Abstract: Most exiting array signal processing techniques for direction finding rely heavily on the far-field assumption. When the sources are located close to the array, these techniques may no longer perform satisfactorily. Most modulated communication signals exhibit cyclostationarity. Many cyclostationarity-based methods have been proposed to improve signal detection capability. In this paper a new algorithm for near-field sources localization is presented based on second order cyclic cumulant. Under the cases of existing interference and different distributed noises, the simulation results indicate that the proposed method can remove cyclostationary interference with different cyclic frequency from the desired signals. Furthermore, it can effectively suppress additive stationary noises with any distribution.

Keywords: near-field source, cyclostationarity, noise

Introduction: An important problem in a wide variety of application such as radar, sonar, speech, communication, etc. is locating in space the sources of signals received by an array of sensors. Most attention has been restricted to the model of far-field. In 2-D far-field source localization, a source's location is characterized only by its bearing angle because the wavefront from a far-field source can be assumed to be planar for a uniform linear array of sensor (ULA). However, when sources are located close to the array the assumption is no longer valid. The wavefront in such cases is spherical. In 2-D near-field source localization, a source's location is characterized by its bearing angle and range. The increase in number of parameter caused the algorithm based on the far-field assumption can no longer be directly applied so substitution must be found. Some work addressing the near-field problem in 2-D has been performed. Spatial Wigner-Ville[1] algorithm proposed by A.L. Swindlehurst et al. can locate the near-field sources but it requires large array if high resolution is needed. Huang et al.[2] proposed the 2-D MUSIC algorithm which requires 2-D grid search on the bearing angle and range to identify the peak of MUSIC spectrum. This algorithm is easily to simulate but it is computed inefficiently. Weiss et al. [3] estimated source locations using a polynomial rooting method thereby reducing the computational burden to some extent, however, the algorithm still requires bulk of computations. Starer et al. [4] and Lee et al. [5] proposed pass-following algorithms to reduce the computational burden with two 1-D searches. In wireless communications, since most signals are non-Gaussian and cyclostationary due to modulation [6], Lee et al. [7] proposed a GIJE method to estimate the near-field cyclostationary signals, but it can only estimate the bearing and can not estimate the range parameter. In this paper we present a MUSIC algorithm based on second order cyclic cumulant which can not only remove cyclostationary interference with different cyclic frequency from the desired signals but also can suppress additive stationary noises with any distribution. Furthermore, it can effectively estimate the bearing and range of near-field multiple sources.

Results: 1. 2-D localization signal model and assumption

2-D near field source localization geometry model is shown in figure 1
Consider $I$ near-field, narrowband, independent sources observed by a ULA of $M = 2p + 1$ sensors with interelement spacing $d$. Let the array center be the phase reference point. The signal received by the $m$th sensor is expressed as

$$x_m(t) = \sum_{i=1}^{I} s_i(t)e^{-j \omega_i \tau_{mi}} + n_m(t), -p \leq m \leq p$$

where $s_i(t)$ denotes the $i$th source signal, $n_m(t)$ is the additive noise, $\omega_i$ is the carrier angle frequency of the $i$th source signal, and $\tau_{mi}$ is delay association with the $i$th source signal propagation time between sensor '0' and sensor $m$. By simple geometry, $\tau_{mi}$ is

$$\tau_{mi} = \frac{r_{mi} - r_i}{c} = \frac{r_{mi} - r_i}{\lambda f_i} = \frac{2\pi(r_{mi} - r_i)}{\lambda \omega_i}$$

where $r_{mi}$ is the distance between the $i$th source and the $m$th sensor, $r_i$ is the distance between the $i$th source and the 0th sensor, $c$ is the signal propagation speed and $f_i$ is the carrier frequency of the $i$th source signal

$$r_{mi} - r_i = r_i \left(1 + \frac{m^2 d^2}{r_i^2} - \frac{2md \sin \theta_i}{r_i} - 1\right)$$

so

$$\tau_{mi} = \frac{2\pi r_i}{\lambda \omega_i} \left(1 + \frac{m^2 d^2}{r_i^2} - \frac{2md \sin \theta_i}{r_i} - 1\right)$$

$\theta_i, r_i$ are the bearing and range of the $i$th source and $\lambda$ is its wavelength. Using binomial expansion and the so-called Fresnel approximation, we can write:

$$\omega_i \tau_{mi} \approx \frac{2\pi r_i}{\lambda} \left(\frac{m^2 d^2}{2r_i^2} - \frac{md \sin \theta_i}{r_i} - \frac{m^2 d^2 \sin^2 \theta_i}{2r_i^2}\right) = \phi_i m + \varphi_i m^2$$
where the parameter $\phi_i$ and $\varphi_i$ are nonlinear function of azimuth $\theta_i$ and range $r_i$ of the $i$th source:

$$\phi_i = -2\pi \frac{d}{\lambda} \sin(\theta_i), \quad \varphi_i = \pi \frac{d^2}{\lambda r_i} \cos^2(\theta_i)$$

thus, the signal model can be approximately expressed as:

$$x_m(t) \approx \sum_i s_i(t)e^{j(\phi_m+\varphi_m^2)} + n_m(t)$$

It can be expressed using vector notion as following:

$$\mathbf{x}(t) = \mathbf{A}(\theta, r)\mathbf{s}(t) + \mathbf{n}(t)$$

where

$$\mathbf{x}(t) = [x_{-p}(t), x_{-p+1}(t), \Lambda, x_0(t)\Lambda, x_{-p-1}(t), x_p(t)]^T$$

$$\mathbf{s}(t) = [s_1(t), s_2(t), \ldots, s_I(t)]^T$$

$$\mathbf{n}(t) = [n_{-p}(t), n_{-p+1}(t), \Lambda, n_0(t)\Lambda, n_{p}(t), n_{p-1}(t)]^T$$

$$\mathbf{A}(\theta, r) = [\mathbf{a}(-1, r_1), \mathbf{a}(1, r_2), \ldots, \mathbf{a}(-1, r_I)]$$

$$\mathbf{a}(\theta, r_i) = [e^{j(-\phi + \varphi^2)}, \Lambda, 1, \Lambda, e^{j(\phi + \varphi^2)}]$$ is array manifold.

The following assumption are considered to hold through the work:

**H1** The desired source has zero-mean with the cycle frequency $\alpha (\alpha \neq 0)$ and is cyclically independent with other signals for this cycle frequency.

**H2** The noises are stationary and spatially uncorrelated with the covariance $\sigma^2$.

**H3** $d = \lambda / 4$ and $M > 2I$.

The task of 2-D near field source localization is to estimate the parameter pair \{\(\theta_i, r_i\)\}$_{i \in ISL}$.

### 2. Second order Cyclic MUSIC algorithm based 2-D localization

From the definition of the cyclic correlation function[6], and under the assumptions for the source signals and noise, we have the cyclic array covariance matrixes (CACM)

$$R^\alpha_s(\tau) = \left\{ \mathbf{x}(t + \tau/2)\mathbf{x}^H(t - \tau/2) \cdot e^{-j2\pi\alpha} \right\}$$

$$= \mathbf{A}(\theta, r)\left\{ \mathbf{s}(t + \tau/2)\mathbf{s}^H(t - \tau/2) \cdot e^{-j2\pi\alpha} \right\}\mathbf{A}^H(\theta, r)$$

$$= \mathbf{A}(\theta, r)R^\alpha_s(\tau)\mathbf{A}^H(\theta, r)$$

where \(\left\{ \right\} = \lim_{N \to \infty} (1/N) \sum_{n=0}^{N-1}\left\{ \right\}, H$$ denote Hermitian transpose, and \(R^\alpha_s(\tau)\) is the cyclic source covariance matrix (CSCM)

when the source signals are mutually not cyclically correlated \(R^\alpha_s(\tau)\) will be diagonal.
\[ \mathbf{R}_s^\alpha (\tau) = \text{diag}(\mathbf{R}_{s1}^\alpha (\tau), \Lambda \ R_{s2}^\alpha (\tau), 0 \Lambda, 0) \]  

(10) 

where \( \mathbf{R}_s^\alpha (\tau) \) is the cyclic autocorrelation function (CACF) of the signal \( s_i(t) \) expressed by 

\[ \mathbf{R}_s^\alpha (\tau) = \langle s_i(t + \tau / 2) s_i^* (t - \tau / 2) e^{-j2\pi\alpha\tau} \rangle \]  

(11) 

\* denotes conjugate transpose. \( \hat{\mathbf{R}}_s^\alpha (\tau) \) can be of asymmetry form and can be get based on the 

\[ \hat{\mathbf{R}}_{sj, sk}^\alpha (\tau) = \frac{1}{N} \sum_{t=0}^{N-1} x(t)x(t + \tau)e^{-j2\pi\alpha\tau} \text{ for } \tau \geq 0 \]  

(12) 

\[ \hat{\mathbf{R}}_{sj, sk}^\alpha (\tau) = \frac{1}{N} \sum_{t=0}^{N-1} x(t)x(t + \tau)e^{-j2\pi\alpha\tau} \text{ for } \tau \leq 0 \]  

(13) 

Rank order the eigen values of \( \hat{\mathbf{R}}_s^\alpha \) in the descending order to obtain 

\[ \lambda_1 \geq \lambda_2 \geq \Lambda \lambda_M \] 

Let eigen vector \( u_{i+1}, u_{i+2}, \ldots, u_M \) corresponding to the (M-I) smaller eigen values, 
\( \lambda_{i+1}, \lambda_{i+2}, \ldots, \lambda_M \), we construct the noise eigen matrix \( \mathbf{G} \) from the eigen vectors as 

\[ \mathbf{G} = [u_{i+1}, u_{i+2} \Lambda u_M] \] 

a(\( \theta, r \)), corresponding to the coordinate pair the \( \{\theta, r\} \) is the continuum of all possible near-field steering vector. 

The spatial spectrum of second order cyclic MUSIC algorithm is 

\[ P_{\text{CMUSIC}(\theta, r)} = |\mathbf{a}^H (\theta, r) \mathbf{GG}^H \mathbf{a}(\theta, r)|^{-1} \]  

(14) 

The peak of the spectrum, \( P_{\text{CMUSIC}(\theta, r)} \) are the location of the source, \( \{\theta_i, r_i\} i = 1, 2, \Lambda I \). 

**Discussion:** Simulation of remove cyclostationary interference signal and suppress additive stationary noises any distribution 

According to [8], the range of the near field is approximately \( r \leq 2L^2 / \lambda \) where \( L = (M - 1)d = 4\lambda \) is the array's aperture. So in this paper \( r \leq 32\lambda \) is the near-field region. 

In order to show the effectiveness of the proposed algorithm, computer simulations for testing signal selectivity, noise suppression are conducted. We consider seventeen-element array spaced as in Figure1. The sensor outputs are collected at the rate \( f_s = 4\text{MHz} \), the speed of propagation is \( c = 3 \times 10^8 \) m/s. The length of samples is \( N = 128 \) and the signals of
interesting (SOI) from \(20.0^\circ, 15.0^\circ\) and \(40.0^\circ, 20.0^\circ\) are all BPSK signals with 2MHz baud rate and interference from \(60.0^\circ, 25.0^\circ\) is also BPSK signals with 1.6MHz baud rate. So the SOI’s cyclic frequency is \(\alpha = 0.5\), and the interference’s cyclic frequency is \(\alpha = 0.4\). The SNR is 20 db. Based on the proposed method the spatial spectrum contour figures are shown in figure 2 and figure 3 with 20 times monte carlo simulation each.

In Figure2 the noise is allow exponential distribution, In Figure3 the noise is allow logarithmic normal distribution, From the experiment results we can see that the cyclostationary interference with different cyclic frequency is removed and the noise is suppressed with the proposed algorithm.

**Conclusion:** In this paper we present an algorithm based on second order cyclic cumulant which can remove cyclostationary interference with different cyclic frequency from the desired signals and can suppress additive stationary noises with any distribution. Furthermore, it can effectively estimate the bearing and range of near-field multiple sources.

![Fig 2 Remove cyclostationary interference and suppress additive exponential noise with near-field cyclic MUSIC algorithm](image1)

![Fig 3 Remove cyclostationary interference and suppress additive exponential noise with near-field cyclic MUSIC algorithm](image2)

**Reference:**


