the solutions for some practical problems may be wavelet transform of AE signals. Wavelet analysis is a new method, though its mathematical underpinnings date back to the works of Joseph Fourier [3] in the nineteenth century. Fourier laid the foundations with his theory of frequency analysis, which proved to be enormously important and influential [4]. Researchers gradually turned from frequency-based analysis to scale-based analysis, when it became clear that measuring average fluctuations at different scales proved to be less sensitive to noise [5]. One of the first records of what we now discuss as a "wavelet" seems to be published in 1909, in the paper published by Alfred Haar [6].

The concept of wavelet in its present form was proposed by Jean Morlet [7] and the team [8]. The method of wavelet analysis has been developed mainly by Y. Meyer and his colleagues [9], who have ensured the methods' dissemination. The main algorithm dates back to the work by Stephane Mallat [10]. Ingrid Daubechies [11], Ronald Coifman [12], Victor Wickerhauser [13] and other researches have recently been actively working in this field.

Modern signal processing allows to present acoustic signals in time and frequency domain as one graphical image of wavelet transform.

It is possible to consider two different types of wavelets, which are presented in the research paper [14]:

- continuous wavelet transforms, which could be presented as model below

$$CWL(scale, position) = \int_{-\infty}^{\infty} f(t) \cdot \Psi(scale, position) dt \quad (5)$$

where  $f(t)$  - signal in time domain;  $\Psi(scale, position)$  - specific wavelet function;

- discrete wavelet transformation, which could be presented for discrete data form below

$$DWL(a, b) = \sum_{n \in Z} S(n) \cdot \Psi_{j,k}(t) \quad (6)$$

where  $a = 2^j, b = k \cdot 2^j, S(n)$  – discrete signal data and  $\Psi_{j,k}(t)$  – discrete wavelet function.

This work will emphasis to continuous wavelet transforms for digital analysis of AE data, received from experimental investigation of low temperature asphalt testing.

## 2. Application of Wavelet Analysis to Acoustic Emission Data

Is it possible to mention several types of wavelets, which could successfully applied to AE analysis: Haar wavelets, Dauvechies wavelets, Meyer wavlets, Gaussian wavelets, Mexican hat wavelets, Shanon wavelets, Morlet wavelets, and Complex Frequency B-Spline Wavelets. Description of some of them could be found below.

Complex Gaussian wavelets are representing the “family” of wavelets based on Gaussian function for interpretation of experimental data  $x$

$$f(x) = C_p \exp(-ix - x^2) \quad (7)$$

by taking the  $p$  derivative of  $f(x)$ . The integer  $p$  is the parameter of this family and in the previous formula,  $C_p$  is such that  $\|f^{(p)}\| = 1$

where  $f^{(p)}$  is the derivative of  $f(x)$ .

Complex Morlet Wavelets: complex Morlet wavelet is defined by

$$\Psi(x) = \frac{1}{\sqrt{\pi f_b}} \exp(2i\pi f_c x - \frac{x^2}{f_b}) \quad (8)$$

depending on two parameters:  $f_b$  - bandwidth parameter;  $f_c$  - wavelet center frequency.

Complex Frequency B-Spline Wavelets: complex frequency B-spline wavelet is defined by

$$\Psi(x) = \sqrt{f_b} (\sin c(\frac{f_b x}{m}))^m \exp(2i\pi f_c x) \quad (9)$$

depending on three parameters:  $m$  - integer order parameter ( $m \geq 1$ ).  $f_b$  - bandwidth parameter.  $f_c$  - wavelet center frequency

Complex Shannon wavelets: this family is obtained from the frequency B-spline wavelets by setting  $m$  to 1 and defined by

$$\Psi(x) = \sqrt{f_b} \sin c(f_b x) \exp(2i\pi f_c x) \quad (10)$$

depending on two parameters:  $f_b$  - bandwidth parameter.  $f_c$  - wavelet center frequency. The continuous wavelet transform of an arbitrary function  $f(t)$  as given by Strang and Nguyen [15] is defined by

$$A_\psi(a,b) = \frac{1}{\sqrt{a}} \int f(t) \Psi(\frac{t-b}{a}) dt \quad (11)$$

Including analyzing function

$$\Psi_{a,b}(t) = \frac{1}{\sqrt{a}} \Psi(\frac{t-b}{a}) \quad (12)$$

as wavelet functions, with the position variable  $b$  and the scale variable  $a$ . The scaling is a primary characteristic of the wavelet analysis. The Gabor function [16] is used as the analyzing wavelet because it provides a small window in the time as well as in the frequency domain. The Gabor function is defined by

$$\Psi_{GABOR}(t) = \frac{1}{\sqrt[4]{\pi}} \sqrt{\frac{\omega_0}{\gamma}} \exp\left[-\frac{(\omega_0/\gamma)^2 t^2}{2} + i\omega_0 t\right] \quad (13)$$

Following [16], the positive constant  $\gamma = \pi \sqrt{\frac{2}{\ln 2}}$  and frequency  $\omega_0 = 2\pi$ .

Thus, Gabor function could be presented as complex sinusoidal function with Gaussian type of window. To applying Parseval's theorem, spectrum could be presented as formulae

$$A(a,b) = \frac{\sqrt{a}}{2\pi} \int_{-\infty}^{\infty} f(\omega) \exp(ib\omega) \Psi_{GABOR}(a\omega) d\omega \quad (14)$$

and Gabor function in frequency domain is

$$\Psi_{GABOR} = \frac{2\pi}{\sqrt[4]{\pi}} \sqrt{\frac{\gamma}{\omega_0}} \exp\left[-\frac{(\gamma/\omega_0)^2}{2} (\omega - \omega_0)^2\right] \quad (15)$$

All these models of wavelet transforms could be programmable and used for new imaging of AE data for better presentation of inner patterns for fracture mechanism in materials.

### 3. Cold Cracking Asphalt Acoustic Emission And Wavelet Data Analysis

Wavelet transforms were applied for visualization of AE during cold cracking investigations of asphalt specimens under mechanical loads. Time-frequency scales were used for separating AE and noise in space and time, that allowed clarifying AE patterns for cold cracking process identification.

An example of AE signal is presented in the Figure 1.

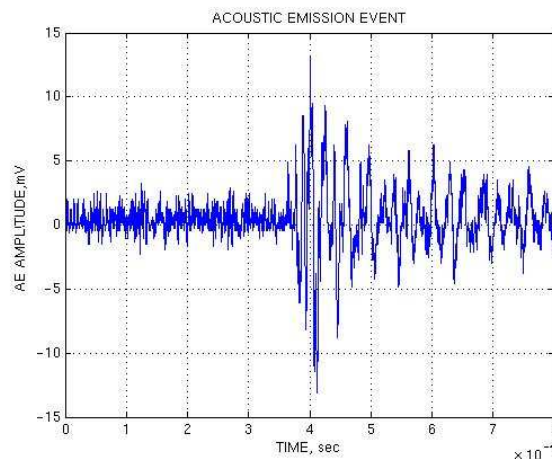


Figure 1. AE pulse obtained during experimental studies of cold asphalt cracking

Application of Gausse wavelets for this AE pulse pattern is shown in the Figure 2.

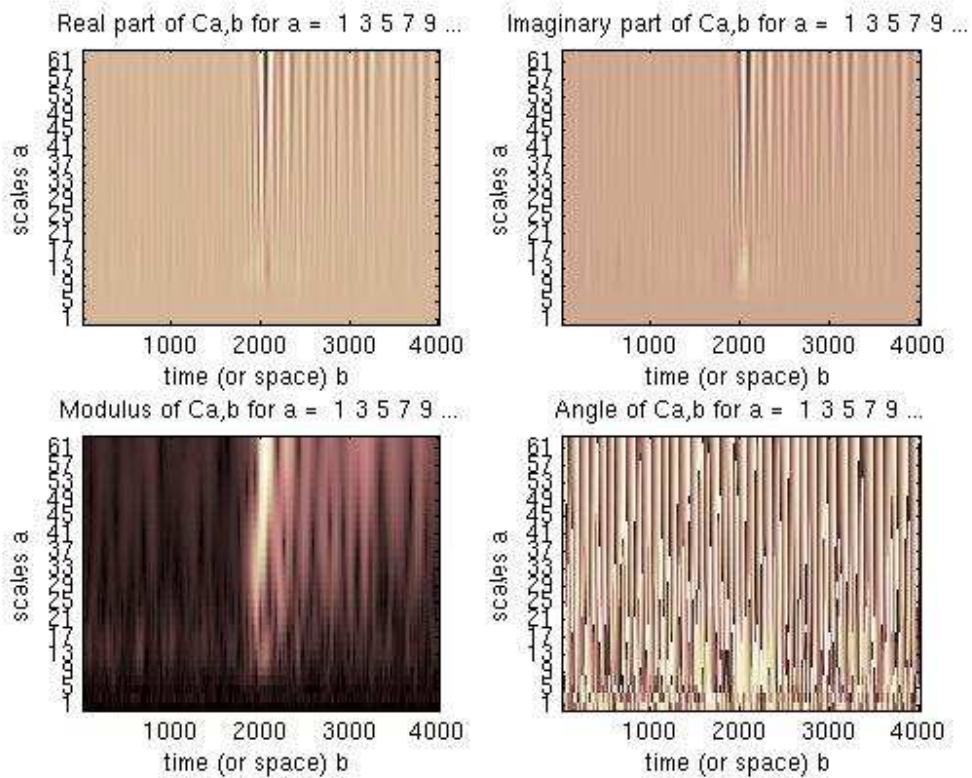


Figure 2. Presentations of Gausse wavelets as real part, imaginary part, modulus and angle of scale coefficients versus time.

Different imagines can be obtained using Shannon wavelets for the same AE pulse pattern (Figure 3)

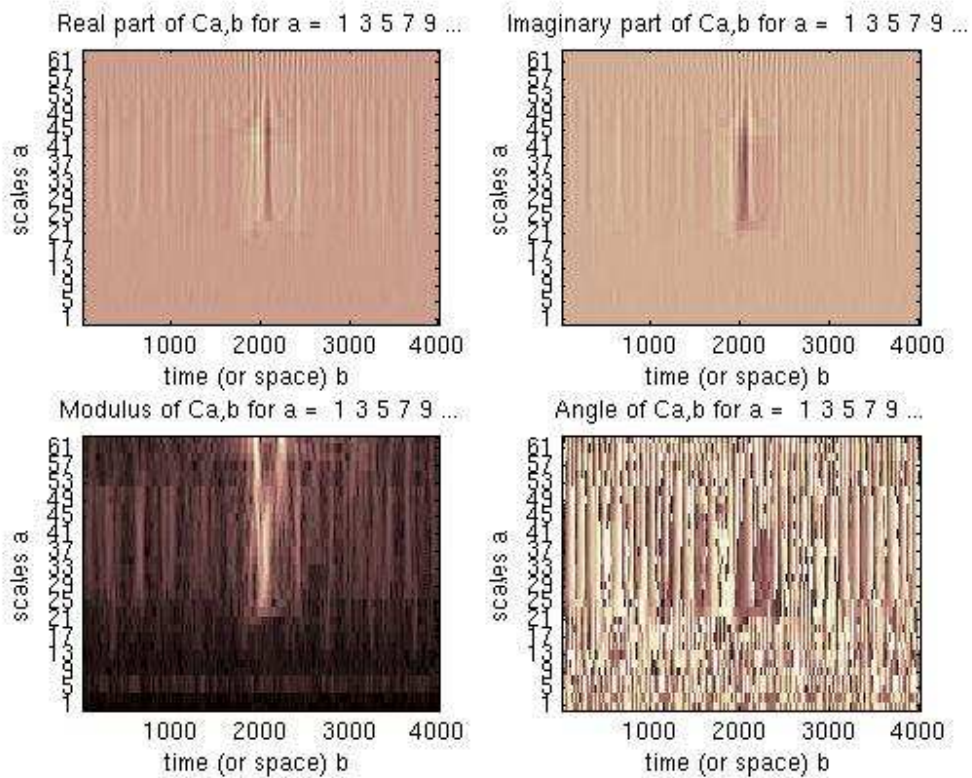


Figure 3. Presentations of Shannon wavelets as real part, imaginary part, modulus and angle of scale coefficients versus time

It is possible to observe insignificant differences in the wavelet patterns received by using Haar (Figure 4) and Meyer (Figure 5) models for processing of AE pulses.



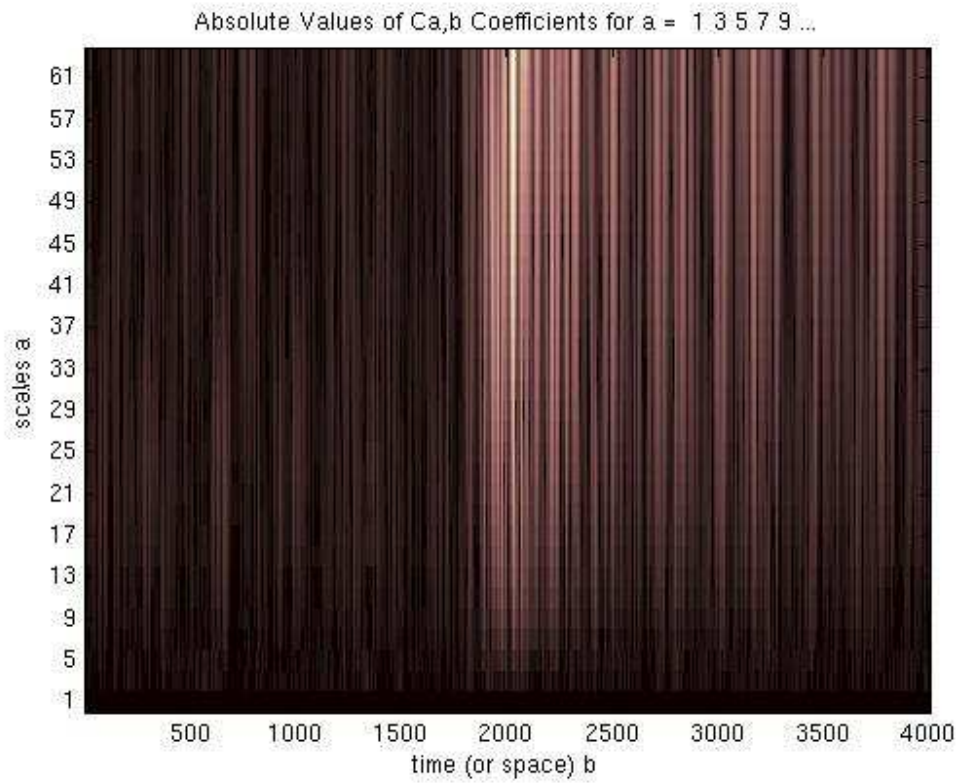


Figure 4. Application of Haar transform for AE pulse imaging.

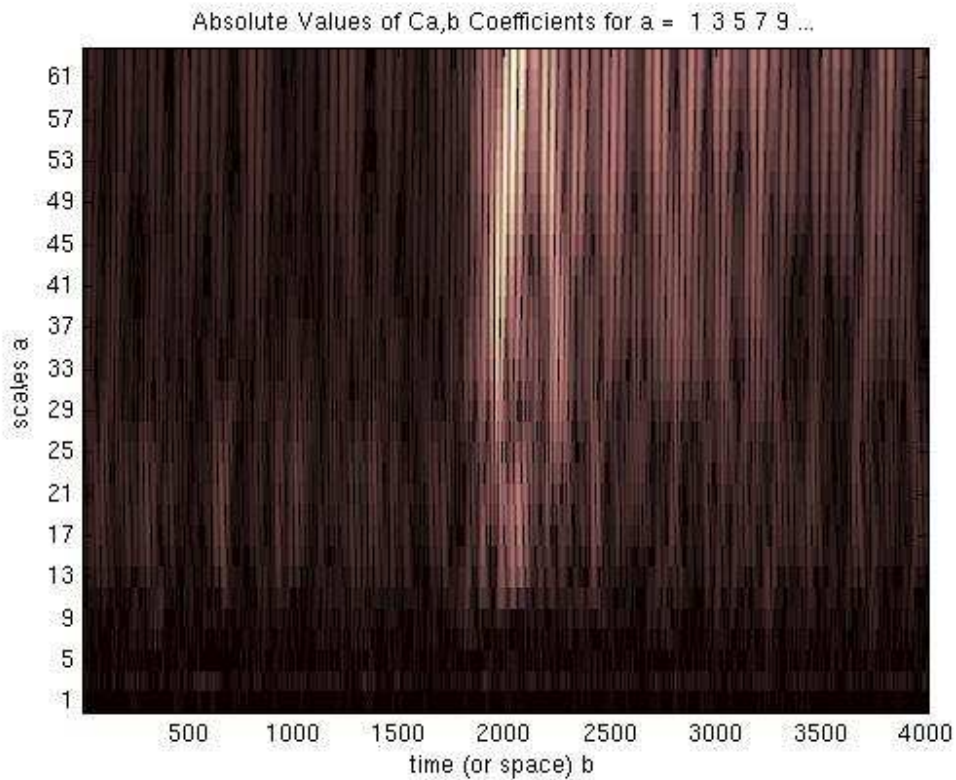
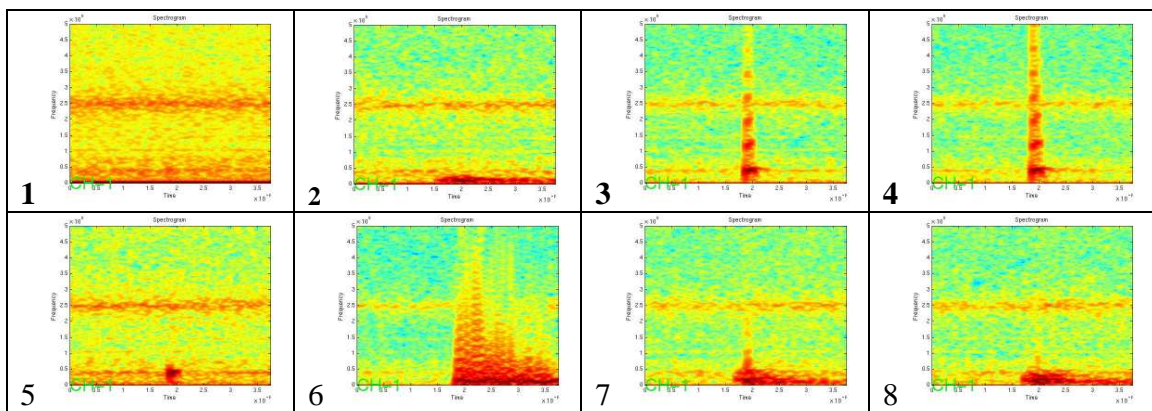
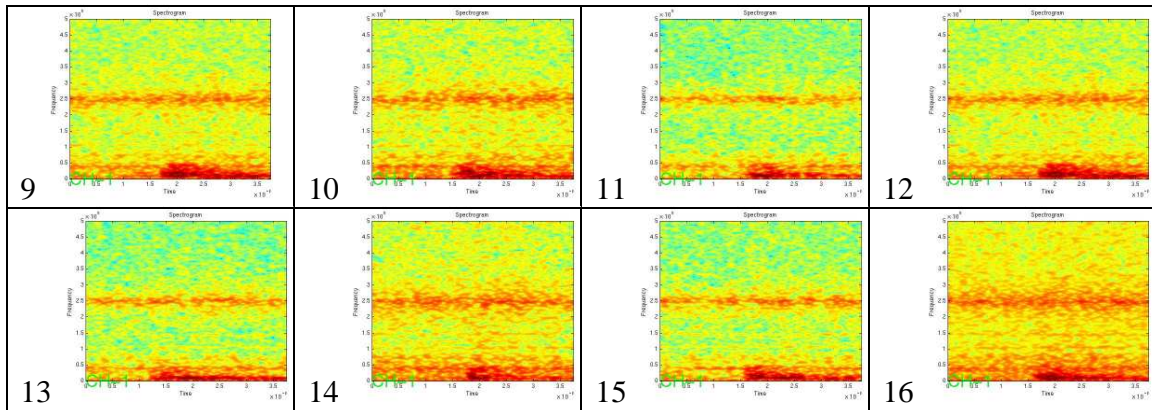


Figure 5. Application of Meyer transform for AE pulse imaging.

Application of Gabor wavelets is considered as classical approach for signal and noise separations in the time and frequency domains. An example of application of Gabor wavelets for analysis of different levels of loads to asphalt specimens is presented in the figures in the Table 2 below. Numbers 1,2...16 indicated stages of loading procedure sequentially.

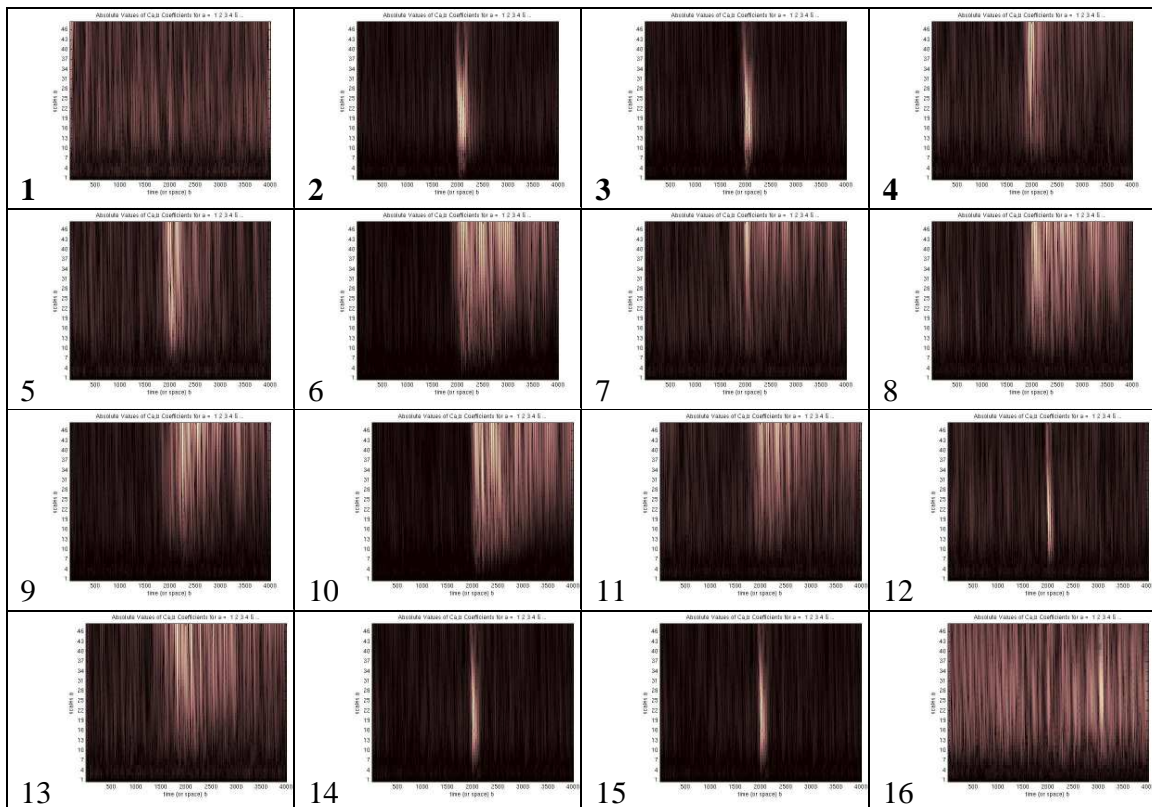
Table 2





Applications of Daubichies (mode 4) wavelets to analysis of another specimen loading are presented in the figures in the Table 3 below

Table 3



There are wavelet pictures of different AE pulses randomly extracted from AE data at each stage of specimen loading. It is possible to recognize different patterns of AE process, such as noise, spark types of AE and wide band of AE pulses similar to those presented in the Figure 9.

## 4. Future Development and Possible Solutions

Presented experimental data of wavelet analysis of cold cracking AE gave some hope that this contemporary approach to AE data analysis will allow to identify evolution of cracking and to find a new imaging for AE presentation and pattern recognition. It is necessary to underline that presented types of wavelets are not optimal solutions for AE data analysis. It should be adjusted for better understanding of AE process. Optimal results could be obtained by developing existing and exploring new types of wavelets for AE analysis and pattern recognition.

## 5. Conclusions

Experimental investigation of cold cracking of asphalt by AE method gave an opportunity to discover a new avenue for nondestructive analysis of pavement materials. The proposed wavelet tools seem to have some advantages over spectral analysis. Wavelets with their flexible structures allow finding optimal designs for better recognition and identification of signals. This flexibility may give an opportunity to custom build new tools for practical tasks of testing of materials and structures. The obtained results could be considered as an early step to the problem study. It may be far from immediate practical applications, but it may show a direction for development of new types of wavelets with their optimization for specific needs of AE applications.

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