REFLECTION AND TRANSMISSION OF LAMB WAVES AT DISCONTINUITY IN PLATE

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Abstract: Lamb waves can be used for testing thin plate and pipe because they provide great potential for improving inspection efficiency and sensitivity. In this paper, numeric analysis implemented by boundary element methods (BEM) and the normal mode expansion technique is used to study the Lamb wave reflection and transmission coefficients at discontinuity in plate. It is considered the interaction of Lamb waves with two basic discontinuities—thickness step and thickness incline. Interaction of individual Lamb wave modes with discontinuity in a plate is investigated by calculating the reflection and transmission coefficients for varying incident Lamb wave modes, frequency, and discontinuity shape. A0 and S0 Lamb mode in aluminum plate incident at frequency to 1.4 MHz, the plate thickness variation from 0.5 mm to 2.0 mm and incline angle from 0° to 60° at the point of discontinuity are considered. The mode conversion phenomena through step and incline discontinuity in a plate are also given. Results in this paper can be used to improve inspection sensitivity and penetration power for a variety of practical NDE applications, notably those in which thickness variation is found.

Introduction: Ultrasonic Lamb wave inspection is an elegant and promising technique whereby an ultrasonic pulse is sent to scan a long path through the structure, such as plate, tube and pipe. Lamb wave propagates in the direction along the surface of plate or in the axial direction of pipe, and as the material like metal has very little damping, the energy of the waves is practically not absorbed as they travel, and thus they can travel over long distances. Therefore a large area of the tested structure can be inspected from a single location. Many work has been published on the use of Lamb waves for inspection purposes because this approach provides great potential for improving inspection efficiency and sensitivity compared with the conventional bulk wave techniques.
However, previous studies show that even simple structural configurations can lead to complex response signals for lamb waves. Hence, it is still needs further study and validation before it can be accepted in the natural industry. On the other hand, theoretical analysis of Lamb wave reflection and transmission (or scattering) at discontinuity in structure is extremely difficult because of the complex multi-mode conversion phenomena. Analytical Approach already has been done for simple geometries with identical thickness and perfect specimens without defects (for example see [1]]). However, these equations become intractable for more complicated geometries or for a non-perfect specimen. Many researchers have tried to tackle Lamb wave scattering problems using numerical methods, e.g. The finite difference method (FDM)[2], the finite element method (FEM)[3] and the boundary element method (BEM) [4-5]. The FDM was the first numerical method to be applied to investigate the propagation of stress waves. The FEM is sometimes very cumbrous to use for more complex shaped waveguide problems due to the nature of its domain type formulation. And the BEM has the advantage that just the surface of the specimen needs to be discretized; the numerical problem itself is therefore reduced by one dimension.
A hybrid BEM (HBEM) combined with the normal mode expansion technique has been used to study Lamb wave interaction with surface breaking defects. This technique is here applied to study of Lamb wave reflection and transmission at step discontinuity and inclined discontinuity in plate. Interaction of individual Lamb wave modes with discontinuity is investigated by calculating the reflection and transmission coefficients in scattered fields for varying incident modes, frequency, and the step elevations or the angles of the incline. Lamb wave propagation in inhomogeneous plates whose thickness varies at definite position is interesting problem but have been reported rarely[6].

HBEM numerical modeling: The calculation model in this study considers only the plate with spatially varying thickness without considering the defect in plate (e.g. notch or crack in plate). This paper presents two HBEM models for step discontinuity (thickness step) and inclined discontinuity of plate respectively. Fig.1a shows the calculation model of step discontinuity with thickness variations. The calculation model for the inclined discontinuity is shown in Fig.1b. Boundary $\Gamma 2$ and boundary $\Gamma 4$ are two real free surfaces of the plate, and there is a typ of discontinuity on $\Gamma 4$. $\Gamma 1$ and $\Gamma 3$ are two created boundary for HBEM calculation. An incident time harmonic Lamb wave is considered propagating in the positive *x* direction. The mode incident onto the discontinuity results in both reflected and transmitted waves of all orders of the propagating.

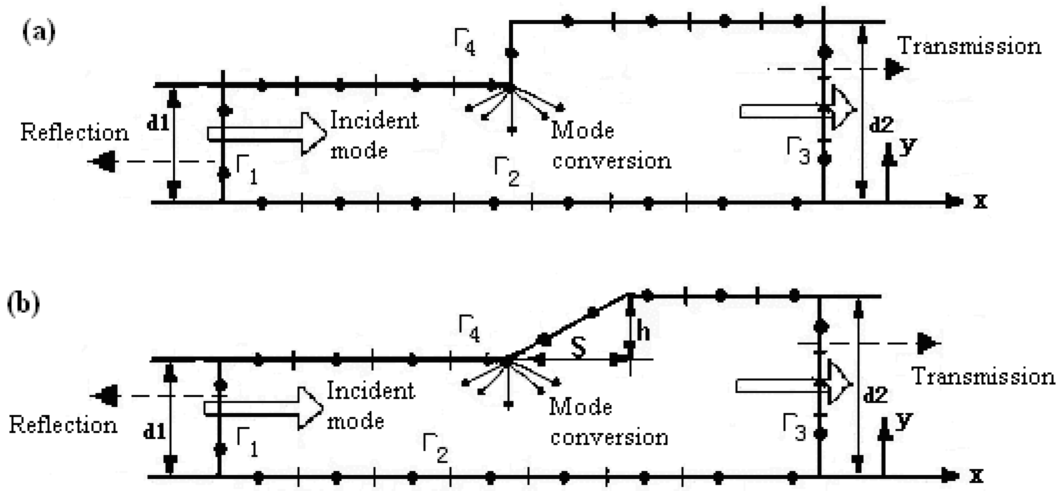

Fig 1: HBEM modeling statement of Lamb wave reflection and transmission in plate with (a) step discontinuity; (b) inclined discontinuity

Unlike the conventional BEM, the boundary element displacement or strain on boundary $\Gamma 1$ and $\Gamma 3$ can not be expressed by certain definite values. The total displacement and surface traction on $\Gamma 1$ and $\Gamma 3$ are the superposition of all of the incident and scattered propagating Lamb wave fields which can be obtained by Lamb wave expansion technique [5]. The *j*th mode Lamb wave reflection coefficient R^j and the *m*th mode Lamb wave transmission coefficient T^m can be calculated as follows:

$$R^j = \alpha^j / \mathrm{A}^I$$

$$T^m = \beta^m / \mathrm{A}^I$$

where A^{I} is the incident Lamb wave amplitude, α^j and β^m denote the amplitude of the *j*th Lamb wave traveling in the negative x directions on $\Gamma 1$ and the amplitude of the *m*th Lamb wave traveling in the positive x directions on $\Gamma 3$ respectively. Detailed Description for HBEM can be found in [5] and will not be iterated here.

In this study, the aluminum plate was considered for simulation. The material properties for aluminum were chosen as: longitudinal wave velocity $C_L = 6260 m/s$, the transverse wave velocity $C_T = 3045 m/s$ and density $\rho = 2.7 g/cm^3$.

Results and Discussion: Matlab codes for two-dimensional HBEM were developed for calculating the reflection and transmission coefficients of Lamb waves on plate with arbitrary shaped internal or surface defects [7]. A simple case Lamb wave scattering, reflection from a free edge in a semi-infinite plate was already successfully solved by various experimental and numerical methods [4,8]. As a comparison, the reflection coefficient curves of plate edge reflection for the case of A0 and A1 incident are show in Fig. 2a and Fig 2b respectively. The plate thickness here is identical with 2mm (d1=d2=2mm in Fig.1). These curves are similar with previous experimental and numerical results. Because the HBEM simulation is a closed form solution, it is confirmed that the HBEM codes developed in this study can be successfully applied to solve Lamb wave scattering problems.

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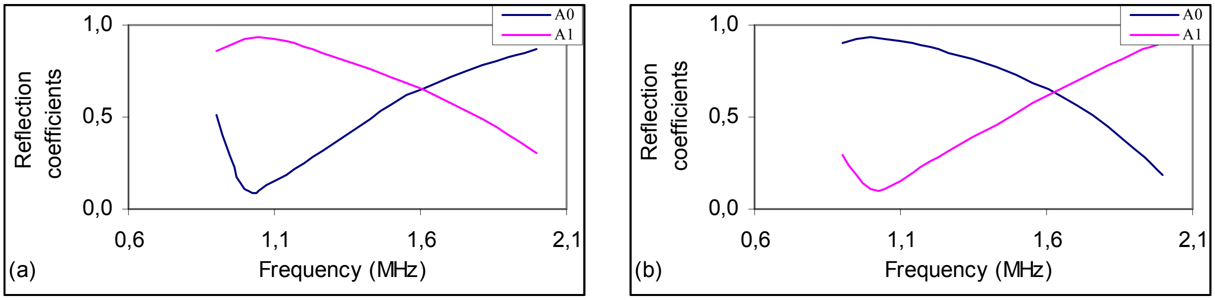

Fig. 2. The Lamb wave reflection curve for a free edge in semi-infinite aluminum plate with thickness 2mm. (a) A0 incident; (b) A1 incident

The HBEM calculated Lamb wave reflection and transmission coefficients versus frequency are plotted in Fig.3 and Fig.4 for the S0 mode and A0 mode incident onto the step discontinuity (d1=1mm and d2=2mm) in an aluminum plate respectively. As seen in Fig.3 and Fig.4, there are at least two wave modes in plate, and three wave modes within the range of high frequency in a plate with thickness 2mm. They show that the reflection and transmission coefficients for any mode under both of S0 mode or A0 mode incident are strongly dependent on frequency. However, at low frequencies(e.g. below 0.5 MHz), it can be seen that the transmission coefficients of S0 mode under S0 incident remain on high level, but the transmission coefficients of A0 mode are relative lower. Fig.5 and Fig.6 show the reflection and transmission curves for the S0 mode and A0 mode incident onto the step discontinuity (d1=2mm and d2=1mm). This similar phenomenon is also shown in Fig.5 and Fig.6.

Figs. 7- 10 show the S0 reflection and transmission curves under S0 incident, the A0 reflection and transmission curves under A0 incident, for several step discontinuity, while Figs. 7-8 show the Lamb waves propagate from thin side to thick side in plate, and Figs. 7-8 show the Lamb waves propagate from thick side to thin side in plate. Note that in these four figures, the reflection and transmission coefficients do not increase or decrease monotonically with the difference between d1 and d2 by most situation, only it is clear that the reflection and transmission coefficients of A0 at low frequencies have the monotonic trend.

Figs. 11- 14 show the reflection and transmission coefficients under S0 or A0 incident for inclined discontinuity with $|s|=|h|$=1mm, and d1=1mm or d2=1mm. Compared with the Figs. 3-6, it is seen that the influence of two type of discontinuity is relatively small. Furthermore, the S0 reflection and transmission curves under S0 incident, the A0 reflection and transmission curves under A0 incident, for different several inclined discontinuity, are given in Figs. 15- 18. Here is the inclined degree *arc**tan***(s/h=0.57735)=30°; *arc**tan***(s/h=1)=45° and *arc**tan***(s/h=0.57735)=60° respectively. It is also found that the influence of different inclined degree (up to 60°) for reflection and transmission curves is also not very large.

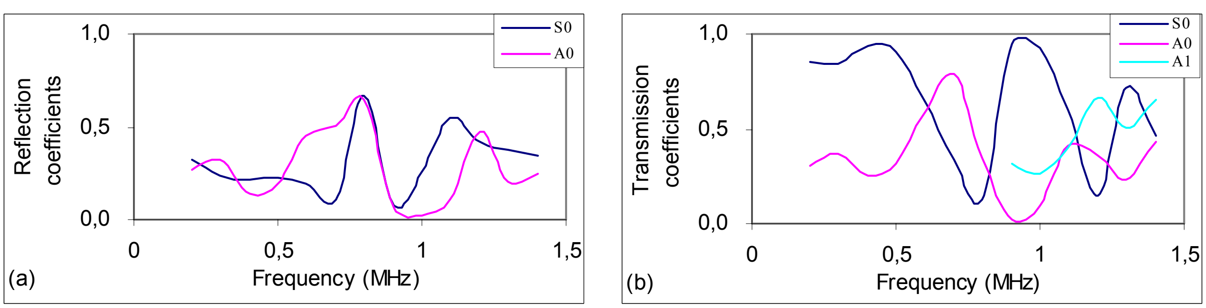

Fig. 3. (a) Reflection and (b) transmission coefficients under S0 incident for aluminum plate with step discontinuity d1=1mm and d2=2mm

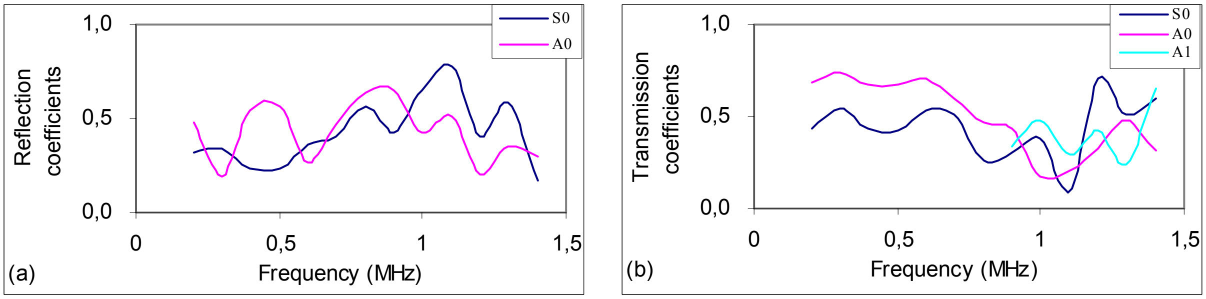

Fig. 4. (a) Reflection and (b) transmission coefficients under A0 incident for aluminum plate with step discontinuity d1=1mm and d2=2mm

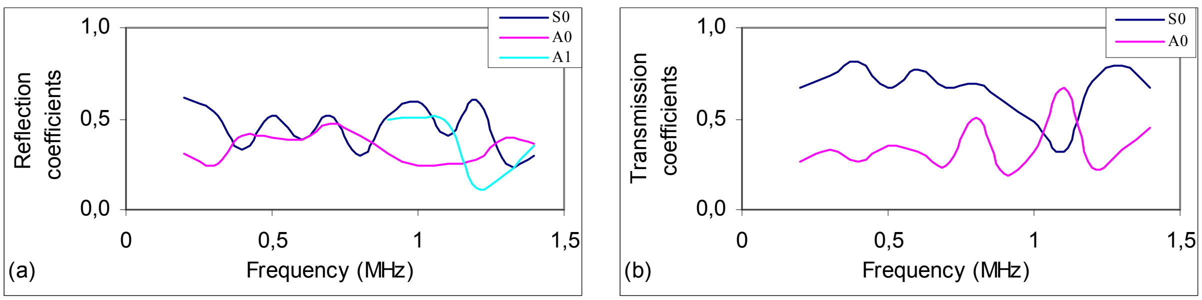

Fig. 5. (a) Reflection and (b) transmission coefficients under S0 incident for aluminum plate with step discontinuity d1=2mm and d2=1mm

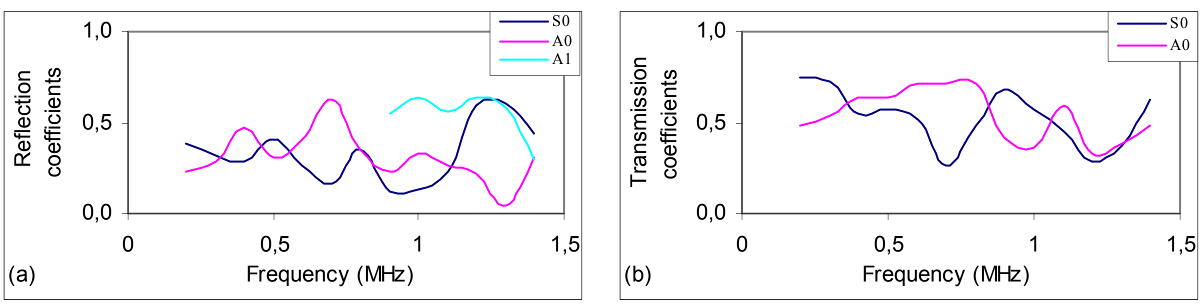

Fig. 6. (a) Reflection and (b) transmission coefficients under A0 incident for aluminum plate with step discontinuity d1=2mm and d2=1mm

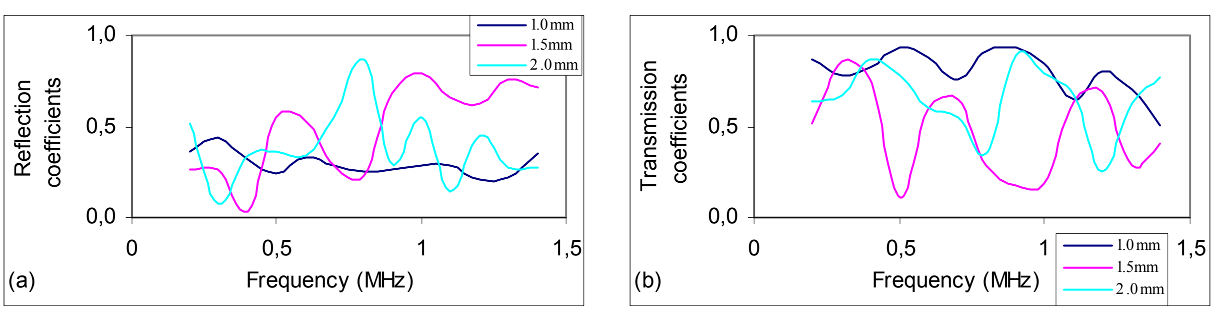

Fig. 7. (a) Reflection and (b) transmission coefficients for S0 mode under S0 incident for aluminum plate with step discontinuity d1=0.5mm and d2=1mm, 1.5mm, 2.0mm

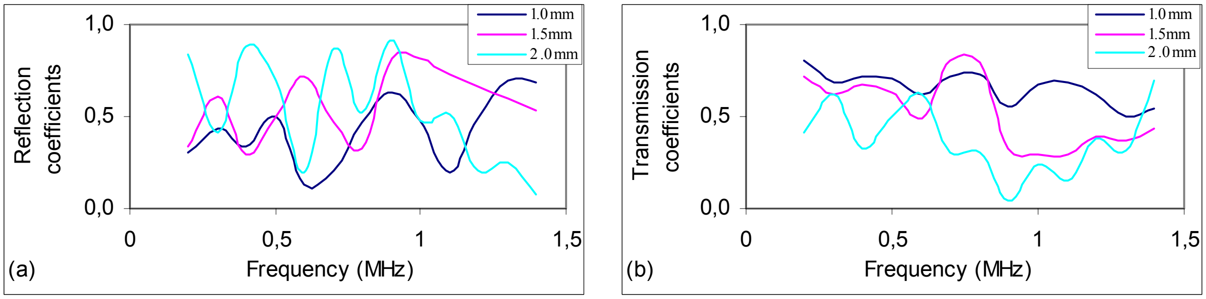

Fig. 8. (a) Reflection and (b) transmission coefficients for A0 mode under A0 incident for aluminum plate with step discontinuity d1=0.5mm and d2=1mm, 1.5mm, 2.0mm

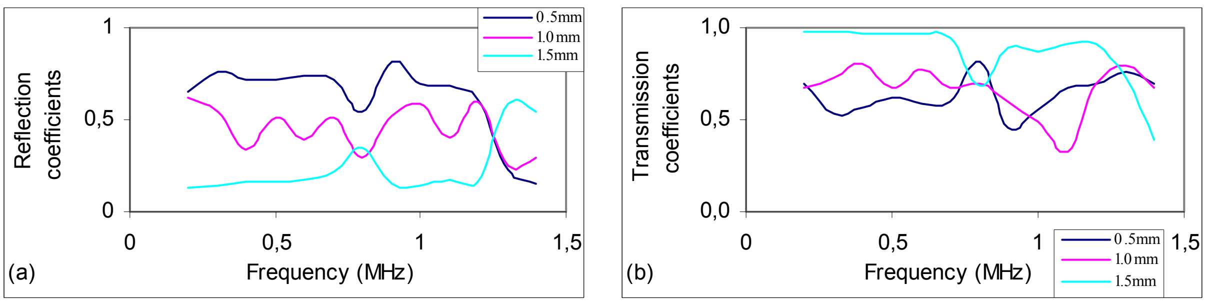

Fig. 9. (a) Reflection and (b) transmission coefficients for S0 mode under S0 incident for aluminum plate with step discontinuity d1=2mm and d2=0.5mm, 1.0mm, 1.5mm

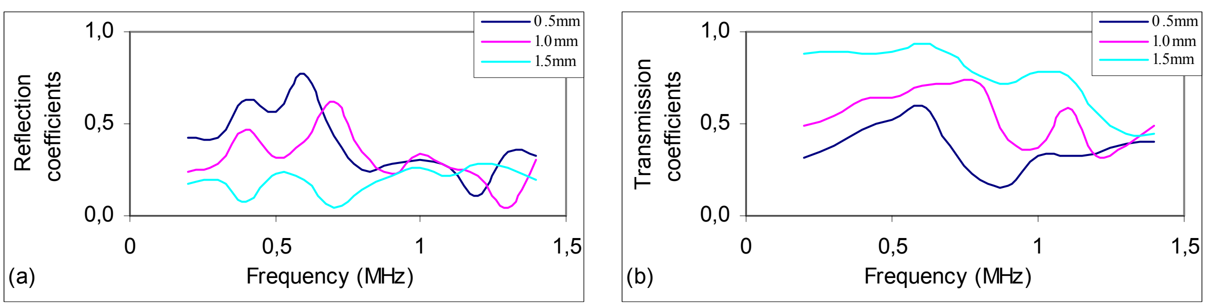

Fig. 10. (a) Reflection and (b) transmission coefficients for A0 mode under A0 incident for aluminum plate with step discontinuity d1=2mm and d2=0.5mm, 1.0mm, 1.5mm

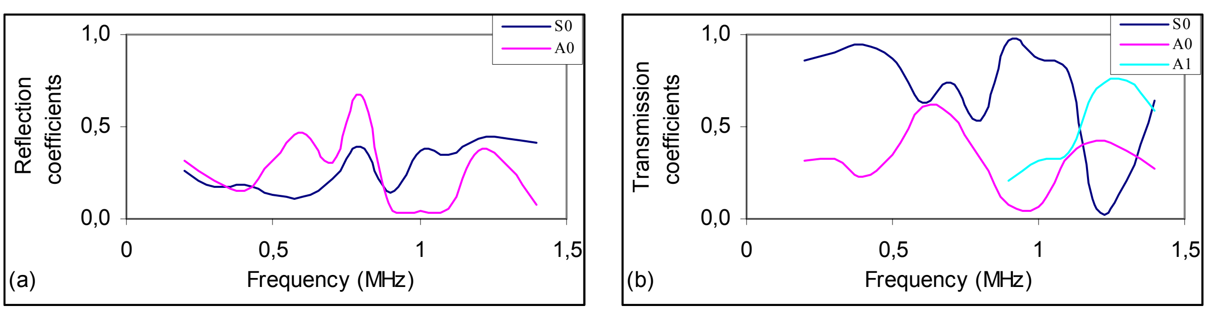

Fig. 11. (a) Reflection and (b) transmission coefficients under S0 incident for aluminum plate with inclined discontinuity d1=s=h=1mm and d2=2mm

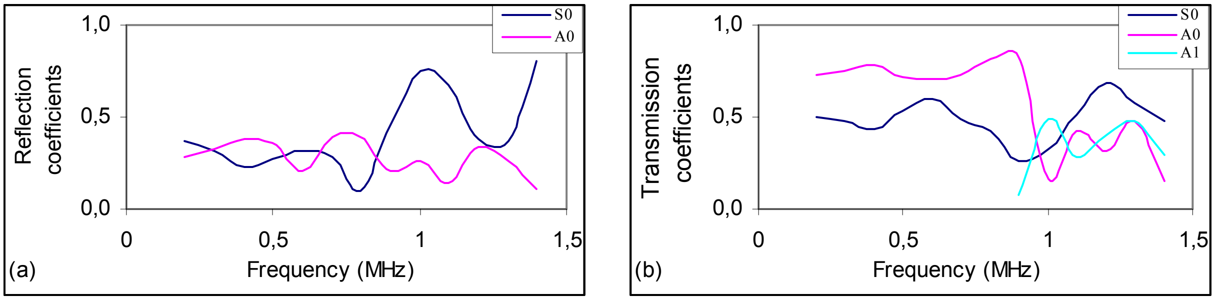

Fig. 12. (a) Reflection and (b) transmission coefficients under A0 incident for aluminum plate with inclined discontinuity d1=s=h=1mm and d2=2mm

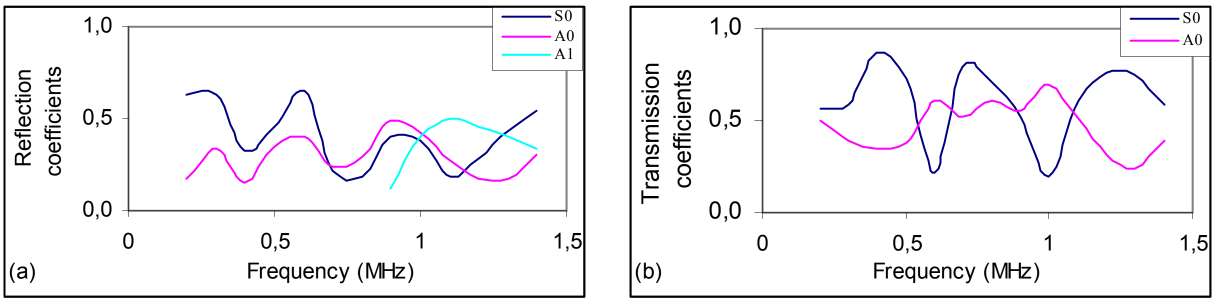

Fig. 13. (a) Reflection and (b) transmission coefficients under S0 incident for aluminum plate with inclined discontinuity d1=2mm and d2=s=1mm; h= –1mm

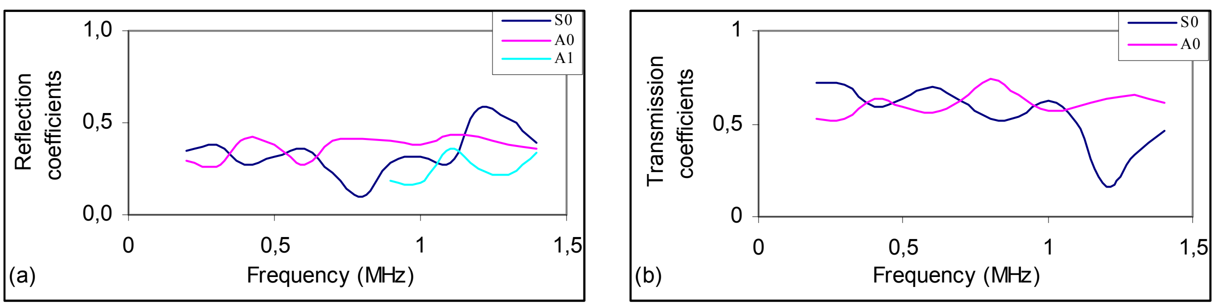

Fig. 14. (a) Reflection and (b) transmission coefficients under A0 incident for aluminum plate with inclined discontinuity d1=2mm and d2=s=1mm; h= –1mm

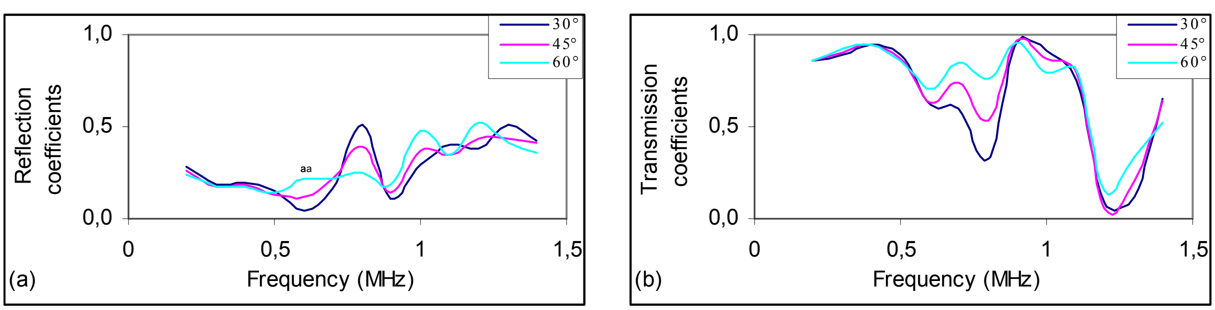

Fig. 15. (a) Reflection and (b) transmission coefficients for S0 mode under S0 incident for aluminum plate with inclined discontinuity d1= h=1mm; d2= 2mm; and s=0.57735mm, 1.0mm, 1.73205mm

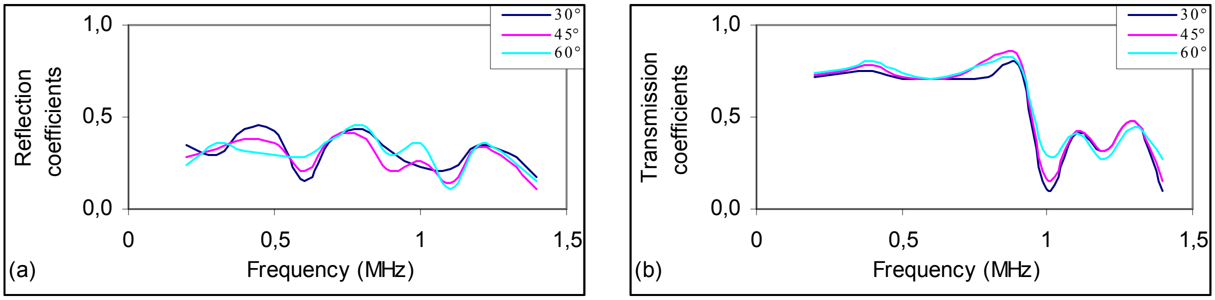

Fig. 16. (a) Reflection and (b) transmission coefficients for A0 mode under A0 incident for aluminum plate with inclined discontinuity d1= h=1mm; d2= 2mm; and s=0.57735mm, 1.0mm, 1.73205mm

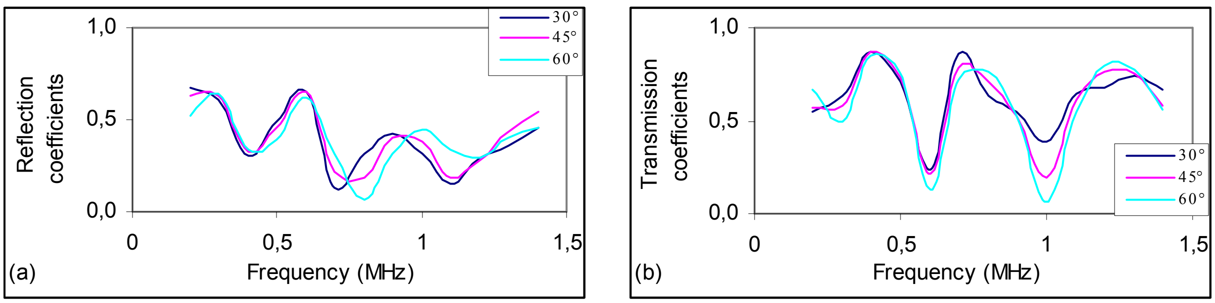

Fig. 17. (a) Reflection and (b) transmission coefficients for S0 mode under S0 incident for aluminum plate with inclined discontinuity d1=2mm; h= –1mm; d2= 1mm; and s=0.57735mm, 1.0mm, 1.73205mm

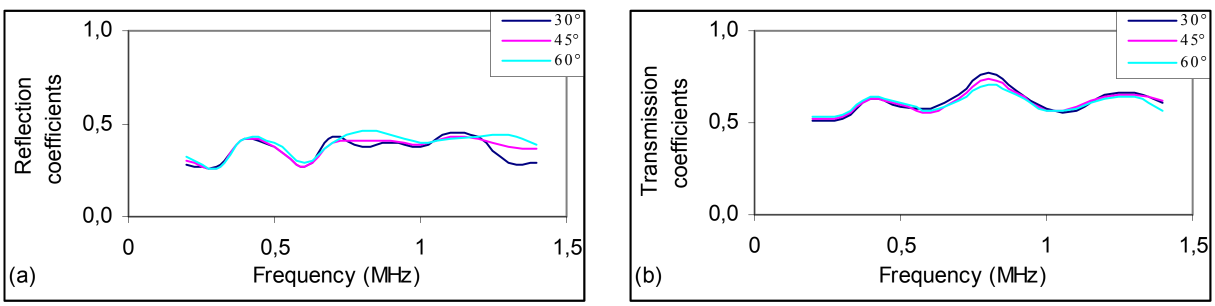

Fig. 18. (a) Reflection and (b) transmission coefficients for A0 mode under A0 incident for aluminum plate with inclined discontinuity d1=2mm; h= –1mm; d2= 1mm; and s=0.57735mm, 1.0mm, 1.73205mm

Conclusions: The HBEM can be used to treat the Lamb wave scattering onto discontinuity in plate. Serial numerical results show the complex for Lamb wave reflection and transmission at discontinuity in plate. According to the calculation, both of S0 and A0 mode Lamb wave have a good penetration through step discontinuity and inclined discontinuity at low frequency, and the sensitivity of inclined degree is also not large (up to 60°). But they have different range of frequency. On other hand, the reflection and transmission coefficients for any mode under both of S0 mode or A0 mode incident are strongly dependent on frequency, particularly at high frequency. The present HBEM is capable of optimizing the Lamb waves and the range of frequency for NDE of plate or pipe with different discontinuity.

References:

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