MEASUREMENT OF OBJECT SPATIAL DISPLACEMENT BY ULTRASONIC SPECKLE CORRELATION METHOD (No.56)

H. Zhu[1], W. Zheng[1], and Z Huang[1]
[1]Dept. of Mechanics, Huazhong University of Science and Technology, Wuhan, P. R. China

Abstract: When an object moves, ultrasonic speckles back-scattered from its surface will follow the object to move. From the Kirchhoff diffraction theory and the correlation principles of random signals, the necessary condition for keeping the correlativity between the speckle fields before and after the objective displacement was deduced. Based on this condition, the formulas for the relationship between the speckle displacement and the objective displacement were obtained. Practical measurement was performed. Correlation method was used to measure the in-plane displacement and out-of-plane displacement of an object. The displacement of the objective surface were obtained after the displacement of the speckles were measured. This method can be also used to measure the displacement of an inner objective surface. A mountain-climbing search method was proposed, which enabled us to find the maximum correlation coefficient in the correlation operation quickly and efficiently. The experimental results showed good agreement with the theoretical predictions.

Introduction: When a rough objective surface is insonified by ultrasound, ultrasonic speckles are formed in the back-scattering space[1]. Ultrasonic speckles have many useful features, e.g. ultrasonic speckles can exist within a solid object, and they are not easily affected by bad environment such as dust, frost, etc.. Especially, as the object moves, the ultrasonic speckles will follow the object to move. According to this feature, the displacement of an object can be figured out by measuring the displacement of the speckles in the space. In order to achieve this goal, the formulas for the relationship between the speckle displacement and the object displacement were deduced. Then according to these formulas, practical measurements were made by using correlation method. In the experiment a mountain-climbing search method was employed to find the maximum correlation coefficient.

Results: In order to find the relationship between the displacement of speckles back-scattered in the space and the displacement of an objective surface, the ordinate $O-XYZ$ in the space and the ordinate $O'-X'Y'Z'$ on the objective surface are established respectively as shown in Fig.1.

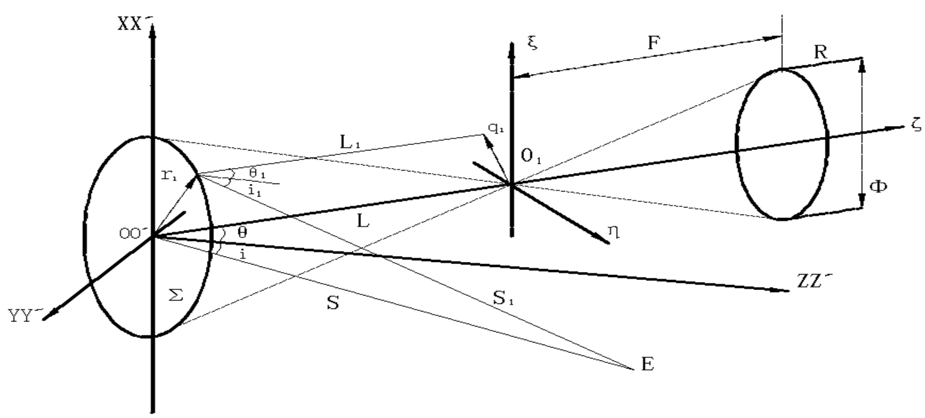

Fig.1 Set-up diagram of the ordinates on the objective surface and in the space

Before the objective displacement, these two ordinates coincide with each other. The sound source E is located in the (*Y-Z*)-plane with $i=\angle EOZ$ being the incident angle. The ordinate $O_1-\xi\eta\varsigma$ is established in the observing space, where the ($\xi-\eta$)-plane is located in the focus plane of a focus probe R for receiving the speckles. The focus length of probe R is F, and the diameter of its head face is Φ. O_1 is located in the (Y-Z)-plane. $O_1\xi$ is parallel to OX, and OO_1 and $O_1\varsigma$ are in the same direction. $\theta = \angle O_1OZ$ is the observing angle and both the θ and i are small quantity. According to the Kirchhoff diffraction theory[2], the complex amplitude of the speckle at $\boldsymbol{q}_1$ in the observing space is:

$$\tilde{A}(\boldsymbol{q}_1) = \int\int_{-\infty}^{\infty} P(\boldsymbol{r}_1)\tilde{A}(\boldsymbol{r}_1)\frac{\exp(\mathrm{j}kL_1)}{L_1}dxdy, \qquad (1)$$

where $k = 2\pi/\lambda$, λ is the ultrasonic wavelength, $\boldsymbol{r}_1 = x\boldsymbol{i} + y\boldsymbol{j}$, $P(\boldsymbol{r}_1)$, the aperture function of the area $\sum$ on the objective surface insonified by the incident ultrasound, is:

$$P(\boldsymbol{r}_1) = \begin{cases} 1 & \boldsymbol{r}_1 \quad \text{inside of } \Sigma \\ 0 & \boldsymbol{r}_1 \quad \text{outside of } \Sigma \end{cases}, \qquad (2)$$

and $\tilde{A}(\boldsymbol{r}_1)$ is the sound complex amplitude on the objective surface:

$$\tilde{A}(\boldsymbol{r}_1) = \frac{D(\boldsymbol{r}_1)}{\mathrm{j}\lambda}\frac{\exp(\mathrm{j}kS_1)}{S_1}\frac{\cos i_1 + \cos\theta_1}{2}, \qquad (3)$$

where $D(\boldsymbol{r}_1)$ is a circular symmetric complex Gaussian random variable, which is dependant on the scattering features of the objective surface. i_1, θ_1, S_1 and L_1 are noted as in Fig.1 and $i_1 \approx i, \theta_1 \approx \theta$. Let the position of a point, which is fixed on the objective surface in the space ordinate $O - XYZ$, be $\boldsymbol{r} = (x, y, 0)^{\boldsymbol{T}}$, where $(...)^{\boldsymbol{T}}$ means the transposed matrix. As the objective surface has small translations u, v, w in the directions of X, Y, Z and small rotation angles α, β, γ about the axes of X, Y, Z respectively, the position of this point in the surface ordinate $O' - X'Y'Z'$ changes to be $\boldsymbol{r}' = (x', y', 0)^{\boldsymbol{T}}$. We have the following relationship between $\boldsymbol{r}$ and $\boldsymbol{r}'$:

$$\boldsymbol{r}' = [M]\boldsymbol{r} - [M][T] \qquad (4)$$

where $[M] = \begin{pmatrix} \cos\beta\cos\gamma & \cos\beta\sin\gamma & -\sin\beta \\ \sin\alpha\sin\beta\cos\gamma - \cos\alpha\sin\gamma & \cos\alpha\cos\gamma - \sin\alpha\sin\beta\sin\gamma & \sin\alpha\cos\beta \\ \cos\alpha\sin\beta\cos\gamma + \sin\alpha\sin\gamma & \cos\alpha\sin\beta\sin\gamma - \sin\alpha\cos\gamma & \cos\alpha\cos\beta \end{pmatrix}$

and $[T] = (u, v, w)^{\boldsymbol{T}}$. After the objective displacement, the ultrasonic complex amplitude $\tilde{A}(\boldsymbol{q}_2)$ at $\boldsymbol{q}_2$ in the observing space ordinate $O_1 - \xi\eta\zeta$ is:

$$\tilde{A}(\boldsymbol{q}_2) = \frac{1}{\mathrm{j}\lambda}\int\int_{-\infty}^{\infty} P(\boldsymbol{r}_2')D(\boldsymbol{r}_2')\frac{\exp[\mathrm{j}k(S_2 + L_2)]}{S_2L_2}\frac{\cos i_2 + \cos\theta_2}{2}dx'dy', \qquad (5)$$

where $\boldsymbol{r}' = x'\boldsymbol{i}' + y'\boldsymbol{j}'$ and i_2, θ_2, S_2 and L_2 are noted as in Fig.1 and $i_2 \approx i, \theta_2 \approx \theta$. Thus, the correlation function of the scattering sound field in the observing space before and after the objective displacement can be written as:

$$R[\tilde{A}(\boldsymbol{q}_1)\tilde{A}^*(\boldsymbol{q}_2)] = \frac{1}{\lambda^2}\iiint\int_{-\infty}^{\infty} < P(\boldsymbol{r}_1)P(\boldsymbol{r}_2') >< D(\boldsymbol{r}_1)D^*(\boldsymbol{r}_2') > \frac{\exp\{-\mathrm{j}k[(S_2 + L_2) - (S_1 + L_1)]\}}{S_1S_2L_1L_2}\frac{(\cos i + \cos\theta)^2}{4}dxdydx'dy', \qquad (6)$$

where * denotes the conjunction operation and $< \Lambda >$ means the assemble average. If the object has a strongly rough surface, we have:

$$< D(\boldsymbol{r}_1)D^*(\boldsymbol{r}_2') >= C\delta[\boldsymbol{r}_1 - ([M]\boldsymbol{r}_2 - [M][T])], \qquad (7)$$

where C is a positive real constant and $\delta(.)$ is a two dimensional Dirac function. Eq.(7) means:

$$<D(\boldsymbol{r}_1)D^*(\boldsymbol{r}_2')>=\begin{cases} C, & \boldsymbol{r}_1=\boldsymbol{r}_2'=[M]\boldsymbol{r}_2-[M][T] \\ 0, & \boldsymbol{r}_1\neq\boldsymbol{r}_2'=[M]\boldsymbol{r}_2-[M][T] \end{cases} \quad . \qquad (8)$$

Substituting Eq.(7) into Eq.(6), we have:

$$R[\tilde{A}(\boldsymbol{q}_1)\tilde{A}^*(\boldsymbol{q}_2)]=\frac{C}{\lambda^2}\int\int_{-\infty}^{\infty}<P(\boldsymbol{r}_1)P([M]\boldsymbol{r}_2-[M][T])> \frac{\exp\{-\mathrm{j}k[(S_2+L_2)-(S_1+L_1)]\}}{S_1S_2L_1L_2}\frac{(\cos i+\cos\theta)^2}{4}dxdy \quad . \qquad (9)$$

When the integral function in Eq.(9) is a real one, i.e.

$$(S_2+L_2)-(S_1+L_1)=0, \qquad (10)$$

$R[\tilde{A}(\boldsymbol{q}_1)\tilde{A}^*(\boldsymbol{q}_2)]$ gets to the maximum, which means that there is a maximum similarity between the two speckle fields produced respectively by the surface before and after its motion. Using the method proposed by Wu[3], we find the rules of the ultrasonic speckle movement in the space, which are similar to that of the laser speckle movement, to be:

$$\begin{cases} U=L(\cos i+\cos\theta)\beta+L(\sin i-\sin\theta)\gamma+u(1+\dfrac{L}{S}) \\ V=-L(1+\dfrac{\cos i}{\cos\theta})\alpha+v(\cos\theta+\dfrac{L\cos^2 i}{S\cos\theta})+w(\dfrac{L\sin i\cos i}{S\cos\theta}-\sin\theta) \\ W=v(\sin\theta-\dfrac{L^2}{S^2}\sin i)+w(\cos\theta+\dfrac{L^2}{S^2}\cos i) \end{cases} \qquad (11)$$

where U, V, W are the translations of the speckles in the ξ,η,ς directions respectively, and L and S are noted as in Fig.1.

Discussion: It must be pointed out that the translations u,v,w and the rotation angles α,β,γ of the object can not be directly figured out from the displacement U, V, W of the speckles in the back-scattering space according to Eq.(11). However, we can see that if i,θ and L are set to be zero, Eq.(11) is simplified to be $U=u, V=v$ and $W=w$. Thus, the displacement of the objective surface can be directly figured out according to the displacement of the speckles on the objective surface. To determine the speckle displacement, the digital correlation method[4] was used. Suppose that before the objective displacement, the amplitude distribution of the speckles in a small sub-field in the space ordinate $O-XYZ$ is $f(x_i,y_j,z_k)$. After the objective displacement, the amplitude distribution of the speckles in the same sub-field in the space ordinate is $f'(x_i,y_j,z_k)$, which can be expressed as $g(x_i',y_j',z_k')$ in the surface ordinate $O'-X'Y'Z'$. The displacement of all the points in this small sub-field can be treated to be uniform. The correlation coefficient of $f(x_i,y_j,z_k)$ and $g(x_i',y_j',z_k')$ is defined as:

$$C(u,v,w,\alpha,\beta,\gamma)=\frac{\sum_{\boldsymbol{i}=1}^{\boldsymbol{m}}\sum_{\boldsymbol{j}=1}^{\boldsymbol{m}}\sum_{\boldsymbol{k}=1}^{\boldsymbol{m}}[f(x_i,y_j,z_{\boldsymbol{k}})-\bar{f}]\cdot[g(x_i',y_j',z_{\boldsymbol{k}}')-\bar{g}]}{\sqrt{\sum_{\boldsymbol{i}=1}^{\boldsymbol{m}}\sum_{\boldsymbol{j}=1}^{\boldsymbol{m}}\sum_{\boldsymbol{k}=1}^{\boldsymbol{m}}[f(x_i,y_j,z_{\boldsymbol{k}})-\bar{f}]^2}\sqrt{\sum_{\boldsymbol{i}=1}^{\boldsymbol{m}}\sum_{\boldsymbol{j}=1}^{\boldsymbol{m}}\sum_{\boldsymbol{k}=1}^{\boldsymbol{m}}[g(x_i',y_j',z_{\boldsymbol{k}}')-\bar{g}]^2}}, \qquad (12)$$

where $\bar{f}=\sum_{\boldsymbol{i}=1}^{\boldsymbol{m}}\sum_{\boldsymbol{j}=1}^{\boldsymbol{m}}\sum_{\boldsymbol{k}=1}^{\boldsymbol{m}}f(x_i,y_j,z_{\boldsymbol{k}})\Big/m^3$, $\bar{g}=\sum_{i=1}^{m}\sum_{j=1}^{m}\sum_{k=1}^{m}g(x_i',y_j',z_k')\Big/m^3$ and

$$C(u,v,w,\alpha,\beta,\gamma)=\begin{cases} 1 & f(x_i,y_j,z_{\boldsymbol{k}})\ and\ g(x_i',y_j',z_{\boldsymbol{k}}')\ being\ \ correlated \\ 0 & f(x_i,y_j,z_{\boldsymbol{k}})\ and\ g(x_i',y_j',z_{\boldsymbol{k}}')\ being\ \ not\ \ correlated \end{cases} \quad .(13)$$

If the object has only an in-plane displacement of translations u and v in the X and Y directions and a rotation angle γ about the Z axis, Eq.(12) becomes to be

$$C(u,v,\gamma)=\frac{\sum_{i=1}^{m}\sum_{j=1}^{m}[f(x_i,y_j)-\bar{f}][g(x_i',y_j')-\bar{g}]}{\sqrt{\sum_{i=1}^{m}\sum_{j=1}^{m}[f(x_i,y_j)-\bar{f}]^2}\sqrt{\sum_{i=1}^{m}\sum_{j=1}^{m}[g(x_i',y_j')-\bar{g}]^2}}, \tag{14}$$

where

$$\begin{cases} x'=(x-u)\cos\gamma+(y-v)\sin\gamma \\ y'=-(x-u)\sin\gamma+(y-v)\cos\gamma \end{cases}. \tag{15}$$

In order to reduce the search time for finding the maximum correlation coefficient and raise the measurement accuracy, a mountain-climbing search method is proposed. The idea is as follows. $C(u, v, \gamma)$ is a trinary function with a peak value. If u and v , or v and γ, or γ and u remain to be constant, $C(\gamma)$, or $C(u)$, or $C(v)$ becomes to be an one dimensional Gaussian function. Moreover, if $f(x_i, y_j)$ and $g(x_i', y_j')$ are correlated, $C(\gamma)$, or $C(u)$, or $C(v)$ is at the peak point of the curve of the one dimensional Gaussian function respectively.

By using this search method, experiment was performed to measure the in-plane displacement of the speckles on an objective surface. The measurement system is shown in Fig.2. Probe R□φ 20mm, f 2.5MHz, F40mm□was perpendicularly placed and focused on the point O on the objective surface in water. R was connected to a three dimensional translation stage T, which was driven by the stepping motors controlled by a PC. Thus, R could move in X, Y, and Z directions simultaneously with a step length of 0.2mm, which equaled to a half of the average lateral size of the speckles[5]. R scanned in a small square area in the (O-XY)-plane with point O being its center and R received the signals one by one. The signals were then amplified, digitized, processed and input to a PC. A total of 11×11 speckle signals were measured and they consisted of a speckle amplitude distribution function $f(x_i, y_j)$,(i, j = 1,2,…,21) after a linear interpolation was operated. After the object had small translations u, v in X,Y directions and a rotation angle γ

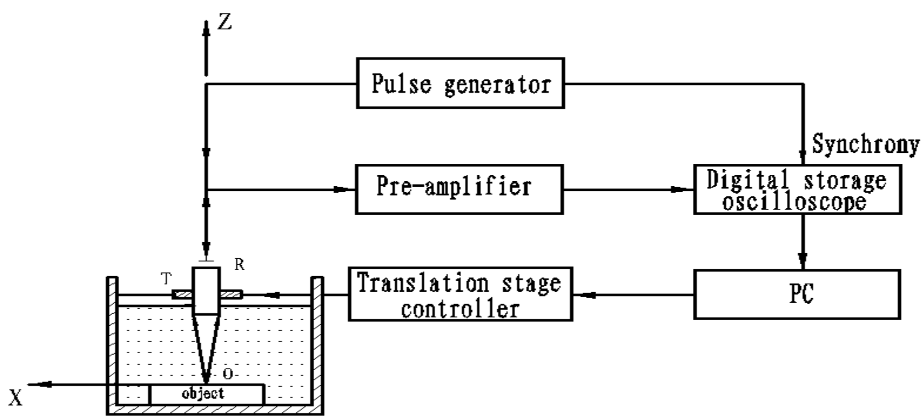

Fig.2 Measurement system

by Z axis, the measurements were repeated in the same way in the same area and function $g(x_i', y_j')$,(i, j = 1,2,…,21) was obtained. Then, the correlation calculations were operated as follows. Firstly, u and v were remained to be zero but γ was changed from -1° to 1° with an increment of 0.1°. After the correlation calculation based on Eq.(14) was done, a peak value of $C(\gamma)$ and the correspondent angle γ_p were obtained. Next, γ was remained to be γ_p and u was remained to be zero, but v was changed from –2.2mm to 2.2mm with an increment of 0.1 mm. v_p could be obtained according to the peak value $C(v)$ after the correlation operation. Finally, γ and v were remained to be γ_p and v_p respectively, but v was changed from –2.2mm to

2.2mm with an increment of 0.1 mm.. u_p was obtained according to the peak value $C(u)$ after the correlation operation. As a result, u_p and v_p were the displacement of the speckle on the measuring point *O,* which equaled to the in-plane displacement of the point *O* on the objective surface to be examined.

The out of plane displacement of an object can also be measured by using this method. In this case, R, which was focused on the point *O* at first, scanned in the *Z* direction. Before and after the out of plane displacement of the object, the speckle amplitude distribution function $f(z_k)$,(k = 1,2,…,21) and $g(z'_k)$ (k = 1,2,…,21) were obtained. After the correlation operation, w_p could be obtained according to the peak value $C(w)$. w_p was the out-of-plane displacement of the point *O*.

In the measurements, the errors between the measuring displacement and the calibrating displacement were 8.7%, 6.2% and 6.9% for *u, v* and *w* respectively.

Conclusions: The relationship between the displacement of an objective surface and the displacement of the speckles back-scattered from the surface was deduced based on the Kirchhoff diffraction theory and the correlation principles of random signals. When a probe, which worked in a self transmitting and receiving mode, was perpendicularly placed and focused on the objective surface, the displacement of the speckle measured by the probe was directly equal to the displacement of the measuring point on the objective surface. Moreover, the displacements of the speckles can be measured by using the correlation method. To verify the theoretical results, a special experimental set-up was built. In the measurement, the mountain-climbing search method, which was employed in the correlation operation, enable us to find the correlation coefficient peak value quickly and efficiently. The experimental results prove that the theoretical results are correct. Since the ultrasonic speckles can exist within a solid object, the method proposed in this paper can be also used to measure the displacement of the inside surface of an object.

References: 1. Abbott J G, Thurstone F L. Acoustic speckle: theory and experiment analysis. Ultrason. Imag., 1979; 1:303-324

2. Goodman J W. 1985 Statistical Optics. (New York: John Wiley and Sons)
3. Wu X P, He S P and Li Z C. Movement of the speckles in the space. Acta. Phys. Sin. 1980; **29** 1142-1150 (in Chinese)
4. Peters W H. Applications of digital correlation methods to digital body mechanics. Opt. Eng., 1983; **22**:738-742
5. Zhu H M and Q H Qin. Statistics of ultrasonic speckles reflected from a rough surface. Archive of Applied Mechanics, 2002; **72**:189-198