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High resolution transforms

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Abstract

The introduction of phased arrays opens-up a range of new possibilities for ultrasonic inspections. Arrays come in various shapes ranging from linear arrays for weld inspection to circular arrays emitting guided waves for permanent monitoring applications[2].
For this kind of applications, the data is best displayed as angle versus time to locate the position a defects. However, the data is normally measured as function of a certain spatial coordinate and time. A transformation is then applied to convert data from the spatial domain into the angle domain. The simplest example is a spatial Fourier transform. Unfortunately the resolution obtained by this kind of transforms is determined by the size of the array compared to the wavelength. The longer the array is compared to the wavelength, the higher is the resolution. The reason for this is the increased phase rotation along the aperture.
A new iterative approach is proposed to overcome the shortcomings of the traditional plane wave decomposition. This is a so-called high resolution transform. The new approach yields at least a five times higher resolution and can deal easily with irregular sampling or missing data. The approach will be illustrated on numerically modeled.

Keywords : Radon transform, beam forming, plane wave, resolution.

1. Introduction

In ultrasonic inspection arrays are regularly applied. The shape of the array varies from simple linear arrays to circular arrays used for permanent monitoring systemss[2]. The array can be used for imaging or directional beam forming. In beam forming, essentially the signal is converted from the space-time domain to the angle domain. This means that the recording signal is projected in the direction from which this signal is returned to the array.

Resolution is obviously an important property when considering arrays. The higher the resolution, the more specific information is obtained from a certain direction. High resolution helps in localizing and interpreting information from defects.
In general the resolution is determined by the size of the array compared to the wavelength. The longer the array, the higher resolution can be obtained due to the increasing phase change along the array.
It is possible to overcome this resolution limit by introducing an iterative process that rapidly converges.

2. Theory

The approach will be explained for linear arrays but is straightforward applicable for any array shape. Let's assume data in the space-frequency domain, $P(x,\omega)$, where x is the spatial coordinate along the array and ω is the angular frequency.

It is possible to define a transformation that maps the data from the space domain to the angle domain (ray-parameter)[1]:

$$P(p_x,\omega) = \int_{-\infty}^{+\infty} P(x,\omega) e^{-i\omega p_x x} dx\,, \qquad (1)$$

where the slowness, $p_x = \frac{\sin\alpha}{c}$. The angle α defines the propagation angle of a wave and c its propagation velocity. This transformation is called a Radon transformation.
This transformation is very similar to a spatial Fourier transform, in case of a 2D Fourier transform to the frequency-wave number domain the relevant information ends-up in a cone, while for the Radon transformation the propagating waves are mapped to a rectangular area.
An inverse Fourier transform along the time axis yields data in the (intercept) time domain:

$$P(p_x,\omega) \xrightarrow{FT^{-1}} P(p_x,\tau), \qquad (2)$$

where τ is the intercept time at x = 0. A typical result is shown in Figure 1

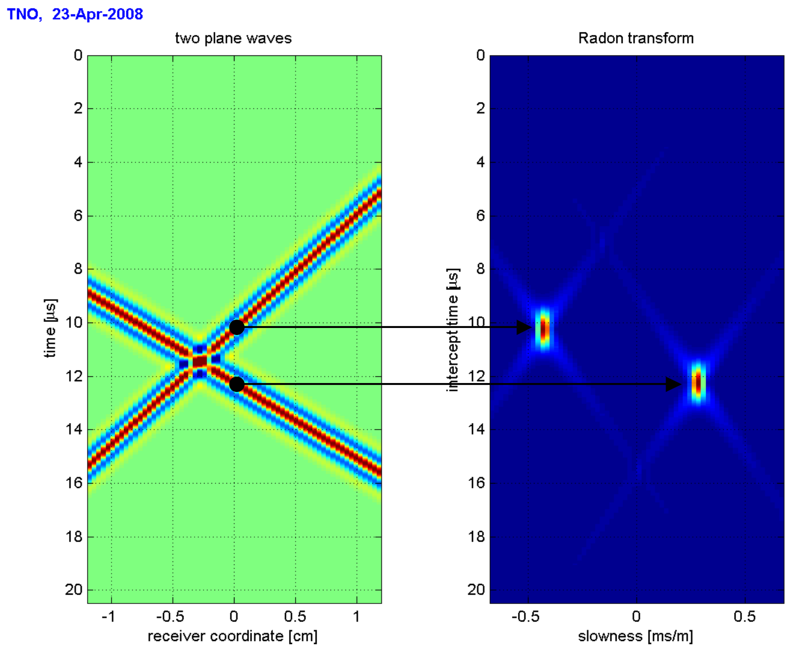

Figure 1 Example of a linear Radon transform, plane waves are mapped to points in the transform domain. The points appear at the apparent velocity (slowness) of the plane wave and the intercept time.

If we now limit the Radon integral, by introducing a window function $A(x)$, such that

$$A(x) = \begin{cases} 0 & \forall |x| > x_{\max} \\ 1 & \forall |x| \le x_{\max} \end{cases}. \qquad (3)$$

Applying a Radon transform on window function A yields:

$$A(p_x) = \frac{2\sin(\omega p_x x_{\max})}{\omega p_x}. \qquad (4)$$

Making use of the convolution property of Fourier-like transforms means that the Radon transform result for infinite bandwidth is convolved by a smearing function, which in this case is a sinc-function. If the input data contains a plane wave, the aperture limitation will cause the peak to smear according to a sinc-function but the peak in the transform domain will remain at the same location.

It turns out that if we take the strongest peak in the Radon domain, transform that peak back to the space domain assuming an infinite aperture and subtract that from the input, we obtain the residue between the finite and infinite aperture transform. If we iterate this process, each time taking the strongest peak in the Radon domain, it turns out that the aperture limitation effects are removed. The iteration process converges rapidly to the 'infinite aperture' result.

We can illustrate the approach, let's assume we have a 32 element linear phased array at a frequency of 1 MHz. The aperture of the array is 3 cm. There are two plane waves impinging on the array, one from an angle of -23.8 degrees and one from 12.3 degrees.

We can compute the direct (equation 1) and the high resolution transforms, the results are shown in Figure 2. The dash-dotted blue line shows the direct transformation result. The sinc-function that smears the result is clearly visible. The high resolution transform clearly produces two spikes, without any artifacts. For this ideal case the convergence is very rapidly. The remaining energy in the input signal is a good indication about the speed of convergence. Figure 3 shows the relative residual energy in the input signal as function of the number of iterations. Within a few iterations, usually less than 10, the high resolution result is obtained.

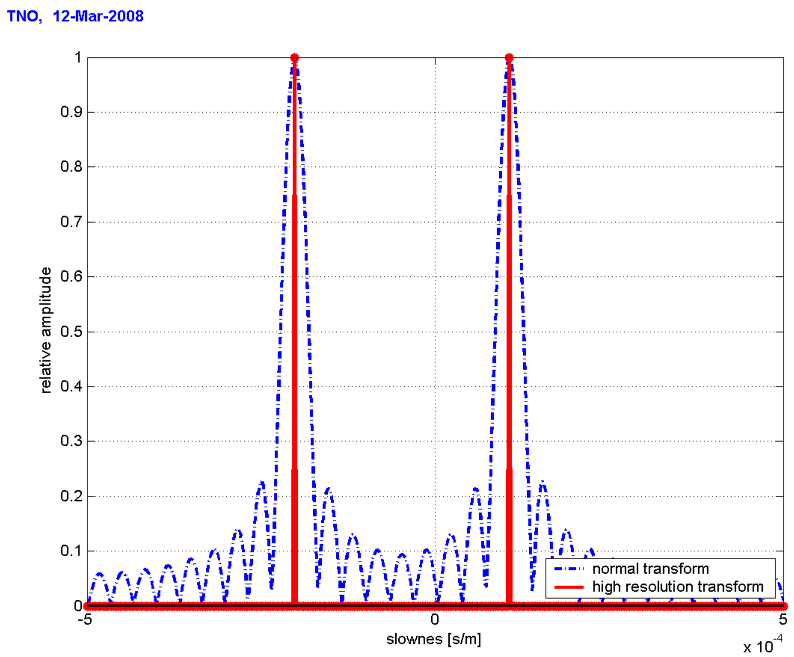

Figure 2 Comparison of conventional and high resolution Radon transform. All edge effects (widening of the main lobe and side lobes) are removed

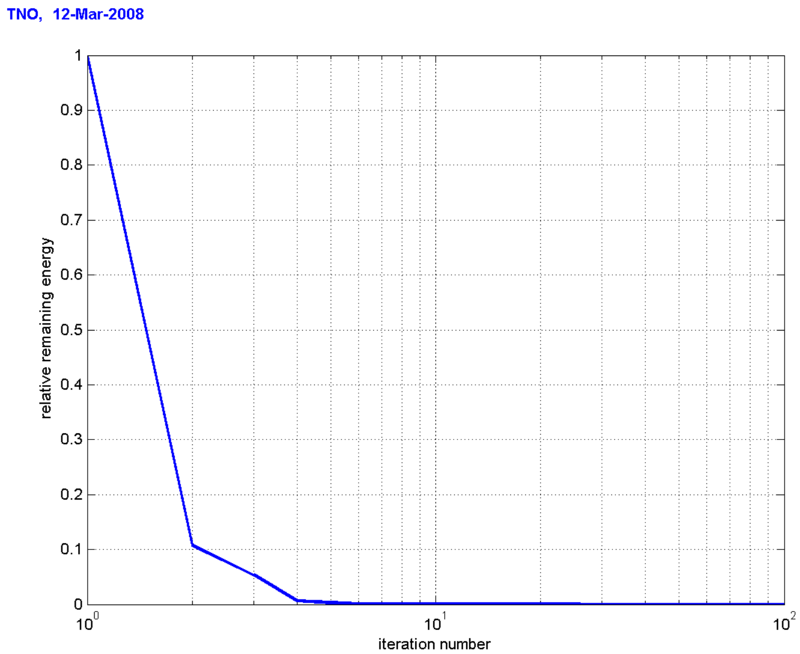

Figure 3 Convergence of iteration process to obtain the high resolution Radon transform. The normalized (relative) energy content is used to quantify the rate of convergence.

The question arises up to what extent two plane waves coming from nearly the same direction can be separated by this transform. If two signals are unresolved, it is not to be expected that with this approach we can resolve them. Figure 4 shows a case where two plane waves are just resolved. In that case the high resolution transform produces two spikes at the correct position. This result is for one single frequency; in practice wide band signals will be used. If a plane wave, with a wide temporal bandwidth, is resolved at the highest usable frequency, then high resolution transform works well for all frequencies producing a high resolution result. In that case the high frequency information is explicitly used for the lower (unresolved) frequencies.

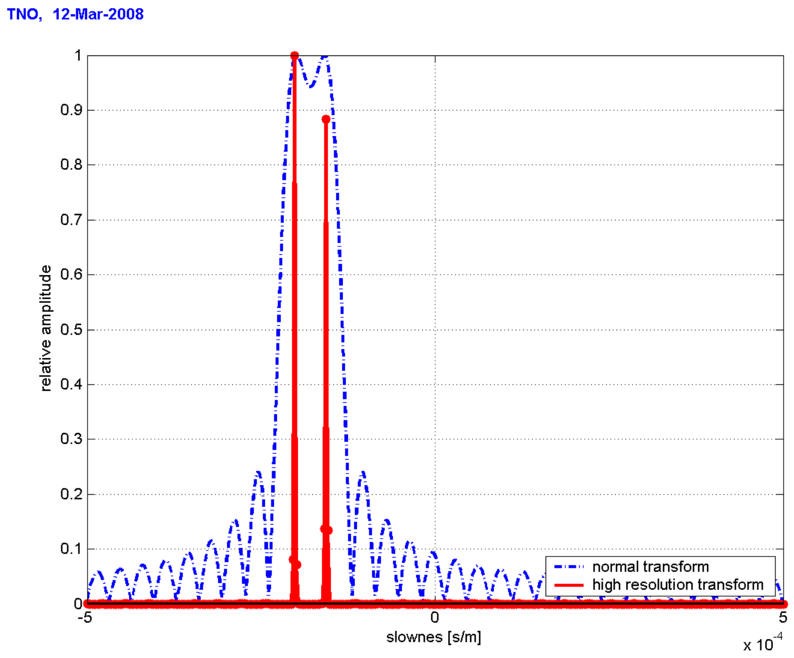

Figure 4 Illustration of resolution limits using the high resolution transform compared to the conventional transform. As long as the events are just resolved for the highest temporal frequency, the high resolution transform is capable of separating the plane waves.

3. Guided wave example

One application where this method may help to improve resolution is for permanent integrity monitoring using guided waves. A circular array is mounted on an object and basically scans the object for defects like pitting and crack growth. For this application non-dispersive guided waves or dispersion-corrected dispersive waves can be used. The design of the array will be a compromise between resolution and complexity of the system.

We will illustrate the concept using a 32-element circular array, one of the elements transmits a guided wave and all elements record the response of the three defects. Figure 5 shows the configuration. The recorded signals are processed using a direct linear Radon transform and using the high resolution transform. The result is shown in Figure 6. The difference in angular resolution is obvious; the indications in the high resolution transform are marked for reference. Clearly all edge effects are removed, which yields three very distinct spikes corresponding to the defects.

We used the plane wave assumption here, which may not be true if the defects are close to the array. Modification of the transform to deal with non-planer arrivals is straightforward.

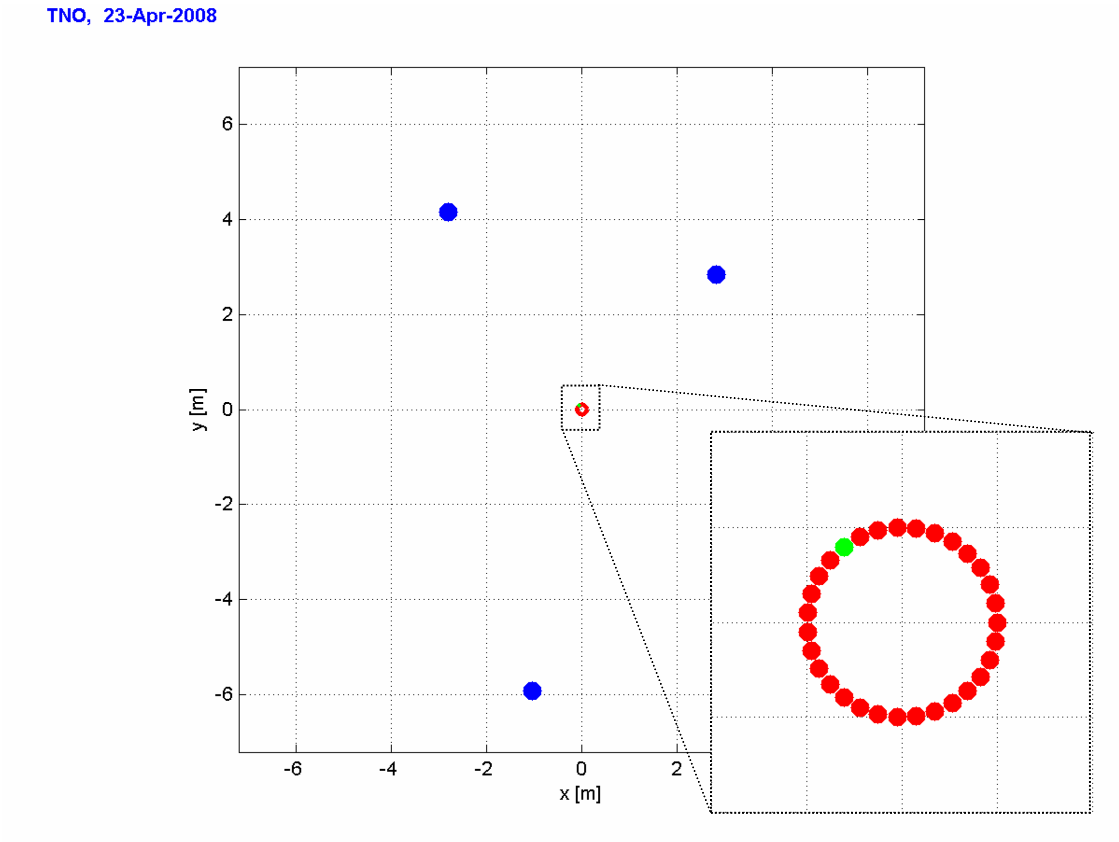

Figure 5 Systematic drawing of acquisition geometry and scatterers, the geometry consists of 32 piezo elements transmitting a desired guided wave mode and three defects

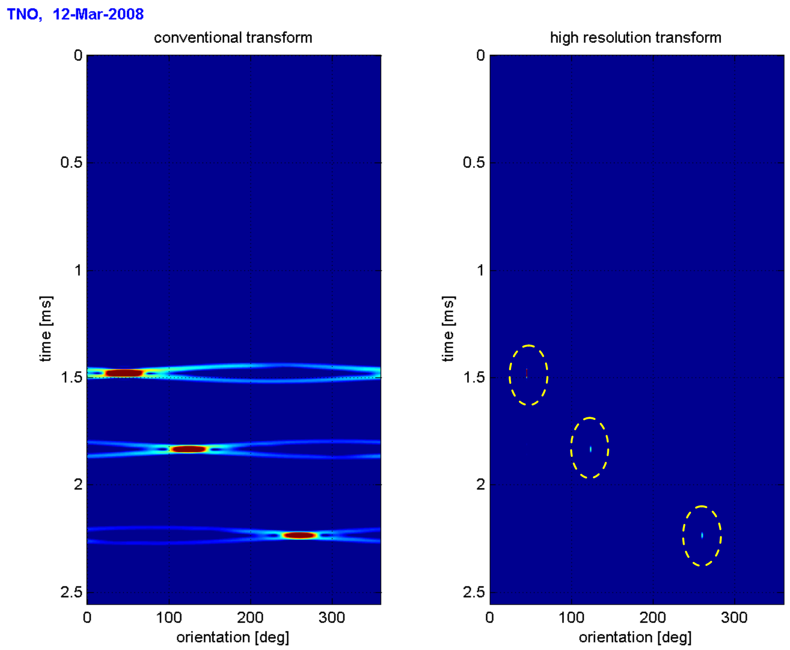

Figure 6 Comparison of the resolution obtained with conventional transforms and with the proposed high resolution transform. The location, in terms of travel time and angle is much better resolved.

4. Conclusion and discussion

Conventional Fourier transformations suffer from finite aperture effects. This causes a smearing (leakage) effect in transform domain. Specially looking at arrays, the array length compared to the wavelength determines the smearing effect. The sorter the array compared to the wavelength, the more pronounced the effect.
We have presented a high resolution transform based on an iterative process subtraction processes. Effectively this process removes the smear effects and yields a much higher resolution. Even if events are barely separable, the algorithm produces high resolution results where after processing the two events are clearly separated. The method is not limited to linear array data but also works on circular arrays as demonstrated on a simple guided wave example. The example was chosen such that the plane wave approximation is valid; if this is no longer the case another transformation operator should be used that matches the input data.

References

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