AN IMPROVED CONE-BEAM FILTERED BACKPROJECTION RECONSTRUCTION ALGORITHM BASED ON X-RAY ANGULAR CORRECTION AND MULTIRESOLUTION ANALYSIS

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Abstract: With the extensive application of industrial computed tomography in the field of non-destructive testing, how to improve the quality of the reconstructed image is receiving more and more concern. It is well known that in the existing cone-beam filtered backprojection reconstruction algorithms the cone angle is controlled within a narrow range. The reason of this limitation is the incompleteness of projection data when the cone angle increases. Thus the size of the tested workpiece is limited. Considering the characteristic of X-ray cone angle, an improved cone-beam filtered back-projection reconstruction algorithm taking account of angular correction is proposed in this paper. The aim of our algorithm is to correct the cone-angle effect resulted from the incompleteness of projection data in the conventional algorithm. The basis of the correction is the angular relationship among X-ray source, tested workpiece and the detector. Thus the cone angle is not strictly limited and this algorithm may be used to detect larger workpiece. Further more, adaptive wavelet filter is used to make multiresolution analysis, which can modify the wavelet decomposition series adaptively according to the demand for resolution of local reconstructed area. Therefore the computation and the time of reconstruction can be reduced, and the quality of the reconstructed image can also be improved.

Introduction: With the development of non-destructive testing, how to efficiently utilize the cone-beam projections to reconstruct 3-D images is more and more important. In practice, it is convenient to control the mechanical system and the complexity of the whole system is reduced when single-circle trajectory is adapted. Therefore the single-circle trajectory is investigated in our research. FDK algorithm [1] is most popular when using single-circle trajectory, which is a approximation reconstruction algorithm including filtering and backprojection. However, FDK algorithm does not satisfy the necessary and sufficient condition for reconstruction which is brought out by B.Smith in 1985 [2]. So there exist two drawbacks of FDK algorithm. One is the lower density resolution of the reconstructed image when comparing with 2-D reconstruction algorithm. The other is that the larger the cone angle is, the worse the reconstructed result is. Many researchers have done much work to resolve these two problems. Minerbo [14] Proposed an orbit consisting of two perpendicular circles. Starting with Kirillov's mathematical theory [15] in 1961, Tuy [16] described an idea of using nonplanar orbits. His main contribution is an exact inversion formula for general orbits satisfying the data sufficiency condition of Smith [2]. In a similar direction, image reconstruction theory of cone-beam tomography was further advanced by Smith [2]. In particular, he made the following important contributions to the problem. First, he derived the completeness condition of the orbit, which can be described as follows. If and only if almost every plane intersecting the object support meets the orbit, the image can be exactly reconstructed without signal extrapolation. A single circle is an incomplete orbit from which complete information about the object cannot be obtained. Second, he derived an inversion formula that is based on the relation between the cone-beam projections and the 3-D Radon transform filtered in the radial direction. Third, he proved that the FDK algorithm coincides with Smith's inversion formula when the orbit is a single circle. An alternative approach permitting a unified treatment of various orbits was proposed by Grangeat [3], where the first derivative of the 3D Radon transform is utilized instead of Smith's intermediate function. The Grangeat algorithm is similar to the Smith algorithm, but has the following advantages when compared with Smith's results.

1) Grangeat's intermediate function is more tractable than Smith's one, because its support is finite when the object has finite extent. This advantage comes from the fact that Grangeat's

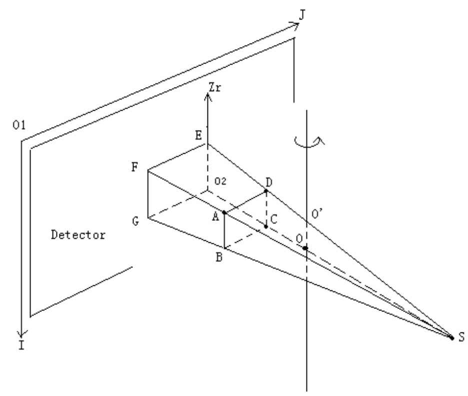
intermediate function is the 3D Radon transform convoluted with a local filter corresponding to the derivative operation.

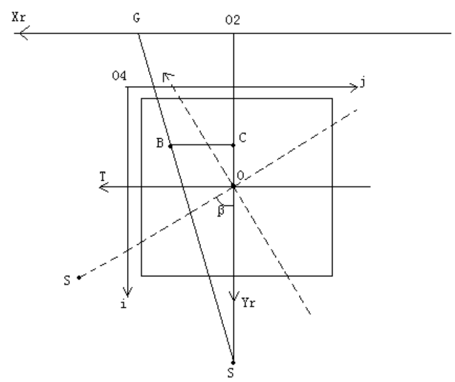
2) The reconstruction algorithm of Grangeat can be implemented with moderate computational requirements, because a procedure for computing the intermediate function is given in an efficient form.

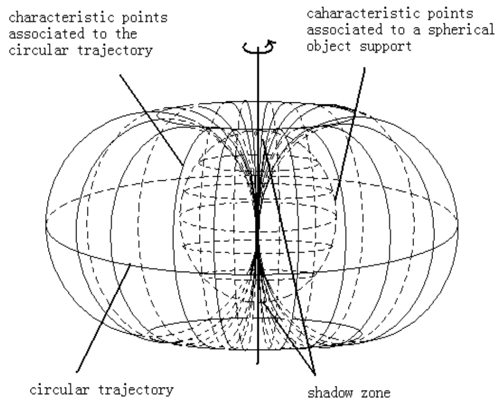
These investigations showed that image degradations due to incompleteness of the orbit can be reduced by using nonplanar orbits. However, in terms of density resolution and artifacts, the image quality is inferior to that of ordinary 2D tomography.

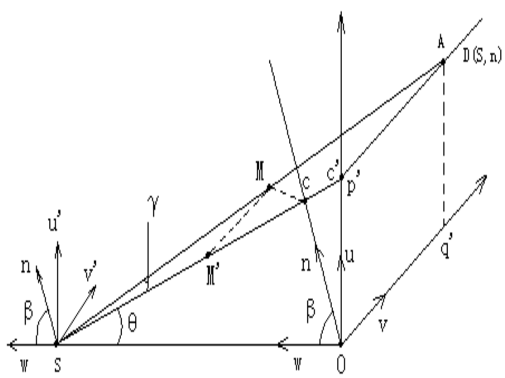
In this paper, we will correct the projection data to improve the reconstructed image of large cone angle under the circumstance of using single-circle trajectory. We will use multiresolution method to reconstruct different region of the reconstructed image with higher resolution in ROI and the lower resolution in other region. Thus the computational cost is reduced greatly and the efficiency is increased.

Results:

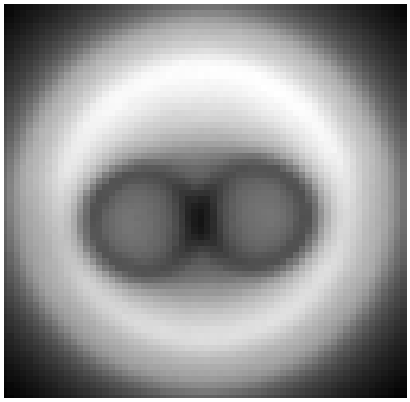


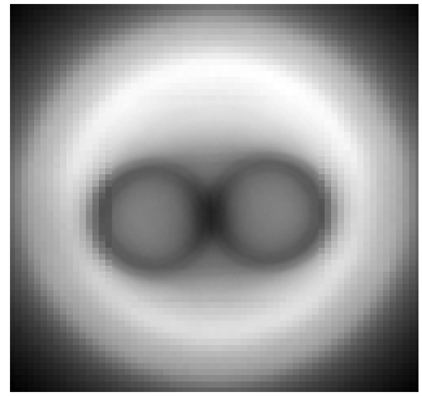


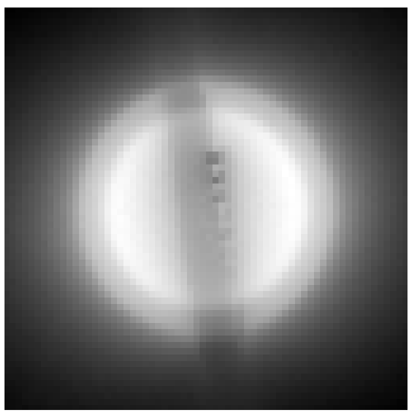




As shown in Fig.3 [3], shadow zone does not exist only in the midplane. The larger the cone angle is, the larger the shadow zone is. Thus with the increasing of cone angle, the Radon value is more and more incomplete which will reduce the quality of the reconstructed image. When the cone angle is larger than 6°, flat effect will appear in the reconstructed image. To lower these artifacts, it is necessary to fill the data by interpolation on this shadow zone. Comparing with Grangeat algorithm, our algorithm is based on FBP algorithm. We correct the cone angle effect by correcting the projection data when the cone angle increases. To get the better reconstructed image, we can change the filter according to different workpieces. Parameters of the fundamental relations are shown in Fig.4 [3], where $u0v$ denotes the virtual plane crossing the original and paralleling to detector plane, M the reconstructed point, A the projection of M in the virtual plane.







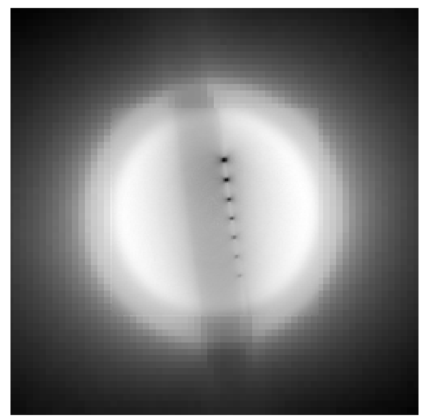


Fig.5 and Fig.7 show low resolution reconstruction of the whole image using different specimen respectively. Fig.6 and Fig.8 show multiresolution reconstruction of the whole image of the two specimen respectively using our method.

Discussion: The single circular trajectory is convenient to design. But it doesn't fulfill the necessary and sufficient condition on the acquisition curves. It is shown in the Smith necessary and sufficient condition [2] that if every plane that intersects the object, there exists at least one cone-beam source point, then the object can be reconstructed. The plane paralleling to the trajectory plane will intersect the object but never intersect the trajectory when adopting single-circle trajectory. Therefore, using single-circle trajectory will not satisfy the Smith's necessary and sufficient condition for exact reconstruction.

How to reconstruct a discrete approximation image at a certain resolution from the original projections? One approach would be to first reconstruct the full-resolution discrete image using FBP, then find the approximation coefficients, and finally find the discrete approximation image. However, it is possible to switch the order of operations listed above: Firstly, we will make multilevel decomposition of the projection data. Three-level decomposition is always used for 2-D information. Then we can make a lower resolution reconstruction of the whole image using the low frequency information of decomposed projection data at a few angles. In the next step, we will make a higher resolution reconstruction of ROI using the original projection data. At last, combining the above two images, we can get the final reconstructed image, which is reconstructed with higher resolution in ROI, but lower resolution in other region. The experiments show that the computation can be reduced about 3~4 times using this method.

Conclusions: In this publication, we have described the framework of our improved cone beam filtered backprojection reconstruction algorithm. The correction of the projection data is to reduce the flat effect when the cone angle increases. Because of the space-frequency localization property of many compactly supported wavelets and because these wavelets remain essentially compactlt supported after being filtered by the ramp filter. Computation can be reduced by performing a multiresolution reconstruction with higher resolution in ROI and lower resolution in other region. To reduce the computation, we use multiresolution reconstruction. The experimental result show the advantage of the algorithm. We feel that the techniques developed here provide a useful tool in industrial non-destructive testing.

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