17th World Conference on Nondestructive Testing, 25-28 Oct 2008, Shanghai, China

Use of Monte-Carlo method for the reconstruction of response functions and radiation spectra in radiography

Mikhail ZUKHOVSKIY, Sergey PODOLIAKO
Keldysh Institute for Applied Mathematics, Russian Academy of Sciences
Miusskaya Square, 4, Moscow, 125047, Russia
Tel: +7-495-250-7825, Fax: +7-495-250-7825
E-mail: zhuk@kiam.ru, psv@kiam.ru

Abstract.

In radiographic testing it is necessary to know the response function of the instrumentation and the energy distribution of the used radiation sources. In particular, it is important for the appropriate interpretation of radiographic images and for the calibration of the measuring equipment. But information on the functions is often incomplete or inaccessible. The developed algorithms based on Monte Carlo method allow designing the operator equations for determination of the response function and the used radiation spectrum. The mentioned operators connect the spectrum of the source radiation and the registration values such as density of the X-ray film or fluorescent screen, spectrum of electrons produced by the used ionizing radiation etc. As a result the systems of integral equations are obtained. The robust methods for solving the corresponding ill-posed problems are developed. The opportunities of the presented tool are discussed in terms of various examples.

Keywords.

Monte Carlo method, reconstruction, radiography, response function, radiation spectrum

Introduction.

Modelling becomes increasingly important in all branches of technical sciences like in system control, workflow optimisation, product design, material sciences, and technical diagnostics. The use of model can help to develop and understand complex systems, to optimise system parameters, and to predict availability, efficiency and reliability of technical systems. The application of modelling can reduce the number of necessary experiments to reach the same goal and therefore can have a great economic impact by reducing costs.

In case of non-destructive testing using X- or gamma rays, to adequately simulate realistic testing scenario the description of the registration installation and the knowledge of radiation characteristics are needed.

In radiography experiments the initial radiation of the source penetrates the object and it is transformed when interaction between the radiation and the object material occurs. Then the penetrated radiation is registered by the detection installation. The spectrum of the detected radiation depends on the initial radiation spectrum and on the experiment configuration (geometry, physical properties of the object material etc). Value of the measured quantity is defined by the characteristics of the registered radiation and by the parameters of the detecting equipment. These parameters are described as a rule by use of so called “response function”.

Thus, the knowledge of the source radiation spectrum and of the response function of the registering equipment is the requisite condition of the adequate mathematical modelling the experiments with the penetrating radiation.

Methods of determinating the spectra and the response functions are based on the registration of various measured values being the functions of the desired functions. Then mathematical handling the measuring results is carried out[1,2]. Choosing the method of developing the operator equation connecting the desired functions and the measured values is of primary importance. The corresponding problems for the equations are the ill-posed problems and require use of special algorithms[3,4].

The developed technique of the radiation spectra reconstruction and the response function determination is described in the paper. The technique includes

(i). the method of constructing the operator equations connecting the desired functions and the measured values;

(ii). the robust algorithm of solving the corresponding ill-posed problems for determining the radiation spectra and the response function of used installation.

The developed approach for computing the operator connecting the desired functions and the measured values takes into account the basic physical processes of interaction between the ionizing radiation and matter (Compton and coherent scattering, photo absorption etc).

1. Constructing the connecting operators.

Building the connecting operators and analyzing the operator properties imply modelling the processes of interacting between the photons and matter of an object and constructing the corresponding functions of the functions characterizing the used radiation (such as spectrum distribution of photons).

Let $f(\varepsilon)$ be energetic distribution of photons of radiation irradiating an object (ε is the energy of photons). After penetrating the object the radiation spectrum $f(\varepsilon)$ is changed to $\varphi(\varepsilon)$, i.e. during the experiment the initial radiation spectrum is transformed to the registered one. This penetrated radiation is registered by the detecting apparatus. Let us consider the mentioned transformation as the operator's A acting in some function space U, i.e. $\varphi = Af$, $f, \varphi \in U$. Then the interaction between the penetrated radiation and registering installation occurs. Measured value F is resulted from this interaction.

We can now write formally the connection between F and φ in the form

$$F(\varphi) = \int g(\varepsilon)\, \varphi(\varepsilon)\, d\varepsilon \tag{1}$$

Here $g(\varepsilon)$ is the response function characterizing the measured value and the way of its measure (type of registering apparatus).

Thus, carrying out the experiment can be divided formally into two stages:

(i). Irradiating the object by penetrating radiation resulting in transforming the initial spectrum $f(\varepsilon)$ into the energetic distribution of the detected radiation $\varphi(\varepsilon)$;

(ii). Measuring the quantity F when the transforming the registered radiation into the measured value occurs.

The first of the stages is described by conversion $\varphi = Af$, the second one is described by transformation $F = P\varphi$, P is determined by the equality (1).

This approach allows describing the registered radiation independently of choosing the measured value and of the way of its measure. Then the transformation $F = P\varphi$ can be built basing on such description for the concrete measured value. The same description of the registered radiation is used for different types of measure at that. The fact is convenient and effective when mathematical modelling the experiments is carried out.

The operator of transforming the initial photon spectrum $f(\varepsilon)$ into the energetic distribution of registered radiation $\varphi(\varepsilon)$ is developed as follows.

Let $f_\delta(\varepsilon,\varepsilon')=\delta(\varepsilon-\varepsilon')$ be the initial photon spectrum ($\delta(\varepsilon-\varepsilon')$ is the Dirac δ -function). The function $f_\delta(\varepsilon,\varepsilon')$ means a photon source of energy equals to ε' . Let $G(\varepsilon,\varepsilon')$ be the result of an action of operator A on the function $f_\delta(\varepsilon,\varepsilon')$, i.e. $A\delta(\varepsilon-\varepsilon')=G(\varepsilon,\varepsilon')$. Function $G(\varepsilon,\varepsilon')$ is the energetic distribution of photons registered when the initial radiation is of energy equals to ε'. It is true equality for arbitrary function $f(\varepsilon)$:

$$\begin{aligned} Af(\varepsilon) &= A\int f(\varepsilon')\,\delta(\varepsilon-\varepsilon')\,d\varepsilon' = \\ &= \int f(\varepsilon')\,A\delta(\varepsilon-\varepsilon')\,d\varepsilon' = \int f(\varepsilon')\,G(\varepsilon,\varepsilon')\,d\varepsilon' \end{aligned} \tag{2}$$

The equality (2) shows that the spectrum $\varphi(\varepsilon)$ of the registered radiation can be found by using the formula

$$\varphi(\varepsilon)=\int f(\varepsilon')\,G(\varepsilon,\varepsilon')\,d\varepsilon', \tag{3}$$

when the source radiation spectrum is equal to $f(\varepsilon)$.

Thus, the construction of the operator A is reduced to calculating the function $G(\varepsilon,\varepsilon')$. Determining the function $G(\varepsilon,\varepsilon')$ is nontrivial problem being solved every time with due regard for concrete physical conditions of the corresponding experiment. The general approach for the calculation of $G(\varepsilon,\varepsilon')$ is the mathematical modelling the processes of transforming the penetrating radiation in materials of the irradiated objects by using the effective computing algorithms[5] based on the Monte Carlo method.

Using the function $G(\varepsilon,\varepsilon')$ permits to reduce a whole number of calculations with different radiation sources to only computing the function G (for instance when $f(\varepsilon)=const$) and then the formula (3) is applied for obtaining the spectrum $\varphi(\varepsilon)$ for different initial spectrum. The function G is used also as an equation kernel when reverse problem of the spectra reconstruction is solved.

Figure 1 illustrates the function G for a case when the spectrum of inverse scattered radiation is measured during an experiment. This function has been computed by use of the effective algorithm[5,6].

A kernel of an operator of transforming the initial photon spectrum into the registered electron spectrum is represented on figure 2. Corresponding computation is carried out by mathematical modelling the processes of transforming the ionizing radiation into electron fluxes taking into account the Compton scattering, the photo absorption and the coherent scattering[7].

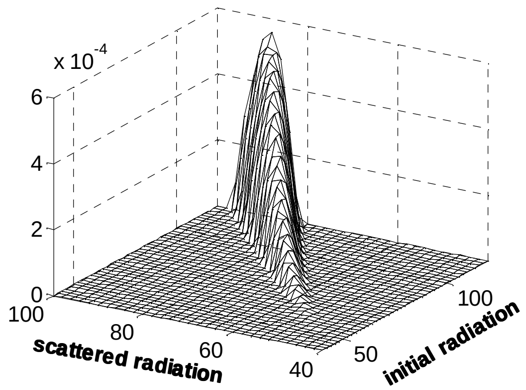


Fig.1. Initial and scattered radiation, keV.

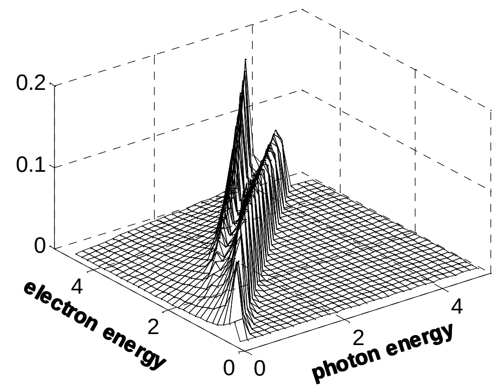


Fig.2. Photon and electron energy, meV.

The transformation of energetic distribution of registered radiation $\varphi(\varepsilon)$ into measured value $F(\varphi)$ is taken place during measuring process. Combining the formulae (1) and (2) we obtain:

$$\begin{aligned} F &= P\varphi = PAf(\varepsilon) = \int g(\varepsilon)d\varepsilon \int f(\varepsilon')G(\varepsilon,\varepsilon')\,d\varepsilon' = \\ &= \iint f(\varepsilon')g(\varepsilon)G(\varepsilon,\varepsilon')\,d\varepsilon' d\varepsilon \end{aligned} \qquad (4)$$

Let $\boldsymbol{p}$ be a set of experiment parameters. Such parameters can be, for instance, characteristics of irradiated object (thickness, material) or of detecting instrumentation. In that case equality (4) can be written as

$$F(\boldsymbol{p}) = \iint f(\varepsilon')g(\varepsilon)G(\varepsilon,\varepsilon',\boldsymbol{p})\,d\varepsilon' d\varepsilon \text{, or} \qquad (5)$$

It follows from (5) that it is possible to obtain an equation (6) for determining the initial spectrum $f(\varepsilon)$ if the response function $g(\varepsilon)$ is known. An equation (7) for determining the response function $g(\varepsilon)$ can be obtained as well if the initial spectrum $f(\varepsilon)$ is known.

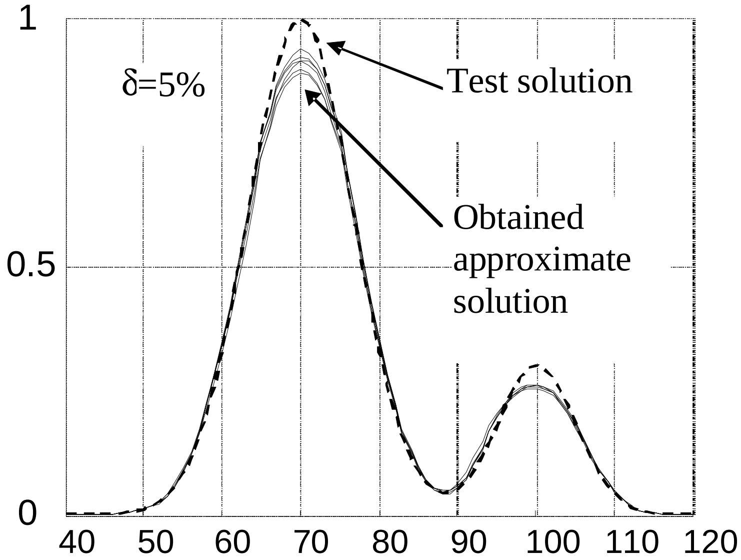
$$F(\boldsymbol{p}) = \int f(\varepsilon')\Phi(\varepsilon',\boldsymbol{p})\,d\varepsilon' \text{, where } \Phi(\varepsilon',\boldsymbol{p}) = \int g(\varepsilon)G(\varepsilon,\varepsilon',\boldsymbol{p})\,d\varepsilon, \qquad (6)$$

$$F(\boldsymbol{p}) = \int g(\varepsilon)\Phi(\varepsilon,\boldsymbol{p})\,d\varepsilon \text{, where } \Phi(\varepsilon,\boldsymbol{p}) = \int f(\varepsilon')G(\varepsilon,\varepsilon',\boldsymbol{p})\,d\varepsilon'. \qquad (7)$$

The robust method of solving the ill-posed problems (6), (7) is developed based on variational principle[3]. Developed method permits to solve systems of operator equation of first type for defining the spectra of used radiation and the response functions of used measuring equipment. Examples are given below.

2. Examples.

A result of spectrum restoration is presented on figure 3. The result is obtained for model experiment when iron wedge is irradiated by a photon flux and energy of propagated radiation is measured by a bar of detectors. The measuring inaccuracy δ is supposed to be equal 5% and the positivity of solution is used as additional information. Bundle of solid lines corresponds to different series of random pointwise error bounded by general level of measuring inaccuracy δ.



Fug.3. Restoration of photon spectrum, energy in keV on X-axis.

Next example illustrates determining the response function. We consider next model experiment. Let collimated flux of photons is falling on the top of a disk detector measuring the absorbed energy. If $f(\varepsilon)$ is the spectrum of incident radiation and $g(\varepsilon)$ is the desired response function we can write

$$F = \int f(\varepsilon) g(\varepsilon) d\varepsilon \qquad (8)$$

where F is measured value.

Let us suppose a set of plates of various thicknesses is used in experiment between the photon source and the detector. The spectrum of radiation falling on the detector is changed in that case subject to plate thickness h, i.e. we have parametric function class $f(\varepsilon, h)$. Equality (8) can be written as the equation

$$F(h)=\int f(\varepsilon,h)g(\varepsilon)d\varepsilon \tag{9}$$

Before description of $g(\varepsilon)$ restoration results we have firstly to obtain test solution $\tilde{g}(\varepsilon)$ of equation (9). As mentioned above the measured value F is absorbed energy of radiation. Therefore

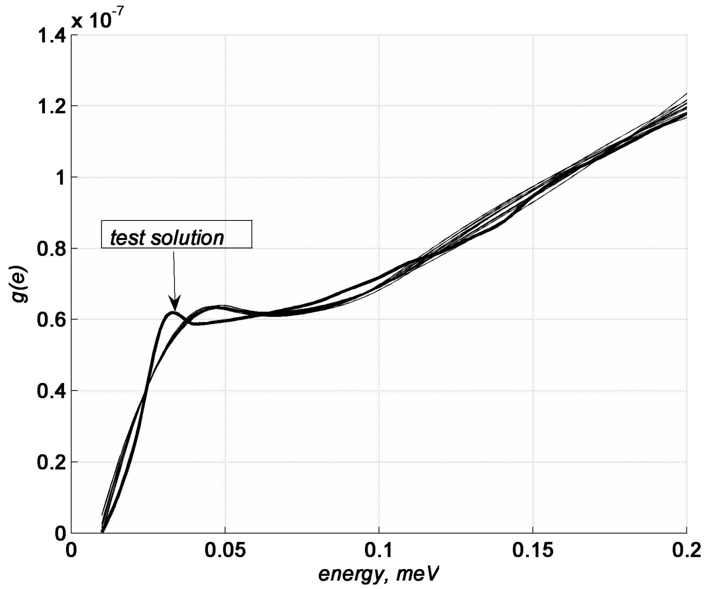
$$F(h)=\int \varphi(\varepsilon,h)d\varepsilon \tag{10}$$

where φ is the spectrum of absorbed radiation. Combining (3) and (10) we obtain

$$F(h)=\int d\varepsilon'\int f(\varepsilon',h)G(\varepsilon,\varepsilon')d\varepsilon \tag{11}$$

Here $G(\varepsilon,\varepsilon')$ is a kernel of operator of transforming the $f(\varepsilon,h)$ into the $\varphi(\varepsilon,h)$. Comparison of (9) and (11) gives formula $\tilde{g}(\varepsilon)=\int G(\varepsilon,\varepsilon')d\varepsilon'$. The function $\tilde{g}(\varepsilon)$ can be used as a test solution of (9) for evaluating the quality of restoration.

The result of response function restoration is illustrated by figure 4.



Fug.4. Determination of response function.

The desired function $g(\varepsilon)$ is obtained as the solution of a system of two integral equations constructed by using the developed technique for model experiments with two sets of aluminium plates and of copper one. Bundle of thin curves corresponds to different series of random pointwise error bounded by general level of measuring inaccuracy.

3. Conclusion.

(1) The effective approach for constructing the operator equation connecting the desired distributions and the measured values is proposed. The corresponding algorithms are

developed basing on statistical modelling the physical processes going with irradiating the objects by ionizing radiation.

(2) The robust methods of determining the spectral distribution of used radiation and the response function of measuring equipment are built. The methods permit to take into account measured values of different kind during the solution of corresponding reverse problem.

References.

[1] G.A.Carlson, A differential absorption spectrometer for determining flash X-ray spectra from 10 to 2000 keV, IEEE Trans. Nucl. Sci., V. 36, 1988, pp. 1255 - 1259.

[2] N.V. Filippov et al, Filippov type plasma focus as intense source of hard X-rays, IEEE Trans. Plasma Sci., V. 24, 1996, pp. 1215 - 1233.

[3] V.P.Zagonov, On an approach to the solution of operator equations with the use of a priori information on the derivative, Soviet Math. Dokl. Vol. 34, No. 1, 1987, pp. 109-112.

[4] A. N. Tikhonov, V. Y. Arsenin, Solutions of ill posed problems. (F. John, Translation Editor) V. H. Winston and Sons (distributed by Wiley, New York), 1977.

[5] G.-R.Jaenisch et al, Monte Carlo Radiographic Model with CAD-based Geometry Description. Insight, № 10, 2006, 618-623.

[6] M. Zhukovskiy et al, On Modelling experiments with ionizing radiation (in Russian), Mathematical modeling (Moscow, Russia), Vol.19, No. 5, 2006, pp. 72-80.

[7] M.B.Markov, M.E.Zhukovskiy. Modelling the radiative electromagnetic field, International Journal of Computing Science and Mathematics, Vol. 2, No.1, 2008.