# EIGEN VECTOR ALGORITHM FOR 3-D NEAR-FIELD MULTIPLE SOURCES LOCALIZATION

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**Abstract:** Most exiting array signal processing techniques for direction finding at present rely heavily on the far-field assumption. When the sources are located close to the array, these techniques may no longer perform satisfactorily. In this paper an eigen vector (EV) algorithm for 3-D near-field multiple sources localization is proposed. New algorithm has low computational quantities on 3-D near-field multiple sources localization by using 2-D near-field multiple sources localization algorithm twice. It also has the properties of high resolution compared with the MUSIC algorithm.
Keywords- near-field sources, eigen vector algorithm, 3-D Localization

***Introduction:*** *An important problem in a wide variety of application such as radar, sonar, speech, communication, etc. is locating in space the sources of signals received by an array of sensors. Most attention has been restricted to the model of far-field. In 2-D far-field source localization, a source's location is characterized only by its bearing angle because the wavefront from a far-field source can be assumed to be planar for a uniform linear array of sensor (ULA). However, when sources are located close to the array the assumption is no longer valid. The wavefront in such cases is spherical. In 2-D near-field source localization, a source's location is characterized by its bearing angle and range. The increase in number of parameter caused the algorithm based on the far-field assumptation can no longer be directly applied so substitution must be found.*

*Some work addressing the near-field problem in 2-D has been performed. Spatial Wigner-Ville[1] algorithm proposed by A.L. Swindlehurst et al. can locate the near-field sources but it requires large array if high resolution is needed. Huang et al[2] proposed the 2-D MUSIC algorithm which requires 2-D grid search on the bearing angle and range to identify the peak of MUSIC spectrum. This algorithm is easily to simulate but it is computed inefficiently. Weiss et al. [3] estimated source locations using a polynomial rooting method thereby reducing the computational burden to some extent, however, the algorithm still requires bulk of computations. Starer et al. [4] and Lee et al. [5] proposed pass-following algorithms to reduce the computational* burden with two 1-D searches.

*Although researchers have shown much interest in 3-D sources localization, the problem has not been widely treated in the literature. 3-D localization involves the estimation of spherical coordinates, namely azimuth, elevation, and range. Hung et al. [6] proposed an algorithm with exact model which is a 3-D extension of algorithm proposed by Weiss et al. But in this algorithm the pairing problem for multiple sources is not addressed. Challa et al. [7] proposed an algorithm using Unitary ESPRIT which solves the paring problem but requires a large amount of computation. Karim et al. [8] proposed a second order statistics based approach and considered two pairing techniques. Lee et al. [9] proposed a 3-D MUSIC algorithm which requires three 1-D searches with three uniform linear subarrays. But it can not solve the paring problem efficiently when the sources number is large.*

*In this paper we presents a 2-D eigen vector algorithm with two uniform linear subarrays to locate the near-field sources in 3-D. It has low computational quantities and the paring problem is well solved. And it has the properties of high resolution, low inutile signal-noise ratio threshold compared with the MUSIC algorithm.*

**Results: 1. 2-D near-field source localization model and assumption**

2-D near field source localization geometry model is shown in figure 1

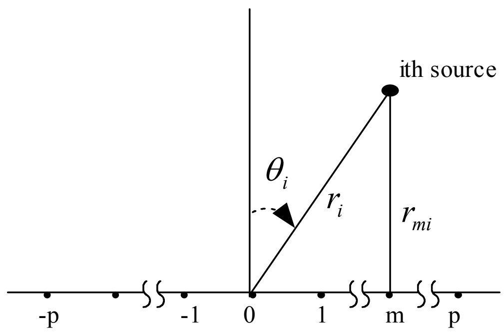


Fig 1 2-D near-field source localization geometry model

Consider $I$ near-field, narrowband, independent sources observed by a ULA of $M = 2p+1$ sensors with interelement spacing $d$. Let the array center be the phase reference point. The signal received by the mth sensor is expressed as

$$x_m(t) = \sum_{i=1}^{I} s_i(t) e^{-j\omega_i \tau_{mi}} + n_m(t), -p \le m \le p$$

(1)

where $s_i(t)$ denotes the ith source signal, $n_m(t)$ is the additive noise, $\omega_i$ is the carrier angle frequency of the ith source signal, and $\tau_{mi}$ is delay association with the ith source signal propagation time between sensor '0' and sensor $m$. By simple geometry, $\tau_{mi}$ is

$$\tau_{mi} = \frac{r_{mi} - r_i}{c} = \frac{r_{mi} - r_i}{\lambda f_i} = \frac{2\pi(r_{mi} - r_i)}{\lambda \omega_i}$$

(2)

where $r_{mi}$ is the distance between the ith source and the mth sensor, $r_i$ is the distance between the ith source and the 0th sensor, $c$ is the signal propagation speed and $f_i$ is the carrier frequency of the ith source signal

$$r_{mi} - r_i = r_i\left(\sqrt{1 + \frac{m^2 d^2}{r_i^2} - \frac{2md\sin\theta_i}{r_i}} - 1\right)$$

(3)

so

$$\tau_{mi} = \frac{2\pi r_i}{\lambda \omega_i}\left(\sqrt{1 + \frac{m^2 d^2}{r_i^2} - \frac{2md\sin\theta_i}{r_i}} - 1\right)$$

(4)

$\theta_i, r_i$ are the bearing and range of the ith source and $\lambda$ is its wavelength. Using binomial expansion and the so-called Freshnel approximation, we can write:

$$\omega_i \tau_{mi} \approx \frac{2\pi r_i}{\lambda}\left(\frac{m^2 d^2}{2r_i^2} - \frac{md\sin\theta_i}{r_i} - \frac{m^2 d^2 \sin^2\theta_i}{2r_i^2}\right) = \phi_i m + \varphi_i m^2$$

(5)

where the parameter $\phi_i$ and $\varphi_i$ are nonlinear function of azimuth $\theta_i$ and range $r_i$ of the ith source:

$$\phi_i = -2\pi \frac{d}{\lambda} \sin(\theta_i), \quad \varphi_i = \pi \frac{d^2}{\lambda r_i} \cos^2(\theta_i)$$

(6)

thus, the signal model can be approximately expressed as:

$$x_m(t) \approx \sum_{i}^{I} s_i(t) e^{j(\phi_i m + \varphi_i m^2)} + n_m(t)$$

(7)

It can be expressed using vector notion as following:

$$\mathbf{x}(t) = \mathbf{A}\mathbf{s}(t) + \mathbf{n}(t)$$

(8)

where

$$\mathbf{x}(t) = [x_{-p}(t), x_{-p+1}(t), \Lambda\ x_0(t) \Lambda\ x_{p-1}(t), x_p(t)]^T$$

$$\mathbf{s}(t) = [s_1(t), s_2(t), \ldots\ldots s_I(t)]^T$$

$$\mathbf{n}(t) = [n_{-p}(t), n_{-p+1}(t), \Lambda\ n_0(t) \Lambda\ n_{p-1}(t), n_p(t)]^T$$

$$\mathbf{A} = [\mathbf{a}(\theta_1, r_1), \mathbf{a}(\theta_2, r_2), \ldots\ldots \mathbf{a}(\theta_I, r_I)]$$

$\mathbf{a}(\theta_i, r_i) = [e^{j(-\phi_i p + \varphi_i p^2)}, \Lambda\ , 1, \Lambda\ e^{j(\phi_i p + \varphi_i p^2)}]$ is array manifold.

The following considered to hold through the work:

**H1** the source signals which allow Rayleigh distribution are mutually independent i.i.d random process , with zero mean;

**H2** The additive noise is a zero-mean spatially complex white process independent from the source signal and with covariance $\sigma^2$;

**H3** $d = \lambda / 4$ and $M > 2I$ .

The task of 2-D near field source localization is to estimate the parameter pair $\{\theta_i, r_i\}_{1 \le i \le I}$ .

**2. 2-D MUSIC & eigen vector algorithm**

Before describing our estimation technique we define the following covariance matrixes

$$\mathbf{R}_s \stackrel{\Delta}{=} E\{\mathbf{s}(t)\mathbf{s}^H(t)\}$$

(9)

$$\mathbf{R}_n \stackrel{\Delta}{=} E\{\mathbf{n}(t)\mathbf{n}^H(t)\} = \sigma^2 \mathbf{I}$$

(10)

$$\mathbf{R}_x \stackrel{\Delta}{=} E\{\mathbf{x}(t)\mathbf{x}^H(t)\} = \mathbf{A}\mathbf{R}_s\mathbf{A}^H + \mathbf{R}_n$$

(11)

In practice we will use the sample covariance matrix

$$\hat{\mathbf{R}}_x = \frac{1}{N} \sum_{t=1}^{N} \mathbf{x}(t)\mathbf{x}^H(t)$$

(12)

as the estimate of $\mathbf{R}_x$.

**(1)MUSIC algorithm[2]**

Rank order the eigen values of $\mathbf{R}_x$ in the descending order to obtain

$\lambda_1 \ge \lambda_2 \ge \Lambda\ \lambda_M$

Let eigen vector $u_{I+1}$ $u_{I+2}$,…, $u_M$ correspond to the (M-I) smaller eigen values, $\lambda_{I+1}, \lambda_{I+2}$,…, $\lambda_M$ ,we construct the noise eigen matrix $\mathbf{G}$ from the eigen vetors as

$\mathbf{G} = [\mathbf{u}_{I+1}, \mathbf{u}_{I+2} \Lambda\ \mathbf{u}_M]$

$a(\theta, r)$, corresponding to the coordinate pair the $\{\theta, r\}$ is the continuum of all possible near-field steering vector.

The spatial spectrum of MUSIC algorithm is

$$P_{MUSIC(\theta,r)} = \left|\mathbf{a}^H(\theta,r)\mathbf{G}\mathbf{G}^H\mathbf{a}(\theta,r)\right|^{-1}$$

(13)

The peak of the spectrum, $P_{MUSIC(\theta,r)}$ are the location of the source, $\{\theta_i, r_i\}$ $i = 1,2,\Lambda\ I$.

**(2)EV algorithm**

The eigen vector algorithm [10] is first used in the DOA estimation. If the number of snapshots is small and /or the SNR of the source is low, the resolution may decrease by using the MUSIC algorithm. In this paper we use the eigen vector algorithm which is a weighted eigenspace algorithm to improve the performance.

Let $\mathbf{U}\mathbf{U}^H = \mathbf{G}\mathbf{W}\mathbf{G}^H$, $\mathbf{W} = [\lambda_{I+1}^{-1}, \lambda_{I+2}^{-1} \Lambda\ \lambda_M^{-1}]$

The spatial spectrum of EV algorithm is :

$$P_{EV(\theta,r)} = \left|\mathbf{a}^H(\theta,r)\mathbf{G}\mathbf{W}\mathbf{G}^H\mathbf{a}(\theta,r)\right|^{-1}$$

(14)

**3. 3-D near-field source localization geometry model and algorithm**

3-D near field localization geometry model is shown in figure 2. 3-D near field localization is to estimate the parameter set $\{\alpha_i, \theta_i, r_i\}_{1\le i\le I}$, $\alpha_i$ is azimuth, $\theta_i$ is elevation and $r_i$ is range. In the traditional method plane array and 3-D search are always used. A 2-D near-field algorithm based 3-D algorithm will be proposed next.

**(1) 3-D near field single source localization**

If there is only one source, the $\{r_x, \beta_1'\}$ and $\{r_y, \beta_2'\}$ of the source can be get with 2-D near-field source localization algorithm twice in the $r_i - y$ axis plane and the $r_i - x$ axis plane, $\beta_1 = 90 - \beta_1'$ and $\beta_2 = 90 - \beta_2'$. After getting $\{r_x, \beta_1\}$ and $\{r_y, \beta_2\}$, then

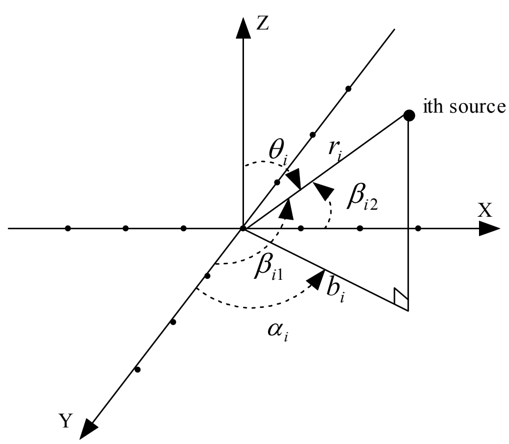



Fig 2 3-D near field localization geometry model

$$r = \frac{r_x + r_y}{2}$$
(15)

with simple geometry we can get the relation that:

$$\alpha = \arctan\frac{\cos\beta_2}{\cos\beta_1}$$
(16)

$$\theta = \arcsin\frac{\cos\beta_2}{\sin\alpha}$$
(17)

so the parameter set $\{\alpha_{i,}\theta_i, r_i\}$ can be get and we can localize the single source in 3-D.

**(2) 3-D near field multiple source localization**

If there are multiple sources, with 2-D near-field source localization algorithm twice we can get the two parameter sets $\{r_{ix}\beta_{i1}\}_{1\le i\le I}$ and $\{r_{iy}\beta_{i2}\}_{1\le i\le I}$ . Pair the two parameter sets between $\{r_{ix}\beta_{i1}\}_{1\le i\le I}$ and $\{r_{iy}\beta_{i2}\}_{1\le i\le I}$ , then use the single sources algorithm the $\{\alpha_{i,}\theta_i, r_i\}_{1\le i\le I}$ can be calculated.

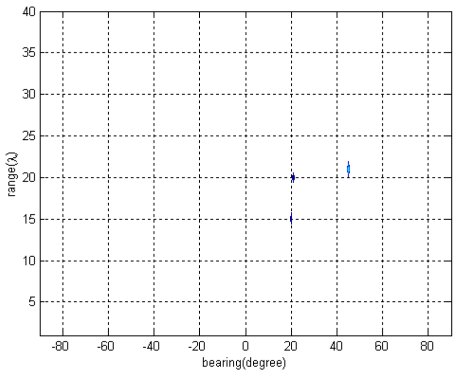
Next we introduce a parameter paring method. Get the $\{r_{ix}\beta_{i1}\}_{1\le i\le I}$ and $\{r_{iy}\beta_{i2}\}_{1\le i\le I}$ estimation, under the assumption that $r_{ix} \ne r_{jx}$ $r_{iy} \ne r_{jy}$ for $i \ne j$ using the above estimation, we can associate for each pair $\{r_{ix}\beta_{i1}\}_{1\le i\le I}$ its corresponding $r_{i'y}, \beta_{i'2}$ as follows:

$$i' = index\{\arg\min_{1\le j\le M} | r_{ix} - r_{jy} |\}$$
(18)

So the parameter sets $\{\alpha_{i,}\theta_i, r_i\}_{1\le i\le I}$ can be get and we can localize the multiple sources in 3-D.

**Discussion: 1Compare 2-D EV algorithm with MUSIC algorithm**

In order to show the effectiveness of the proposed algorithm seventeen-elements ULA is considered. The snapshot number is 128 and the SNR is 20. According to [11], the range of the near field is approximately $r \le 2L^2/\lambda$ where $L = (M-1)d = 4\lambda$ is the array's aperture. So in this paper $r \le 32\lambda$ is the near-field region. In this experiment we consider three sources from $\{30.0°\ 15.0\lambda\}$, $\{31.0°, 20.0\lambda\}$,$\{45.0°\ \ 21.0\lambda\}$impinging on the 1-D array. The spatial spectrum contour figures are shown in figure 3 and figure 4 with 20 times monte carlo simulation. From the experimental results we can see that the EV algorithm has high resolution and we can localize the three sources successfully no matter the bearing is close or the range is close, but under the same condition the MUSIC algorithm can no longer perform satisfactorily.



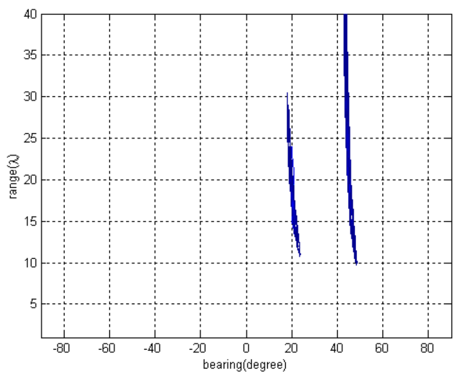




Fig3 EV algorithm

Fig4 MUSIC algorithm

**2 Single source 3D localization**

In this experiment we use two ULA which is the same as the array in 2-D localization. We consider only one source from $45.0°\ \ 45.0°\ \ 15.0\lambda$ impinging on the array showed in figure 2. The $x$ axis and $y$ axis are two ULA with seventeen-elements. By using the 2-D near-field source localization EV algorithm twice with grid search we get the range and bearing estimations which are $60.0°\ 15.0\lambda$ and $60.0°\ 15.0\lambda$, so using (15), (16) and (17) we get $r = 15.0\lambda$, $\alpha = 45.0°$ and $\theta = 45.0°$.

**3 Multiple source 3D localization**

The array is the same as single source 3D localization. In this experiment we consider two sources from $45.0°\ \ 45.0°\ \ 15.0\lambda$ and $30.0°\ \ 45.0°\ \ 20.0\lambda$ impinging on the array and by using the 2-D near-field source localization EV algorithm twice we localize them and with grid search method we get the range and bearing estimations sets□With the x axis array the parameter sets we estimate is $\{60.0°, 15.0\lambda\}$ and $\{75.0°, 20.0\lambda\}$, with the y axis array the set we estimate is $\{60.0°, 15.0\lambda\}$ and $\{75.0°, 20.0\lambda\}$. With parameter pairing method showed in (18) $\{60.0°, 15.0\lambda\}$ is paired with $\{60.0°, 15.0\lambda\}$ and $\{69.3°, 20.0\lambda\}$ is paired with $\{69.3°, 20.0\lambda\}$. With the single sources algorithm (15), (16) and (17) the $\{\alpha_i, \theta_i, r_i\}$ can be calculated: $r_1 = 15.0\lambda$, $\alpha_1 = 45.0°$, $\theta_1 = 45.0°$ and $r_2 = 20.0\lambda$, $\alpha_2 = 45.0°$, $\theta_2 = 30.0°$. So the two sources are localized.

**Conclusion:** From the simulation results we get the conclusion that the EV algorithm has the properties of high resolution, low inutile signal-noise ratio threshold compared with the MUSIC algorithm. New algorithm has low computational quantities on 3-D near-field multiple sources localization by using 2-D near-field multiple sources localization algorithm twice and with two linear array it saves the array elements. Finally, one point we should mentioned that the 2-D EV algorithm is not the only method to localize the sources in 3-D plane and our 3-D localization algorithm can use any 2-D localization algorithm.

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