DECENTERED ANNULAR TRANSDUCER ARRAY FOR GENERATING DIRECTION-VARIABLE ACOUSTIC BEAM

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Abstract: In this paper, we propose a method for generating a direction-variable acoustic wave beam with sharp profile using a decentered annular transducer array. Imaging equipment using acoustic wave is one of the important technology in nondestructive testing, because of features such as noninvasiveness, ease of handling, and so on. In order to obtain a result with a high transverse resolution, it is necessary to use an acoustic beam of which the width is narrow. Bessel Beam is one of the solutions, and we proposed a method for approximately generating acoustic Bessel Beam with a sound source by a concentric annular transducer array, which has the simple structure. The source has the clearance between each neighboring element of the array, so applying these clearances to decenter the array elements, it becomes possible that the direction of radiated beam is made to change. The results of numerical calculations using this method show that the beam radiated to the direction of a maximum of 15 degrees from the perpendicular of the source plane keeps the sharp profile, and the validity of this method is verified. The sound source fabricated by this method has the small, simple and planar structure, so the source has advantages such as high flexibility of installing place and simplicity of manipulation.

Introduction: Measurement equipment and imaging devices using acoustic waves are important technologies in the nondestructive testing, because of their features such as noninvasiveness, ease of handling and inexpensively composition, and widely used in various fields. In order to improve the total performance of these devices, it is very important that the transverse resolution of the output image is improved. However, it was generally difficult for the narrow beam to propagate over a long distance, since the width of the beam extends by the diffraction.

In 1987, a nondiffracted beam was proposed by Durnin[1,2] in the field of optics. This beam is based on a nondiffraction solution to the wave equation. And, it has a narrow intensity profile and propagates for a long distance without diffractive spreading. In acoustics, this beam could be generated from a transducer driven with a normal velocity distribution represented by the zeroth-order Bessel function of the first kind, J_0, so it is known as a "Bessel Beam." It is widely expected that this type of beam will be applied in fields such as measurement and imaging using acoustic waves, and the realization of a nondiffracted beam such as the "Bessel Beam" has been pursued actively using various methods[3-8].

As one of them, a method that uses a concentric annular array, in which each element has a width corresponding to the J_0 function, and which is driven with an antiphase from its neighboring elements and with an equiamplitude, is proposed[9,10]. This method makes the structure much simpler than conventional methods in which each element is driven with an amplitude corresponding to the Bessel function. This type of sound source has a clearance between each neighboring array element. By utilizing this clearance in order to decenter each element, it would appear that the direction of the radiated acoustic beam becomes variable. [11,12].

In this study, the design method and the decentering procedure of the annular transducer array with clearances are presented, and numerical calculations of the radiated sound field using this type of sound source are carried out. By the results of the calculations, it is shown that the acoustic beam radiated to the direction that is deviated from the perpendicular of the source plane keeps the sharp profile. Additionally, it is also shown that the radiation direction of the beam is variable, even if the decentered annular array has a few elements.

Design of Sound Source: At first, a design procedure a concentric annular transducer array for generating a nondiffracted beam. As shown in Fig. 1, let us consider a coordinate system and a source by a concentric annular transducer array located on a plane perpendicular to the z-axis, where r is the radial distance from the z-axis.

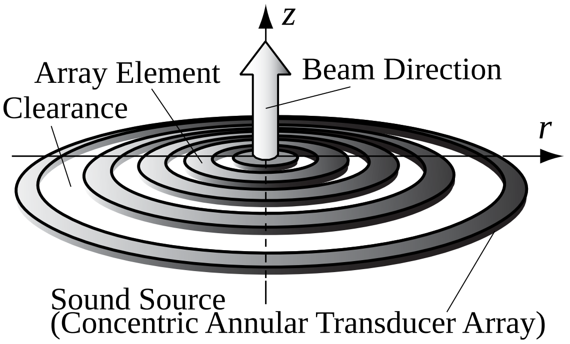

Fig. 1: Geometry of a coordinate system and a sound source by a concentric annular transducer array.

Figure 2 shows a cross-sectional view of an annular array consisting of N elements, a disk element and $(N-1)$ ring elements, and the zeroth-order Bessel function of the first kind, $J_0(x)$. The position and width of each sound element is determined by the J_0 function shown in Fig. 2(a), as the following procedure.

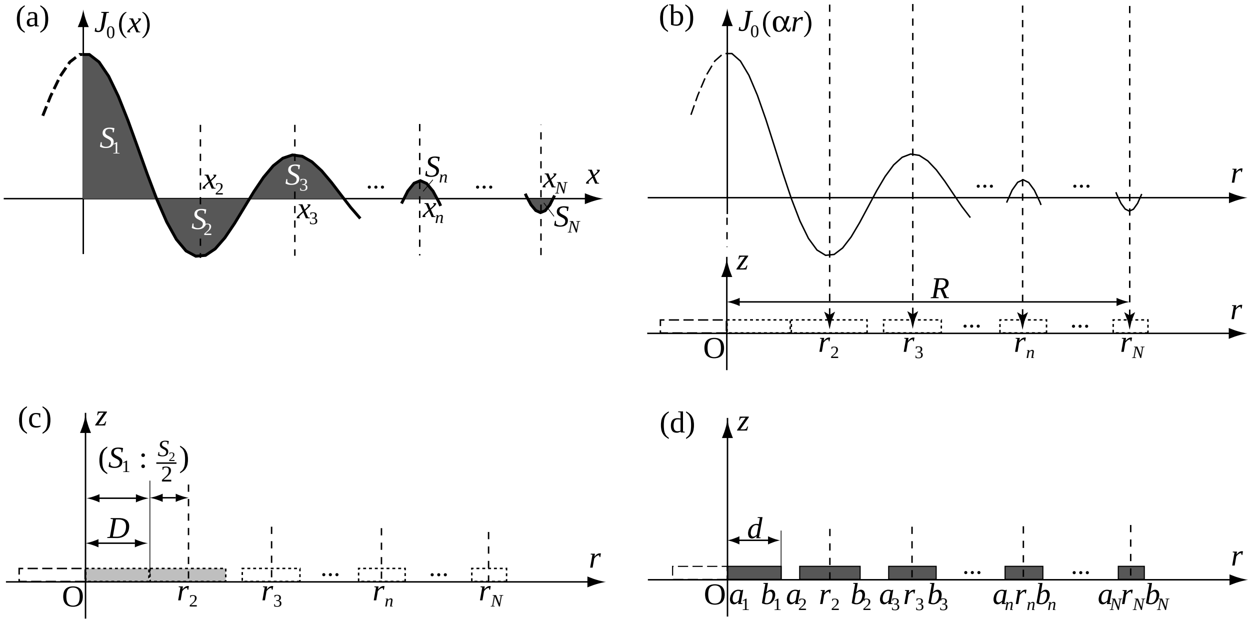

Fig. 2: Design procedure of a concentric annular array for generating a nondiffracted beam. (a) Zeroth-order Bessel function of the first kind, $J_0(x)$, (b) Allocation of the center radius of each element, r_n, (c) Maximum radius of the center element, D and (d) Actual radius of the center element, d, and allocation of the inner radius, a_n, and the outer radius, b_n, of each element.

The center disk element is located at $r=0$, which yields the main extreme value of the J_0 function. As shown in Fig. 2(b), each of the ring elements is located at r, which yields extreme values of the scaled J_0 function, $J_0(\alpha r)$. When the center radius of the nth element from the innermost disk element is r_n and the center radius of the outermost N th element is $R = r_N$, r_n is expressed by the following equation,

$$r_n = R\frac{x_n}{x_N} \quad (n > 2), \tag{1}$$

where x_n and x_N are the values that yield the n th and N th extreme values, $J_0(x_n)$ and $J_0(x_N)$, respectively. And, the scale factor of the J_0 function, α, is given by the following equation.

$$\alpha = \frac{x_N}{r_N}. \tag{2}$$

The relative width of each element is determined by the area S_n in Fig. 2(a), enclosed by the extreme lobe of the J_0 function and the x-axis. As shown in Fig. 2(c), the width of each element becomes the largest possible value when the center disk element and the 2nd ring element are close together. In this situation, the radius of the center disk element D is given by the following equation.

$$D = r_2 \frac{S_1}{S_1 + \frac{S_2}{2}}. \tag{3}$$

The actual radius of the center disk element, d, is given by the following equation,

$$d = KD \quad (0 < K \le 1), \tag{4}$$

where K is the coefficient that determines the absolute width of each array element, which is decided only relatively by the J_0 function. Normally, K is selected appropriately from the range of $0 < K \le 1$, and as a result, the source has a clearance between each neighboring array element.

The inner radius a_n and the outer radius b_n of the n th element are given by the following equations, and an annular array sound source shown in Fig. 2(d) is constructed.

$$a_n = \begin{cases} 0 & (n = 1), \\ r_n - d\dfrac{S_n}{2S_1} & (\text{otherwise}), \end{cases} \tag{5}$$

$$b_n = \begin{cases} d & (n = 1), \\ r_n + d\dfrac{S_n}{2S_1} & (\text{otherwise}). \end{cases} \tag{6}$$

The beam which has a sharp profile is generated by driving by driving a source formed in this way with an antiphase from its neighboring elements and with an equiamplitude.

In the concentric annular transducer array described above, the radiation direction of the beam becomes variable by applying the clearance which exists between each element in order to decenter it. Figure 3 shows the decentering procedure of each array element.

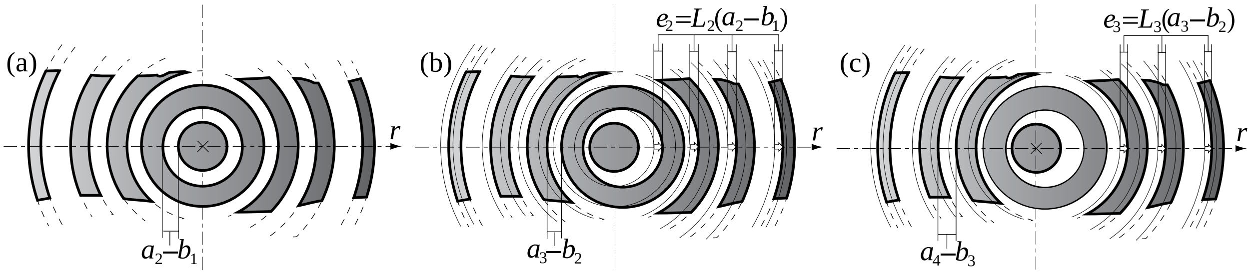

Fig. 3: Decentering procedure of array elements, (a) Step 0: the initial condition, (b) Step 1: all elements except for the center disk element are decentered and (c) Step 2: elements outer than 2nd ring element are decentered.

The initial condition of the annular transducer array is shown in Fig. 3(a) as a top view. The clearances in the array are expressed by the inner radii a_n and outer radii b_n of array elements, and the width of the clearance between $(n-1)$th and nth element becomes $a_n - b_{n-1}$. This is the maximum possible value of decentering, and the actual decentering amount e_n concerning the nth element is given by the following equation,

$$e_n = L_n(a_n - b_{n-1}) \quad (n > 2), \tag{7}$$

where L_n is the coefficient that is determined arbitrarily from the range of $0 \le L_n \le 1$, and it is termed a decentering ratio of each element.

At first, the innermost clearance, $a_2 - b_1$, is considered. The decentering amount $e_2 = L_2(a_2 - b_1)$ is obtained by the use of L_2 which is previously determined, and all elements except for the center disk are decentered at e_2. Figure 3(b) illustrates this operation. Next, with respect to the subsequent clearance, $a_3 - b_2$, $e_3 = L_3(a_3 - b_2)$ is obtained similarly, and array elements that are located more outside than the clearance $a_3 - b_2$ are decentered at e_3. These are shown in Fig. 3(c). By repeating the above operation for the case of $e_N = L_N(a_N - b_{N-1})$ appertaining to the outermost clearance in a sequential order, a decentered annular array transducer consisting of N elements is constructed, as shown in Fig. 4.

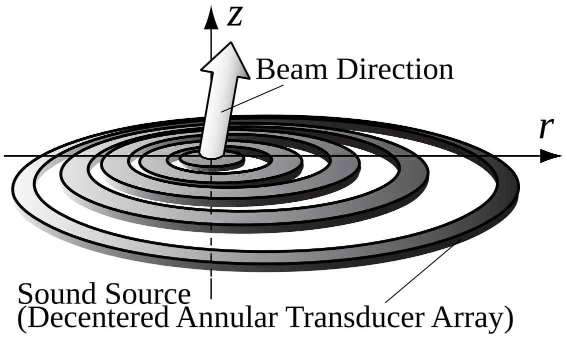

Fig. 4: Geometry of a coordinate system and a sound source by a decentered annular transducer array.

Numerical Calculations: Figure 5 displays numerical calculation results of the pressure distribution in magnitude, when decentered annular arrays are driven by a single-frequency continuous wave.

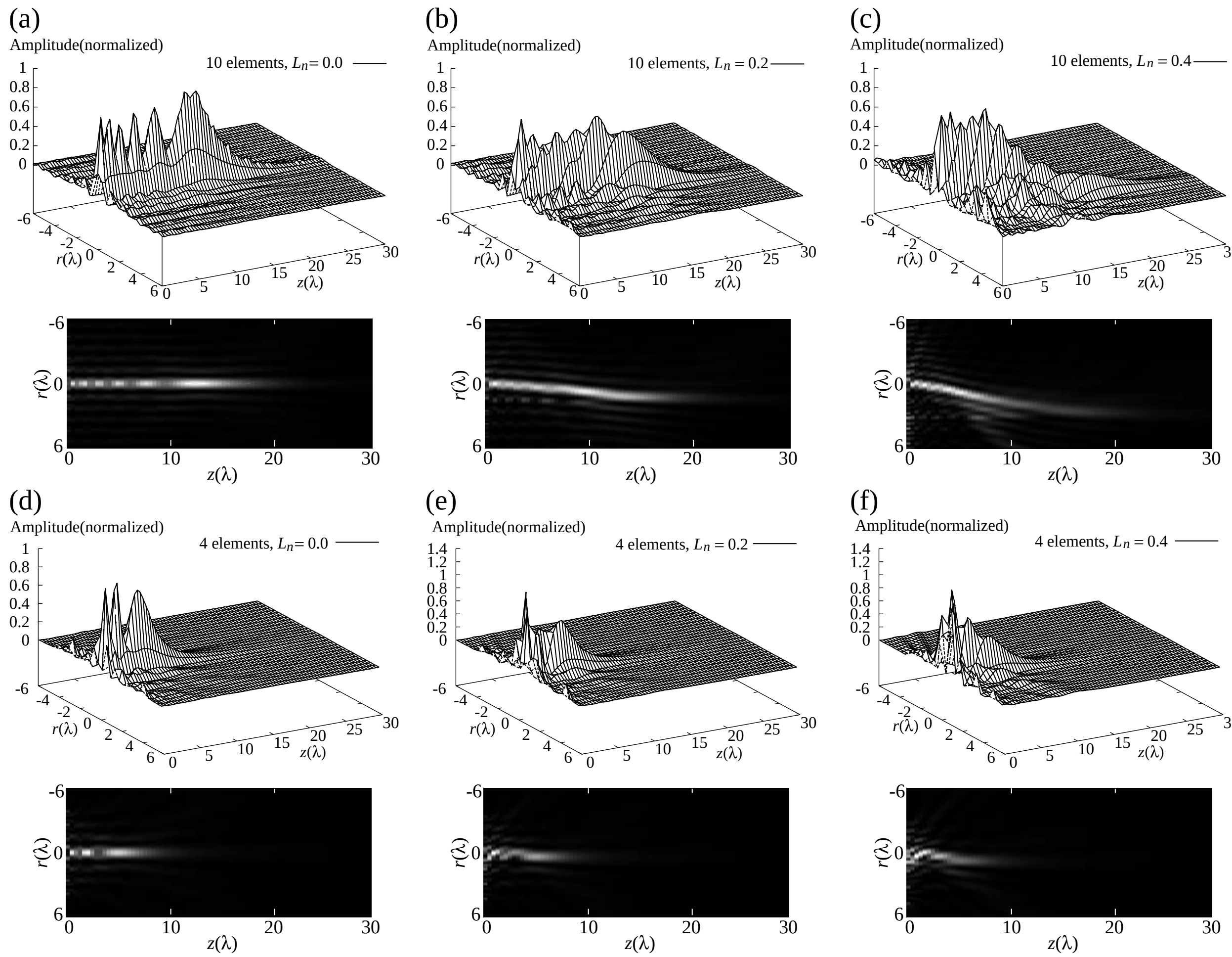

Fig. 5: Calculation results of the radiated pressure distribution from decentered annular arrays, 3-D plot (upper) and intensity distribution (lower); (a) 10 elements, $R = 10\lambda$, $L_n = 0.0$, (b) 10 elements, $R = 10\lambda$, $L_n = 0.2$, (c) 10 elements, $R = 10\lambda$, $L_n = 0.4$, (d) 4 elements, $R = 4\lambda$, $L_n = 0.0$, (e) 4 elements, $R = 4\lambda$, $L_n = 0.2$ and (f) 4 elements, $R = 4\lambda$, $L_n = 0.4$.

The three-dimensional plots are shown in the upper part of the subfigures. The amplitude is normalized by the maximum value of the case in Fig. 5(a), and the calculation intervals are 0.4λ in both r and z-axial directions. In the intensity distributions shown in the lower part of the subfigures, the amplitude is normalized by the maximum value of each calculation result, and the calculation intervals are 0.1λ in the r-axial direction and 0.4λ in the z-axial direction. In Fig. 5(a)-(c), the number of array elements is 10 and the center radius of the outermost element R is 10λ. In contrast, Fig. 5(d)-(f) show the results in the case that the number of array elements is reduced. In Fig. 5(d)-(f), the number of array elements is 4 and the center radius of the outermost element R is 4λ. In each array, K of eq. (4) is 0.6. The decentering ratio L_n in eq. (7) of each array is set to $L_n = 0.0$ for all n (in Fig. 5(a) and Fig. 5(d)), $L_n = 0.2$ for all n (in Fig. 5(b) and Fig. 5(e)) and $L_n = 0.4$ for all n (in Fig. 5(c) and Fig. 5(f)), respectively. Additionally, the decentering directions are identical for all array elements. In all subfigures of Fig. 5, it is shown that the beam in which the width is narrow in comparison with the radius of the source is formed.

In the case of $L_n = 0.2$, as in Fig. 5(b), it is shown that the radiated beams are deviated from the perpendicular of the source plane approximately 5 degrees. Furthermore, the fluctuation of magnitude for the peak of each beam becomes smaller than the non-decentered case (see Fig. 5(a)), and it is apparent that a stable beam is radiated to a certain distance.

In the case of $L_n = 0.4$, as in Fig. 5(c), though a little attenuation of the magnitude is shown in the beam located far from the source plane, the radiation direction of the beam becomes approximately 15 degrees from the perpendicular of the source plane, which is greater than the case of Fig. 5(b).

On the other hand, in the case of the arrays shown in Fig. 5(d)-(f), which have 4 elements and the center radius of the outermost element R is 4λ, it appears that a propagation distance of the beam is shorter than the case of Fig. 5(a)-(c), since the number of elements is reduced and the diameter of the entire sound source gets smaller. However, even if the cases of a few elements like these, it is possible to observe the beam deviations concerning the radiation direction. In the case of $L_n = 0.2$, as in Fig. 5(e), it is shown that the radiated beams are deviated from the perpendicular of the source plane approximately 3 degrees, and in the case of $L_n = 0.4$, as in Fig. 5(f), it is shown that the radiation direction of the beam becomes approximately 7 degrees from the perpendicular of the source plane.

From these results, it is shown that the direction of sharp beam becomes variable within the range of approximately 15 degrees from the perpendicular of the source plane, and that the radiation direction of the beam is variable, even if the decentered annular array has a few elements, such as only 4 elements.

Conclusions: We propose a method for generating a direction-variable acoustic wave beam with sharp profile using an annular transducer array. In this method, an annular array that has a clearance between each neighboring element is used. By applying these clearances to decenter the array elements, the radiation direction of the beam becomes variable. From the results of numerical calculations using sound sources by this method, it is shown that the sharp beam with the direction is up to approximately 15 degrees from the perpendicular of the source plane can be radiated, by the annular array that has 10 elements and the center radius of the outermost element R is 10λ. Furthermore, it is also shown that the radiation direction of the beam is variable in maximum of 7 degrees, even if the small annular array that has only 4 elements and the center radius of the outermost element R is 4λ.

The sound source using this method has a small, simple and planar structure, and it is considered that it becomes electronically controllable by selecting driving elements in the simple switching operations. Therefore, it is also considered that the source has advantages such as high flexibility of installation location and simplicity of manipulation, in comparison with conventional methods such as mechanical scanning or phased array, which require a scanning mechanism or a phase-shifting circuit. We expect that the sound sources based on this technique will be widely applied in the field of nondestructive testing, as the directional acoustic probes, and we will continue to examine the many facets of the practicability of this sound source.

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