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Prediction of side-drilled hole signals captured by a linear phased arrays with steering and focusing beams

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Abstract

A completed ultrasonic measurement model for the phased array system is developed by combining a nonparaxial multi-Gaussian beam model. This provided measurement model can be used to predict the response of a side-drilled hole over a wide range of steering angles. And a reference setup is also presented to determine the system efficiency factor of the phased array system. Furthermore, the accuracy of this provided model will be certified by comparing with the experimental testing.

Keywords: ultrasonic measurement model, phased array, nonparaxial, multi-Gaussian beam model

1. Introduction

An ultrasonic measurement model can give us a deep insight into the complex physical principle of the ultrasonic testing processes and help us evaluate the inspection results quantitatively. In general, a complete ultrasonic model need consider three components including the beam radiation/reception, the flaw scattering processes and the system efficiency factor. Several ultrasonic measurement models with high computational efficiency were individually developed to solve various ultrasonic NDT problems[1-5]. However these ultrasonic measurement models could not be extend to predict the response of the linear phased arrays over a wide range of steering angle. The main problem is due to the beam model based on the paraxial assumption were adopted in most of these ultrasonic measurement models, and the paraxial beam model will lose accuracy in calculation of a steering and focusing beam fields generated by the phased array transducer. Recently, a nonparaxial multi-Gaussian beam (NMGB) model was developed and some simulation examples had shown it was an accurate and efficient tool to calculate the steering and focusing beam fields radiated from the phased array transducer[6]. As a natural extension of this research work, a completed ultrasonic measurement model for the phased array system can be developed by combining the NMGB method.

In this paper, we firstly present some fundamental approaches including a general ultrasonic measurement model, the NMGB model and the scattering models. And then based on these essential components, we develop a complete phased array ultrasonic measurement model which can be used for the system efficiency factor measurement and the calibration signals prediction. Finally, the model prediction of a side-drilled hole (SDH) respond will be compared with the experimental testing signal to prove the accuracy of our provided model.

2. Theory

2.1. A general ultrasonic measurement model

Under the assumptions of linear time invariant (LTI) system and quasi-plane waves incidence, an ultrasonic measurement model for the pitch-catch immersion setup was developed by Schmerr[7], and it can be applied to contact testing setups as well. According to this general measurement model, in frequency domain the received voltage signal for the pulse-echo contact testing can be related to the sound field and scattering field on the flaw surface,

$$V_R(\omega) = \frac{2\pi\beta(\omega)}{-ik_p S_0} \int_{s_f} \left[\hat{V}(\mathbf{x},\omega)\right]^2 A(\mathbf{x},\omega) \exp(ik_p \mathbf{e}\cdot\mathbf{x}) ds \tag{1}$$

where $\beta(\omega)$ is the system efficient factor, it can be determined by a reference experiment. S_0 is the area of the transducer surface and S_f indicates the flaw surface. The $\hat{V}(\mathbf{x},\omega)$ term is normalized velocity field on the flaw surface due to a quasi-plane wave incidence. And the $A(\mathbf{x},\omega)$ term is related to the far-field scattering amplitude,

$$A(\mathbf{x},\omega) = \frac{1}{4\pi\rho c_p^2}\left[\tilde{\tau}_{ij} d_j + \frac{C_{ijkl}}{c_p} d_k e_l \tilde{v}_j\right] n_j \tag{2}$$

where $\tilde{\tau}_{ij}$ and $\tilde{v}_j$ are stress and velocity components respectively, d_j and n_j is polarization components and unit normal components of the flaw, C_{ijkl} is the elastic constants for the solid. More detail introductions of these parameters can be found in Ref.[7].

In order to make this general ultrasonic measurement model be suitable for a phased array system, it needs to combine the proper beam model and the flaw scattering model, also needs a sufficient declaration to meet the assumption of a quasi-plane wave incidence. These components will be discussed individually in the following parts.

2.2. Beam field for a linear phased array transducer

Recently, a nonparaxial multi-Gaussian beam (NMGB) model was developed in order to overcome the limitation that paraxial Gaussian beams models lose accuracy in simulating the steering beam fields generated by phase array transducers[6]. Using this NMGB model, the velocity fields in a solid medium radiated by a rectangular element (length $2a_1$ and width $2a_2$ in x and y direction respectively) can be written as,

$$\begin{aligned} V(\mathbf{x},\omega) = \frac{-i\omega \mathbf{d}_p K_p(\theta)}{2\pi\rho c_p^2} \sum_{n=1}^{10}\sum_{l=1}^{10} \frac{A_n A_l \exp(ikR)}{\sqrt{1+iB_n R/D_1}\sqrt{1+iB_l R/D_2}} \\ \times \exp\left(\frac{-ik}{2}\frac{x^2/R}{1+iB_n R/D_1}\right)\exp\left(\frac{-ik}{2}\frac{y^2/R}{1+iB_l R/D_2}\right) \end{aligned} \tag{3}$$

where $D_1 = ka_1^2/2$, $D_2 = ka_2^2/2$ and $R = \sqrt{x^2+y^2+z^2}$. The ρ and c_p are the density and P-wave velocity of solid medium respectively, $\mathbf{d}_p$ is the polarization vector for P-wave. A_j and B_j ($j = 1,2,\Lambda,10$) are plural coefficients obtained by Wen and Breazeale[8]. Unlike the immersion transducer case, a directivity function $K_p(\theta)$ is still contained in the solid case[7].

For a phased array transducer with N elements, the synthetic beam field can be obtained by superposition of the wave field from all of the elements with corresponding time delays. The P-wave velocity field radiated by a linear phased array transducer can be calculated by

$$\hat{V}(\mathbf{x},\omega) = \sum_{n=1}^{N} V_n(\mathbf{x},\omega)\exp[i\omega t_n] \tag{4}$$

where $\hat{V}(\mathbf{x},\omega)$ indicates the synthetic sound field. $V_n(\mathbf{x},\omega)$ and t_n are the sound fields and the time delay of the n^{th} element. And the calculational approach of time delays for both beam steering and focusing can refer to Ref. [9].

2.3. Scattering model

2.3.1. Kirchhoff approximation method for planar reflectors[7]

With the help of Kirchhoff approximation, the far-field scattering amplitude of planar reflectors can be reduced to a simple expression. Consider the incident direction $\mathbf{e}_i^p$ is parallel to the normal of the plane surface, then the $A(\mathbf{x},\omega)$ term (Eq.(2)) for the pulse-echo mode can been simplified as,

$$A(\mathbf{x},\omega) = -\frac{ik_p}{2\pi}\left(\mathbf{e}_i^p \cdot \mathbf{n}\right)\exp\left[2ik_p\left(\mathbf{e}_i^p \cdot \mathbf{x}\right)\right] = \frac{ik_p}{2\pi} \tag{5}$$

2.3.2 Separation of variable method for a SDH

The far field scattering amplitude of a cylinder can be exactly calculated by the separation of variable method. In the case of a P-wave normal incidence on the SDH with length *L* and radius *b*, the 3-D scattering amplitude of P-wave can be obtained from the 2-D separation of variables solution [3],

$$\frac{A_{3D}\left(\mathbf{e}_i^p;\mathbf{e}_s^p\right)}{L} = \frac{i}{2\pi}\sum_{n=0}^{\infty}\left(2-\delta_{0n}\right)\cos(n\theta)F_n \tag{6}$$

where $\delta_{0n} = \begin{cases}1 & n=0\\ 0 & otherwise\end{cases}$, and $F_n = 1 + \frac{C_n^{(2)}\left(k_pb\right)C_n^{(1)}\left(k_sb\right) - D_n^{(2)}\left(k_pb\right)D_n^{(1)}\left(k_sb\right)}{C_n^{(1)}\left(k_pb\right)C_n^{(1)}\left(k_sb\right) - D_n^{(1)}\left(k_pb\right)D_n^{(1)}\left(k_sb\right)}$.

with

$$C_n^{(i)}(x) = \left(n^2+n-\left(k_sb\right)^2/2\right)H_n^{(i)}(x) - \left(2n\,H_n^{(i)}(x) - x\,H_{n+1}^{(i)}(x)\right)$$

$$D_n^{(i)}(x) = n(n+1)H_n^{(i)}(x) - n\left(2n\,H_n^{(i)}(x) - x\,H_{n+1}^{(i)}(x)\right)$$

here $H^{(i)}(x)$ $(i=1,2)$ is the first or second kind Hankel function for the cylinder.

2.4. Ultrasonic measure models for a phased array system

Due to dimensions of each array element is in a small scale, the radiating beam field from each element is hardly considered as a quasi-plane wave, it is more reasonable to assume as a spherical wave. However, a synthetic beam field of all elements with proper time delays has good collimation toward the steering direction $\mathbf{e}_i$, so the velocity fields can well satisfy the assumption of quasi-plane wave and be expressed as $\hat{V}(\mathbf{x},\omega)\mathbf{d}_p\exp(ik_p\mathbf{e}_i\cdot\mathbf{x})$, where the normalized velocity amplitude $\hat{V}(\mathbf{x},\omega)$ can be determined from the beam model. For this reason, if we treat the beam fields radiated from all the elements as an entity, then the general ultrasonic measurement model can also be put to good use to predict the flaw responds for the phased array system.

According to the above discussion, a phased array measurement model for planar reflector can be obtained by substituting Eq.(5) into Eq.(1),

$$V_R(\omega) = \frac{\beta(\omega)}{S_0}\int_{S_f}\left[\hat{V}(\mathbf{x},\omega)\right]^2 dS \tag{7}$$

In the case of the SDH reflector, if we consider the incident beam is perpendicular to the axis of the SDH, the scattering sound fields are independent of the axis direction of the SDH. Also the beam variations over the SDH cross-section can be neglected for the SDH with small radius. Following the above assumptions, an ultrasonic measurement model[3] for the SDH reflector can be obtained by combining Eq.(1) and Eq.(6).

$$V_R(\omega) = \frac{2\pi\beta(\omega)}{-ik_pS_0}\frac{A_{3-D}(\omega)}{L}\int_L\left[\hat{V}(\mathbf{x},\omega)\right]^2 dl \tag{8}$$

To summarize, here the pulse-echo measurement models are directly applied for the phased array system due to we treat the beam fields generated by a linear phased array transducer as an entity. It also implies the receiving sound fields should be the same with the transmitting sound fields. In the following discussion Eq.(7) and Eq.(8) are used to measure the system efficiency factor of the phased array system and predict the respond of the SDH, respectively.

3. System efficient factor measurement and experimental verification

A schematic diagram of the experimental testing is shown in Fig. 1. The specific transducer used in experiment is 64 elements linear phased arrays with center frequency at 5 MHz. The size of each element is 0.49×10 mm, and the gap between adjacent elements is 0.1 mm. A 1.8 mm diameter SDH is fabricated in a rectangular steel specimen at the location of 30 mm depth. And the width of the testing specimen is 30 mm.

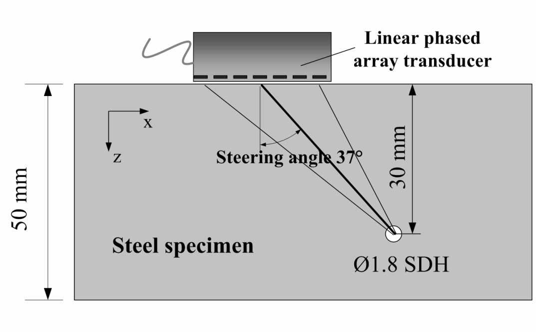


Fig. 1 Schematic representation of phased array ultrasonic testing for the SDH in steel specimen

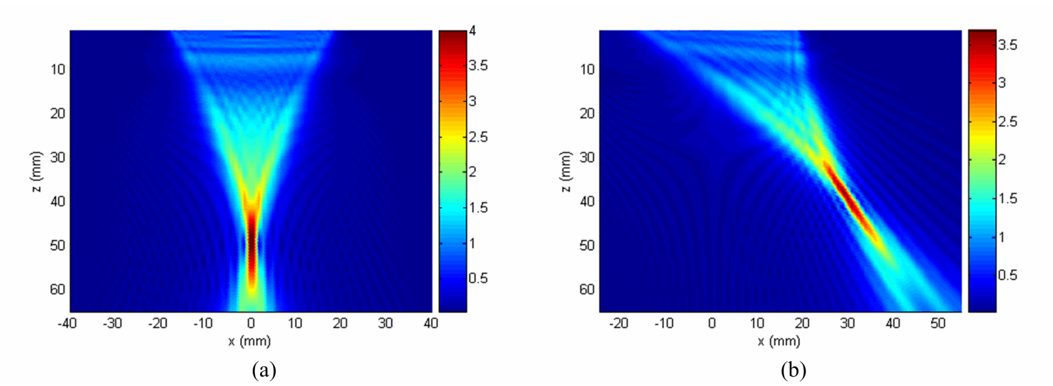


Fig. 2 Beam patterns of linear phased arrays at center frequency (a) Focal length 50 mm without steering (b) Focal length 50 mm with steering angle of 37 degree

As shown in Fig. 2, two types of beam patterns according to the given time delays are calculated by NMGB model. Fig. 2 (a) shows a transmitting/receiving beam field with 50 mm focal length and normal incidence. This beam pattern is used to obtain the reflected signal from the back surface of the specimen. And Fig. 2 (b) shows a transmitting/receiving beam field with 50 mm focal length and 37 degree steering angle, which is used to measurement the response from the SDH.

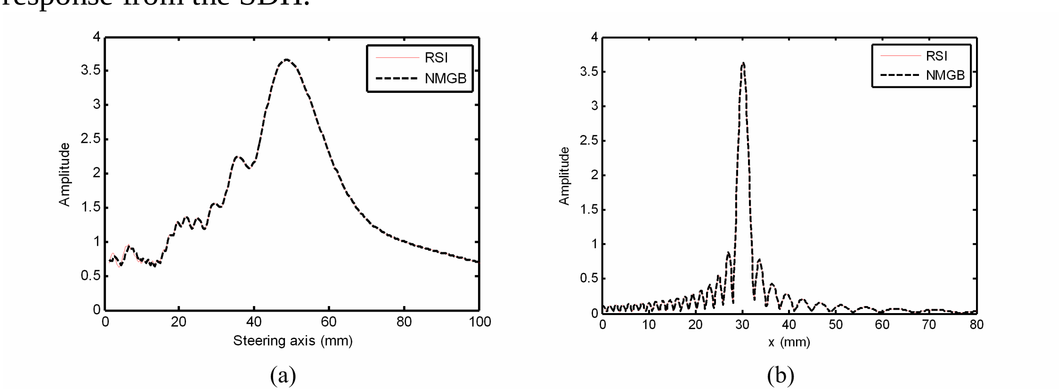


Fig. 3 Comparisons of velocity magnitudes calculated by NMGB model and RSI model with focal length 50 mm and steering angle 37 degree. (a)The on steering axis result (b) The x-axis result (at z=40 mm)

Here, in order to present the accuracy of the NMGB model at the given steering angle, some comparisons of the NMGB to the more exact Rayleigh-Sommerfeld integral (RSI) are shown in Fig. 3. And good agreements can be found in both on-axis and off-axis results.

3.1. System efficient factor measurement

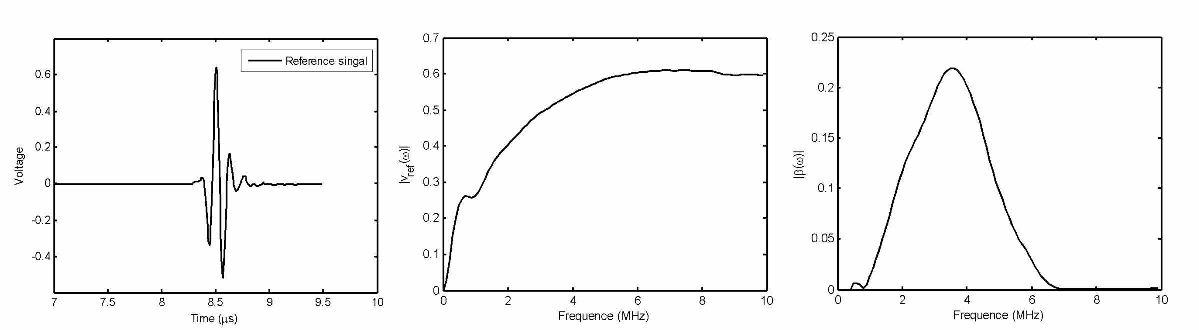
As shown in Fig. 4 (a), the reflection from the back surface of the specimen was regarded as a reference signal to determine the system efficiency factor of the phased array system. With the help of this reference signal and corresponding theory reference model, the system efficiency factor can be determined by a deconvolution method , here a Wiener filter is also used to stabilize the division process,

$$\beta(\omega)=\frac{V_{\mathrm{R}}(\omega)v_{ref}^{*}(\omega)}{\left|v_{ref}(\omega)\right|^{2}+\varepsilon^{2}\max\left\{\left|v_{ref}(\omega)\right|^{2}\right\}} \tag{9}$$

According to Eq.(7), the theory reference model from planar reflector can be written as,

$$v_{ref}\left(\omega\right)=\frac{1}{S_{0}}\int_{S_f}\left[\hat{V}\left(\mathbf{x},\omega\right)\right]^{2}dS \tag{10}$$

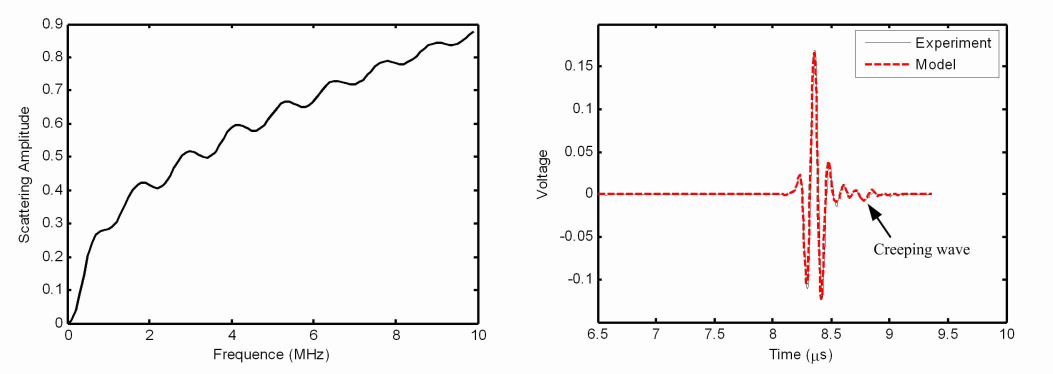
Here the $v_{ref}\left(\omega\right)$ term was numerically evaluated by dividing the reflected plane into many small area elements. The calculational results of $v_{ref}\left(\omega\right)$ and $\beta(\omega)$ are shown in Fig. 4 (b) and (c), respectively.



(a) (b) (c)

Fig. 4 System efficiency factor measurement (a) Reference signal (b) The theory reference model (c) System efficiency factor

3.2. Prediction of the SDH signal and comparison with experiment



(a) (b)

Fig. 5 (a) The scattering amplitude of the SDH (b) A comparison between model prediction and experimental measurement

Once the system efficiency factor has been determined, then Eq.(8) can be used to predict the respond from the SDH. Fig. 5 (a) shows the scattering amplitude of the SDH calculated by separation of variable method. For the convenience of comparison with the experimental result, the inverse Fourier transform was used to invert the frequency domain respond of the SDH into time domain. Fig. 5 (b) shows a comparison between the model predication and experimental measurement. Good agreements can be found in both signal shape and amplitude. Especially, a clear creeping wave can be predicted well since the scattering model is calculated by the exact separation of variable method.

4. Summary

An efficient measurement model for an ultrasonic phased array system was established by combining some analytic methods in both acoustic field and scattering model calculations. With the help of the nonparaxial multi-Gaussian beam model, this provided measurement model can overcome the limitation of paraxial approximation and predict the ultrasonic flaw responses over wide range of steering angle. Also, an excellent agreement between the model prediction and the experimental result from a SDH testing shows the accurate behavior of this ultrasonic measurement model.

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