THERMAL NONDESTRUCTIVE TESTING OF BUILDINGS IN PRACTICE

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Abstract: The general mathematical approach of the solution of inverse problem of nonstationary heat conductivity equation is given. This approach is applied for creating a method of determination of the thermalphysical parameters of an building construction using experimental data and prospective values of these parameters. Advantages of such approach are discussed. An example of successful implementations of this method for the real building structures examination in practice is given.

Introduction: Increasing energetic efficiency is currently a major problem in the industrial countries. Considerable contribution of industrially produced energy is used by buildings of all kinds. The buildings receive the energy in several kinds (e. g. electrical or by means of stream-heat pipe) and at last emit it outside as warmth. One of the problems of thermal nondestructive testing of the building refers to the determination of the building’s speed of the heat emission.

This problem is especially actual in northern countries where thermal resistance of the building’s walls determines the conveniences of houses and economical efficiency of building heating.

The solution of the problems described above consists of two phases: the experimental and the theoretical ones. The experimental phase runs at the considerable object. The first part of the work includes the measurements of temperature’s series of inner and outer air and temperature of inner and outer surfaces of the fragment of wall under consideration. One also needs to make a thermal imaging of all surfaces of outer guarding construction of the building. The received data is processed at the second phase of the investigations. The requirement of rapid examination of the considerable construction and the huge field of the application of such investigation leads to the necessity of the creation of the certified practical methodizes and the calculation methods.

The first lets discuss the general mathematical approach of the solution of inverse problem of nonstationary heat conductivity equation needed for the solution of the first phase. Solution of inverse problem in general form bases on solution of direct problem [1] viewed in the following meaning: considerable object parameters need to be selected so that a calculated function (some function of time $U(\tau)$) becomes closer to the measured reaction function $U_0(\tau)$.Therefore, inverse heat conductivity problem is reduced to problem of probability functional extreme search (integration is carried out at range $(0,t)$):

$$\Phi[U]=\int_0^t \left(U_0(\tau)-U(\tau)\right)^2 d\tau \ , \qquad (1)$$

The solution’s existence and uniqueness of inverse problems (maybe, inside a bounded region) is proved [2], so the extreme (1) existence is guaranteed.

We pick out the reaction function $U_0(\tau)$ dependency of parameters, turning it into a function, which has a minimum in the space of it’s parameters can be determined. Functional (1) turns into the view:

$$\Phi(\Theta) = \int_0^t \left(U_0(\tau)-U(\tau,\Theta)\right)^2 d\,\tau\,, \qquad (2)$$

where Θ is the selected set of the parameters. Function (2) extreme can be found using numerous methods. We are using downhill gradient method [3]:

$$\frac{d\Theta_i}{d\,\tau}=-\frac{\partial\Phi(\Theta)}{\partial\Theta_i}, \qquad (3)$$

where the Θ coordinate set is an aggregate of parameters (thermalphysic characteristics, see below) and τ carries out the role of time. For practical usage of (3) all derivatives (partial and total) need to be determined for (3) to give an iteration procedure for minimum search.

With the accordance to the oscillation theory classification [4] the system mentioned above is an autonomous dissipative dynamic system. If a solution (1) exists, the system (3) must have a stationary attracting critical point. Therefore the right part of equation (3) can be factorized up to the first term near the critical point:

$$\frac{d\Theta_i}{dt}=-\frac{\partial^2\Phi(\Theta^0)}{\partial\Theta_i\partial\Theta_j}\left(\Theta_j-\Theta_j^0\right). \qquad (4)$$

Summation is carried out here by the iterant indexes, the upper index 0 marks the coordinates of the critical point. Hessian (the matrix at the right part) is symmetrical and so his proper values are real. According to minimum's existence for (3) Hessian is nonsingular and positively defined. The system dissipation (4) is positive and equals to the sum of Hessian's proper values, taken with the inverse sign. This means unique compression of the phase volume, natural in this case.
The system (4) is replaced by some other system of the lower dimension. This can be achieved by finding of the explicit function (2) view of some parameters. This dependency can be found either analytically or by guessing this dependencies using understanding theoretical background of stated physical problem. The determination of explicit functional (1) dependency of some parameters means choosing of a probe function.
Let's pick out of the Θ set of parameters a subset, an explicit dependency of which is known. This set is marked φ and the other variables – Ψ. Therefore $\Phi(\Theta)$ becomes a new function $\Phi(\varphi,\Psi)$. According to minimum search:

$$\frac{d\varphi_i}{d\tau} = -\frac{\partial\Phi(\varphi,\Psi)}{\partial\varphi_i} = 0. \qquad (5)$$

Thereby the dimension of given system is effectively reduced by a number of variables, analytical dependency of which is now known. At the language of the variation calculations this means that with the passage from (3) to (4) a number of bonds is stated. The initial system (3) needs to be solved numerically (taking into account results (4)) for other variables Ψ. Calculation time profit with numerical solving of (3) can achieve several orders.
Well known that the statement of direct problem of heat conductivity is bases on the law of conservation of energy, written as equation of continuity [5] (let's discuss equation bounded body without a source):

$$\frac{\partial Q(\vec{r},t)}{\partial t} + div\vec{J}(\vec{r},t) = 0, \qquad (6)$$

where $Q(\vec{r},t)$ and $\vec{J}(\vec{r},t)$ are correspondingly thermal energy and thermal flux volume densities, both determined by

$$Q(\vec{r},t) = \rho(\vec{r})C_\rho(\vec{r})T(\vec{r},t) \qquad \vec{J}(\vec{r},t) = \lambda(\vec{r})\frac{\partial T(\vec{r},t)}{\partial\vec{r}}, \qquad (7)$$

where $T(r,t)$ – temperature, $\rho(r)$ – medium density, $C_\rho(r)$ – it's specific thermal capacity, $\lambda(r)$ – heat conductivity. At general the three last values are thermalphysic local characteristics, in practice it is usually determined heat conductivity. Usually equation (6) is considered at 1d and 2d approximations taking into account the symmetry of the problem and chosen direction of the heat flux, e. g. "from the outer space to the inner".
The initial and boundary conditions need to be added for heat conductivity equations to be restraint. In practice they must be determined experimentally. The boundary conditions in linear approximation look like Newton's Law [5]:

$$-\lambda_0 \left.\frac{\partial T(t)}{\partial n}\right|_0 = \alpha_0\left(T(t) - T_0(t)\right), \qquad (8)$$

where the thermal flux at the object's surface stays at the left part (time partial derivative is used in direction of surface's normal), α_0 – coefficient of the thermal emission, $T(t)$ – known temperature of the environment (air), $T_0(t)$ – object surface's temperature.
In practice the initial conditions mostly have no meaning: on the expiry of the specific time there contribution into general solution (6) would be insignificantly small. This specific time (e. g. for single-component outdoor structure) is determined using formulae:

$$T_d = \left(\frac{L}{\pi}\right)^2 \frac{\rho C_\rho}{\lambda}, \qquad (9)$$

Therefore, the extreme is found by solving the direct equation of thermal conductivity (6) and substituting it into (2) with parameters determined as described above.

Results: The approach discussed above can be applied for an examination of a building's outdoor structure. Having some experimental data and building's projected characteristics, one can calculate different heat-engineering parameters of such object. As it was pointed above the investigation procedure has two stages.

First of all, one needs to calculate a thermal resistance of chosen fragment. The calculation algorithms essentially use one-dimensional approximation of the heat problem. That is why one needs to select for investigation a fragment of the guarding constructions with a homogeneous distribution of temperature field on its surface. The linear dimensions of the selected fragment must be few times greater than its thickness in order to provide the one-dimensional approximation to be correct. In order to carry out the investigation one needs to measure four time series of temperatures: the temperatures of inner and outer air and temperatures of inner and outer surfaces of the selected fragment of wall. In our investigations we register required temperature time series for 4-5 days for the non stationary regime. A time interval between successive measurements must be few time shorter than characteristic time of outer air temperature variation. In our investigation we use 5 minute interval between the successive measurements. As analysis shows that several “minimal sets” of data can be picked out of the total set of temperature time series. Such minimal set provides enough information for the examination procedure. It includes two variable series, measured indoors and outdoors (for the solution of the direct heat conductivity problem) and one more series measured either indoors or outdoors (for the selection of the thermalphysic parameters, implementing (2) extreme), i.e. 3 temperature time series altogether.

Having these 3 temperature time series it is possible to advert to direct algorithm of heat conductivity value selection. Using 2 temperature series at the bounds of the outoor structure direct one-dimension heat conductivity problem (6) is solved analytically or numerically taking into accounts boundary conditions of the first kind (within (8) heat emission factor is tends to infinity), i.e. it is solved with given temperatures at the surfaces. Using obtained solution, i.e. temperature profile of the outdoor structure at any point of time $T(\lambda,\alpha_0,x,t)$ where x is coordinate, the heat flow through the surface is calculated at the same bound with known air temperature. Using boundary condition (8) it is easy to express temperature value, which could be reached if the selected set of thermalphysic characteristics took place:

$$T_n(\lambda,\alpha_0,t)=\frac{J_n(\lambda t)}{\alpha_0}+T_0(t)\ , \qquad (10)$$

where $J_n(\lambda,t)$ is calculated heat flow, $T_0(t)$ – measured temperature at the same surface. It is clear that information about heat conductivity λ is found in the calculated flow $J_n(\lambda,t)$. Furthermore, there is a measured curve of air temperature dependent of time $T_a(t)$ and a set (α_0,λ) which would be discussed as real if they provide the closeness of $T_n(\lambda,\alpha_0,t)$ to $T_a(t)$, i.e. calculated and measured air temperatures.

This way, the initial problem turned to kind of (1), i.e. to problem of “probability functional” $\Phi[T_n]$ extreme search. The functional gives the calculated air temperature distance degree from it’s value calculated by solving direct heat conductivity problem (6):

$$\Phi[T_n(\lambda,\alpha_0,t)]=\int_0^t (T_n(\lambda,\alpha_0\tau)-T_a(\tau))^2\,d\tau \qquad (11)$$

If one can solve problem of determination $T_n(\lambda,\alpha_0,t)$ analytically (direct problem of heat conductivity), functional (11) becomes a function of thermalphysic parameters (2). The characteristics of our interest determined by means of global minimum search. However for multicomponent constructions the analytical temperature profile expression tends to be ponderous and so in practice numerical heat conductivity equation solutions is preferred. In this case (11) turns into a function (cause temperature is determined only at the mesh points, i.e. at finite number of space and time points), however, we would not mention this meaning possibility to substitute integration at (12) by summation by mesh points for simplicity reasons.

According to above-stated, the boundary condition (8) is discussed as the additional data about view of explicit functional dependency of some parameters (α_0 in this case) or as the same, the probe variation function. Taking into account (10), (11) is written as following:

$$\Phi[T_n(t)]=\frac{1}{\alpha_0^2}\int_0^t J_0^2(\lambda,\tau)+\frac{2}{\alpha_0}\int_0^t J_n(\lambda,\tau)(T_0(\tau)-T_a(\tau))d\tau+\int_0^t (T_0(\tau)-T_0(\tau))^2\,d\tau \qquad (12)$$

Let’s determine (12) extreme of variable α_0 (this extreme is conditional in fact) by means of equating the corresponding argument’s partial derivative to zero taking (5) into account. Extreme is reached with the condition

$$\alpha_0 = -\frac{\int_0^t J_n^2(\lambda,\tau)d\tau}{\int_0^t J_n(\lambda,\tau)(T_0(\tau)-T_a(\tau))d\tau}, \qquad (13)$$

This way the problem has been reduced to $\hat{O}(\lambda,\alpha_0(\lambda))$ function's minimum search. The global minimum is achieved by the solving direct heat conductivity problem for sufficiently large set of λ values. If this cannot be done for one of the surfaces the described procedure need to proceed for both outdoor structure surfaces, what would provide the global minimum of the "general probability functional". It is a linear combination of (11) functionals for both surfaces of the outdoor structure. Linear combination factors' values are situated at the $(0,1)$ interval, a conclusion of "probable interval" inclusive the true value of heat conductivity can be done using them.

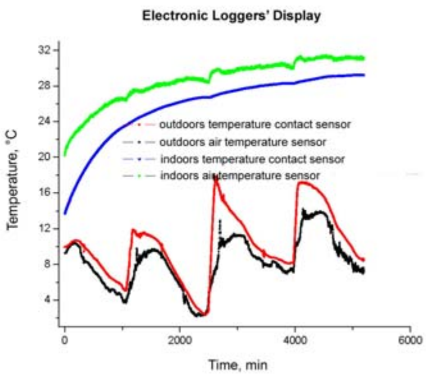

Fig. 1. Typical temperature time series of the selected fixed zone.

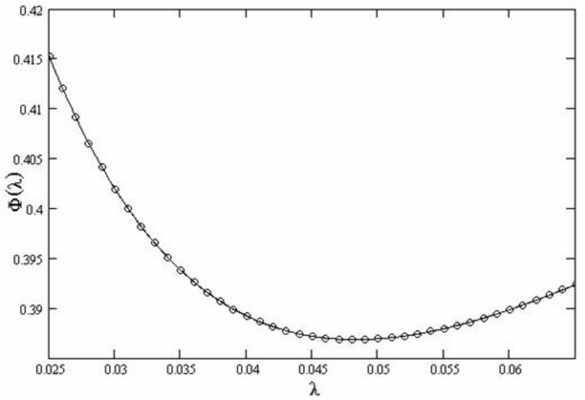

Fig. 2. "Probability functional" (11) implementation taking (12) into account within heat conductivity factor.

The control duration of a typical 10-floor residential structure demands about 2-3 hours and so meets claims of [6]. The computer measurement data processing and report preparation, including time of thermal images assembling, requires about 5-7 hours dependently on object complexity, on location measurement quality, existence of complete building project documentation and so on.

Using algorithm given above a search of one of heat conductivity factors is carried out. Numerically obtained "probability functional" is presented at fig. 2 and heat transfer factors are presented at fig. 3.

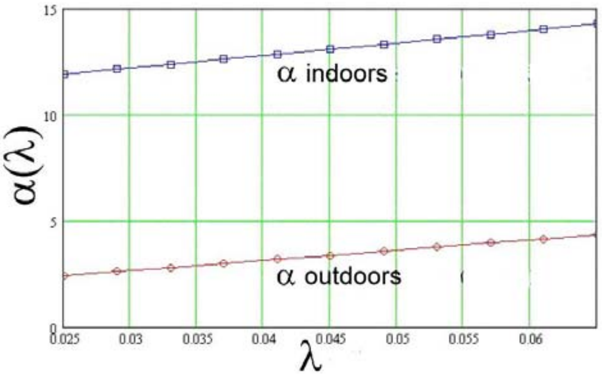

Fig. 3. Outdoor structure's heat emission factors of the inner (with given value 3.7) and outer (given value 13.7) surfaces with their dependency of the heat conductivity, calculated using (12). Probability functional for this case is presented at fig. 2.

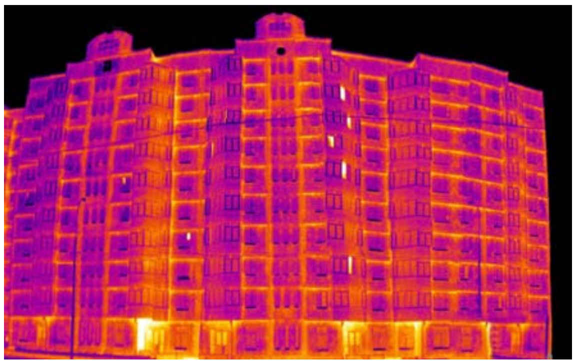

Fig.4. A typical thermogramm of outdoor guarding construction of a building.

The dependence of the probability functional on λ for a wall consisting of three layers is shown at figure 2. It follows from figure 2 that correct value of coefficient of thermal conductivity of heat insulation layer λ is equal to 0.047 Wt/(K*m). Having determined coefficient λ (and therefore all thermo physical parameters of materials constituting the fragment under investigation) and of heat transfer coefficients $\alpha_{in,out}$ one can straightforwardly calculate the thermal resistance of the wall's fragment:

$$R = \frac{1}{\alpha_{in}} + \frac{1}{\alpha_{out}} + \sum_{n=1}^{N} \frac{l_n}{\lambda_n} \tag{14}$$

where N is a total number of layers in the fragment.

In order to calculate an average thermal resistance along the surface of the building one needs a spatial distribution of temperature on the surface of outer guarding construction of the building. To accomplish this task one must image a thermogram of the whole surface of the building by means of IR camera. The main problem with converting intensities of different points of surface into values of temperature at these points is that function providing such a conversion essentially depends on angle of shooting of a fragment and on the distance between the point of shooting and the fragment. This problem can be solved by setting thermosensors on few points of surface and following calibration of IR camera with known procedures.

One has to use IR snapshots of the surface of guarding construction of the building in order to calculate its average thermal resistance. A thermogramm of surface of a typical investigated building is shown at figure 4.

One can derive the thermal resistance $R(\bar{r})$ of an arbitrary fragment of constructions from the value of temperature of its outer surface T(r) by means of the formula:

$$R(\bar{r}) = (T_{in}^{air} - T_{out}^{air})(\alpha_{out}\left(T(r) - T_{out}^{air}\right))^{-1} \tag{15}$$

Using formula (15) we can easily calculate an average thermal resistance of guarding construction of the building under investigation:

$$R_{av} = \int_S R(\bar{r})ds(\int_S ds)^{-1} \tag{16}$$

Here integration is extended on the all elements of surface of outer guarding construction and ds is the area of the element of surface.

The average thermal resistance of guarding construction of building is one of most significant characteristics of a building because it is this characteristic that defines how qualitatively the buildings preserve heat energy in winter.

Discussion: Let's estimate reliability of applying such approach to determine heat engineering parameters of an abstract outdoor structure and discuss the field of it's application.

Laboratory investigations with usage of warmth chambers are usually used to solve this problem: a fragment of considerable outdoor structure is installed at the bound of cold and warm tanks of the chamber and significant temperature modes are created. But during apparent advantages of such approach it has some important defects. Let's consider solution of this problem by means of analytical (statistical particularly) examination of error with using of experimental data.

Building constructions' thermal NDT (the sampling includes more than 100 objects) is carried out in typical seasons – winter months, when required temperature pressure – air temperature difference indoors and outdoors is more than 10 degrees – is provided.

Reduces heat conduction for the whole building (formulae 14) depends only of specific weights of different sorts of walls and windows in consisting of the outdoor structure, formulae for it's calculation can be represented in form of (usual averaging for conductivities – magnitudes inverse for the resistances):

$$\frac{1}{R} = \sum_{i=1}^{n} \frac{w_i}{R_i} \qquad \sum_{i=1}^{n} w_i = 1 \ , \tag{17}$$

where summation is carried out for all sorts of walls and windows with their specific weights. n is total number of windows' and wall's types. The last stating (17) is a normalization condition.

But in practice a deviation from (14) are observed and more than that heat conduction resistance R_k becomes a function of location at the wall. This takes place as a result of changes in building technologies, different exploitation modes and so on. At the same time the numerous factors exert influence onto the process of measurement, their regular account is practically impossible (for example flaw, indoors draught and so on). Therefore the reasons for real reduces resistance value difference from the project value (i.e. from (14)) and rise of measurement errors carry poorly controllable character. Because of this let's bring in an assumption that both procedures: measured and projected values have accidental characteristics. Further let's consider measurement

procedure of heat engineering characteristics of external outdoor structure and procedure of indication of their prior values as accidental processes.
Consider following probabilistic model. Let probabilistic value ξ be a measured value of heat conductivity resistance (17), η – the same magnitude, declared by projecting organization. Allocation of fluctuating components (accounted as independent of the real value R_0^r) caused by necessity of statistical error account and does not depend of concrete object properties. So let's take into account that errors statistics is independent of the object, i.e. their existence is a property of measurement and projecting control methods. Let's consider the probability density of measurement results equals $P(\xi)$, the same probability density for the value of reduces resistance, declared by projecting organization is marked through $Q(\eta)$. Probability density acquired while measuring result ξ and simultaneously for the project organization declares value of the reduced resistance η is stated through joint probability density $W(\xi,\eta)$:

$$W(\xi,\eta) = W(\xi \mid \eta)Q(\eta), \qquad (18)$$

where density of conditional probability $W(\xi \mid \eta)$ is introduced – one can be achieved as the result of ξ measurement under condition of declared reduces resistance equals η. Apparently relation between $P(\xi)$ and $W(\xi,\eta)$ (and cause to (18) and $Q(\eta)$) is following:

$$P(\xi) = \int W(\xi,\eta)d\eta = \int W(\xi \mid \eta)Q(\eta)d\eta \qquad (19)$$

where integration is carried out by admissible values of corresponding arguments. Within extreme case total independence conditional probability density is simply an another randomized value distribution function. Random values ξ and η are independent as showed below. It is simple that distribution functions (18) and (19) are normalized to unity.
Let's determine concrete form of functions situated in (19). Conditional probability density can be estimated using mentioned experimental data. Let's build a histogram of measured resistances turning out of declared ones, i.e. $(\eta - \xi)$. Difficulty with respect to difference between real resistances R_0 for different objects is excluded this way – they would cancel by subtraction (fig. 5).

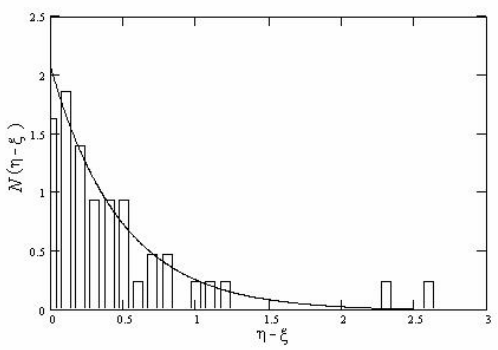

Fig. 5. Normalized histogram of the measured resistances' tunes off from the declared ones and exponential distribution (3.5) approximation for it.

Let's mention that histogram on fig.5 averages conditional probability density $W(\xi \mid \eta)$. Assuming fluctuations do not depend on true value of R_0^r, it is stated that there are no selected resistance values and conditional probability depends only on $(\eta - \xi)$ difference. It means that assembly averaging result does not differ from of measurements took place on the same object, averaged for many times. This way, the method can be considered analyzing data of a set of objects. Further, the measurement procedure provides one with $W(\xi \mid \eta)$. After all, data about resistance η, stated at controllable object's project, is available before thermal testing take place.
Let's approximate fig.5 histogram with exponential distribution:

$$W(\xi \mid \eta) = \frac{1}{u}\exp\left(-\frac{\eta - \xi}{u}\right)\Theta(\eta - \xi), \qquad (20)$$

where u is typical decrement scale and $\Theta(x)$ is a step function (1 at $x > 0$ and 0 with other argument's values).

By means of numerical analysis, a value $u = 0.49$ has been found. Now it can be shown the mentioned absence of randomized values dependency – statement (20) depends on η, i.e. joint probability density (18) is not factored out, there is a correlation between random values.
Let's bring in the following assumptions for the probability distribution function $Q(\eta)$ form. Let one be analogous the $W(\xi \mid \eta)$ form. This way $Q(\eta)$ can be written as:

$$Q(\eta) = \frac{1}{\nu}\exp\left(-\frac{\eta - R}{\nu}\right)\Theta(\eta - R), \tag{21}$$

where ν is a typical decrement scale.
To determine one the following reasoning can be done. Let the probability of the resistance be declared within $\left[R_0^r - R_\delta, R_0^r + R_\delta\right]$ range equals ε, where δ is an admissible deflection. Then one can find ν with the explicit form integrating (21) with the mentioned range:

$$\nu = -\frac{\delta R}{\ln(1-\varepsilon)}, \tag{22}$$

For the numerical estimation let's use $\delta = 0.2$, $\varepsilon = 0.75$ and $R = 3$ parameter values (these values are close to the often used ones). (22) results $\nu = 0.43$. Mentioned above parameter values would be used further for the numerical estimations.
This way, calculating integral (19) taking (20) and (21) into account, the explicit measurement results' distribution function's explicit form for the $P(\xi)$ measurement results can be found as following:

$$P(\xi) = \begin{cases} (u+\nu)^{-1}\exp\left(\dfrac{R-\xi}{\nu}\right), & \xi > R \\ (u+\nu)^{-1}\exp\left(\dfrac{-R+\xi}{\nu}\right), & \xi \le R \end{cases}. \tag{23}$$

It is clear that (23) function means nothing with argument values below zero. But caused by this normalization error is small.
Further let's find average values and dispersions of the (21) and (22). For the declared project values of the resistances one can get following average and dispersion values from the (21):

$$\langle \eta \rangle = R + \nu \qquad \sigma^2(\eta) = \nu^2 . \tag{24}$$

For the resistance values taken from the measurements the average and dispersion values can be written as following using (23):

$$\langle \xi \rangle = R + \nu - u \qquad \sigma^2(\xi) = u^2 + \nu^2 . \tag{25}$$

From the (24) and (25) comparison it follows that outdoor structure TNC gains more accurate averaged results compared with the declared resistance value (in the meaning of the averaged value taken from measurements is less shifted from the true value – the constant bias is smaller). But the (25) dispersion is a bit bigger than the (24) dispersion, and thus unitary measurements can gain a sufficient inaccuracy.
Formulae (24) and (25) allow us estimate TNC method reliability using calculation of the ratio error:

$$E = \frac{|\nu - u|}{R}. \tag{26}$$

Ratio error for the parameters' values mentions above equals $E = 0.02$. Value $1 - E$ is used wider and equals 0.98 for the considered parameter values. Thus, formulae (24), (25) and (26) permit to estimate reliability of the thermal nondestructive control method of heat engineering characteristics of the residential constructions' and industrial buildings' outdoor structures.
Conclusions: The discussed approach for the inverse heat conductivity problem solution gains ability for determination of different heat engineering characteristics of the controllable object. The method proved to be reliable and high-productive for non-destructive analysis of objects of different types. The primary advantage of this method is the possibility of examination in practice.
Method described above is developed by Technological institute of energetic investigations, diagnostic and nondestructive testing "WEMO". This method was used for inspection of more than 200 buildings by request of Moscow Government. Corresponding technique have been certified by "State Standard" of Russian Federation and agreed with Department of Power Engineering of Russian Federation and "State City Technical Supervision".

References: The following references are related to subject discussed in this article.

1 Budadin O. N., Abramova E. V., Rodin M. A., Lebedev O. V., Thermal defectoscopy of buildings and building constructions. – Defectoscopy #5, 2003, 77-94.

2 Budadin O. N., Potapov A. I., Kolganov V. I. and others. Thermal nondestructive testing. – Moscow, Science, 2002, 476p.

3 Bahvalov N. S., Zhidkiv N. P., Kobelkov G. M. Numerical methods. Moscow, Base knowledge laboratory, 2002.

4 Andronov A. A., Vitt A. A., Haikin C. E. Oscillations theory. Moscow, Physical and mathematical literature Government Publishers, 1959.

5 Tihonov A. N., Samarsky A. A. Mathematical physics equations. Moscow, Science, 1977.

6 Budadin O. N., Abramova E. V., Lebedev O. V., Rodin M. A. Residential structures' and industrial building's heat-engineering charasteristics determination and nondestructive testing devices. – Building materials, devices, XXIth century technologies – #9(44), 2002, 21-23.

6 Budadin O. N., Abramova E. V., Rodin M. A., Lebedev O. V., Thermal defectoscopy of buildings and building constructions. – Defectoscopy #5, 2003, 77-94.

6 Budadin O. N., Potapov A. I., Kolganov V. I. and others. Thermal nondestructive testing. – Moscow, Science, 2002, 476p.

6 Bahvalov N. S., Zhidkiv N. P., Kobelkov G. M. Numerical methods. Moscow, Base knowledge laboratory, 2002.

6 Andronov A. A., Vitt A. A., Haikin C. E. Oscillations theory. Moscow, Physical and mathematical literature Government Publishers, 1959.

6 Tihonov A. N., Samarsky A. A. Mathematical physics equations. Moscow, Science, 1977.

6 Budadin O. N., Abramova E. V., Lebedev O. V., Rodin M. A. Residential structures' and industrial building's heat-engineering charasteristics determination and nondestructive testing devices. – Building materials, devices, XXIth century technologies – #9(44), 2002, 21-2