The Application of Time-Frequency Transform in Mode Identification of Lamb Waves

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Abstract

This paper is focused on the problem of mode identification of Lamb waves in ultrasonic testing, and the principle and process of two time-frequency transform method, smooth pseudo Wigner–Ville distribution and two-dimensional Fourier transform, were discussed. The experimental work were carried out in aluminium plates, and the single and multimode Lamb waves were excited by 5MHz oblique incidence longitudinal wave in 0.82mm and 1.32mm thick plates respectively. The experimental results show that these two time-frequency transform methods can identify not only single Lamb wave mode but also multimode effectively, and furthermore the first method can provide position information of different Lamb wave modes in time domain. Comparing these two methods, smooth pseudo Wigner-Vile distribution is desired to be used in the case where there is no real time required for its long time of calculation, while two-dimensional Fourier transform method is suit for automatic testing since more points of sampling were required.

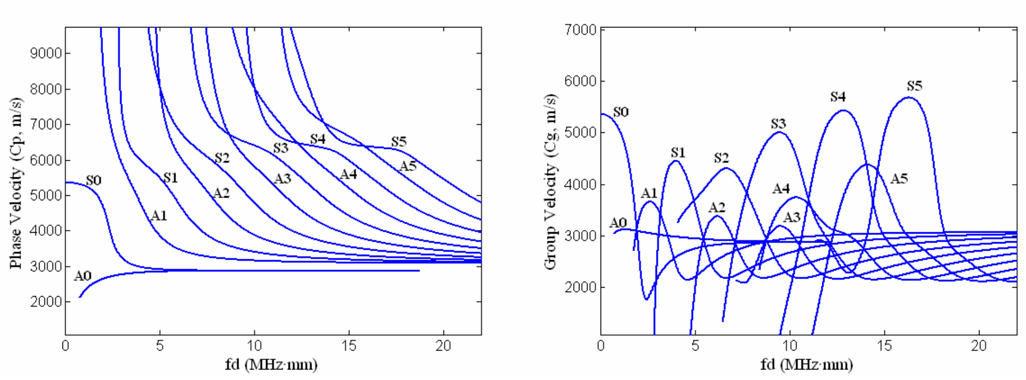
Key words: Lamb wave, Mode identification, Ultrasonic testing, Time-frequency transform

1. Introduction

Lamb waves refer to perturbations propagating in a solid pate (or layer) with free boundaries, for which displacements occur both in the direction of wave propagation and perpendicularly to the plane of the plate [1]. Compared to volume waves, Lamb waves are very fit for nondestructive testing of large and thin plate structure, for its long propagation distance and high efficiency of testing. Presently, Lamb waves have been used in many nondestructive testing fields, such as metal plate testing, composites testing, bond structure testing, and honeycomb structure testing. While for its dispersive nature, Lamb waves may be multimode in its propagation which increases much difficulty of estimation on testing results. So it's very important to identify Lamb modes correctly for Lamb wave testing.

The most used method for mode identifying is to measure velocity of Lamb waves in testing structure. For example, in aluminium plate, different modes of Lamb waves propagate with different group velocity, such as shown in Figure 1, so the mode of Lamb wave can be identified by measuring its group velocity in plate. Since the wave packets of multimode Lamb will be superposition in time domain, it's difficult to measure the group velocity accurately, and this method is suit for single mode identification which has little dispersion but not for multimode [2]. Considering that pulse ultrasonic is composed of many sine waves with different frequency which lead to different Lamb modes and the signal received by transducer is unstable, it is an effective method to identify Lamb wave modes by translating the ultrasonic signal from time domain to frequency domain that is time-frequency transform.

In this paper two methods of time-frequency transform, smoothed pseudo Wigner-Ville distribution (SPWVD) and two-dimensional Fourier transform (2DFT), were used to identify the modes of Lame waves in aluminium plates with different thickness. The basic theory and process of the two methods were discussed. The experiment results shown that these two methods could identify mode of Lamb waves effectively not only single mode but also multimode. Finally some advices on the application of the two methods were given.



(a) (b)

Figure 1. (a)Phase velocity dispersion curves for aluminium plate. (b)Group velocity dispersion curves for aluminium plate. fd is the product of ultrasonic frequency and plate thickness

2. Basic Theory and Process

2.1 Smoothed Pseudo Wigner-Ville Distribution

Wigner-Ville distribution (WVD) is a time-frequency analysis method which interprets a given signal in time domain and frequency domain synchronously. It can exam how frequency content changes as a function of time, and the output of the distribution is the energy intensity of various frequency components of the signal at given points in time. Because of the existence of the nonzero interferences term in WVD which are bad for the explanation for the results, in practice, it could be replaced by smoothed pseudo Wigner-Ville distribution (SPWVD), in which some window functions are convolved with WVD to restrain and decrease the effect of the interference terms. Supposing x is the time domain signal, then SPWVD of x is given by [3]

$$S(t,f)=\iint h(\tau)g(s-t)x(s+\frac{\tau}{2})x^*(s-\frac{\tau}{2})e^{-j2\pi f\tau}dsd\tau \quad (1)$$

where x^* is complex conjugate of x, h and g are time and frequency smooth window function respectively. The result of SPWVD could be shown in t-f plane with different color according to the values of $S(t, f)$, which represents the distribution of the signal energy in time domain. Different mode has different energy distribution, so Lamb wave modes can be identified by comparing experimental result to analytical solution.

The analytical energy distribution of the SPWVD in t-f plane can be obtained by translating the coordinate in Figure 1(b) using follow formulas.

transverse coordinate: $$t = \frac{2L}{C_g} + \Delta t \quad (2)$$

longitudinal coordinate: $$f = \frac{fd}{d} \quad (3)$$

where fd and C_g is the transverse and longitudinal coordinate respectively of the group velocity dispersion curves in Figure 1(b), d is the thickness of the plate, L is the sound path length in the plate and Δt is the delay time including system delay time and probe wedge delay time.

2.2 Two-dimensional Fourier Transform

Suppose Lame wave propagates in the plate along direction of x, then the displacement of the points on the plate surface is given by formula (4)[4]:

$$u(x,t) = A(\omega)e^{j(\omega t - kx - \theta)} \quad (4)$$

where $A(\omega)$ is a constant of the amplitude associating with frequency, x is the coordinate on the direction along the wave propagation, k is wave number, $\omega = 2\pi f$ is angle frequency and θ is initial phase. The two-dimensional Fourier transform of formula (4) is

$$H(k, f) = \iint u(x,t)e^{-j(2\pi ft + kx)}dxdt \quad (5)$$

In practice a two-dimensional Fourier transform (2-D FFT) may be used. The Fourier transforms of t and x expresses the transformation from time domain to frequency domain and spacial domain to wave number respectively. Different Lamb mode has different k and f corresponding to different $H(k, f)$ which is a three-dimensional graph usually, and projects the contour of $H(k, f)$ onto the f-k plane then the actual f-k curves can be obtained. Because different f-k curves express different Lamb waves, the Lamb mode can be identified by comparing the actual curves to analytical curves. The formula (6) and (7) could be used to calculate coordinate of the analytical curves.

transverse coordinate: $$f = \frac{fd}{d} \tag{6}$$

longitudinal coordinate: $$k = \frac{2\pi f}{C_p} \tag{7}$$

where fd and C_p is the transverse and longitudinal coordinate respectively of the phase velocity dispersion curves in Figure (1a), d is the plate thickness.

According to formula (5), the two-dimensional Fourier transform of $u(x,t)$ includes the Fourier transform of special coordinate x, so it's necessary to sample a series of points on equally spaced position along the direction of Lamb wave propagating. The more points are sampled, the better resolution of space is.

3. Experiment

The experiment was carried out on the 0.82mm and 1.32mm thick aluminium plates respectively. A schematic diagram of the plate and the position of the transducer is shown in Figure 2, where L is the distance from incidence point to the end of the plate. The transducer used was an available angle probe whose center frequency was 5MHz, and the bandwidth was 2.1MHz. The transducer excites pulse ultrasonic and receives the ultrasonic reflecting from the plate end. The signal was processed by Matlab (Version7.0) software.

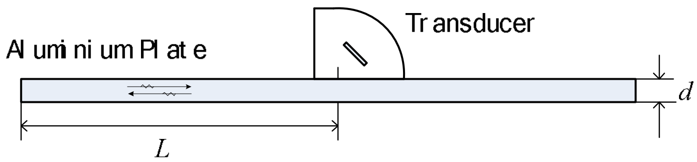
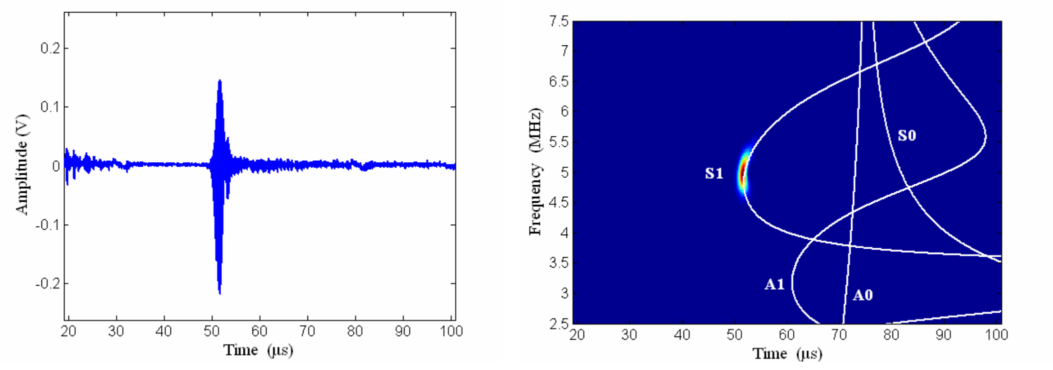


Figure 2. Schematic diagram of the experiment.

For the 0.82mm thick aluminium plate, the incidence angle of the probe was 27 degree, and L was 95 mm. The time domain response is shown in Figure 3(a). Figure 3(b) is the results of the mode identification by SPWVD using Equation (1). Obviously, the practical results is accord to the theoretical energy distribution of S1 mode, and the corresponding frequency is 5MHz approximately which is agreed to the probe center frequency, so the Lame mode in the plate could be identified as S1 mode.

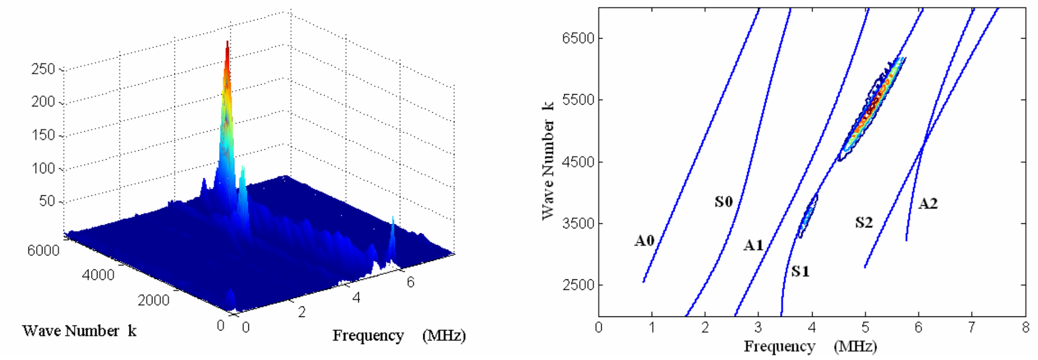
When using 2-D FFT method, L was changed from 95mm to 63mm with 0.5mm step, and 64 points were sampled. These data were processed using Equation (5). The result of 2-D FFT is shown in Figure 4(a). Figure 4(b) is the projection of Figure 4(a) onto the f-k plane, and in order to get a clear map, the part where the value of $H(k, f)$ were less than 50 had been removed. From Figure 4(b), it could been seen that the practical f-k curve is accord to the theoretical curve of S1 mode, and the corresponding frequency is 5MHz approximately also, and this indicates that the Lamb mode in the plate is S1 mode. This result is agreed to that using PSWVD.



(a) (b)

Figure 3. (a) Time domain signal in 0.82mm thick aluminum plate.

(b) SPWVD of time-domain signal, plus analytical solution (solid line)



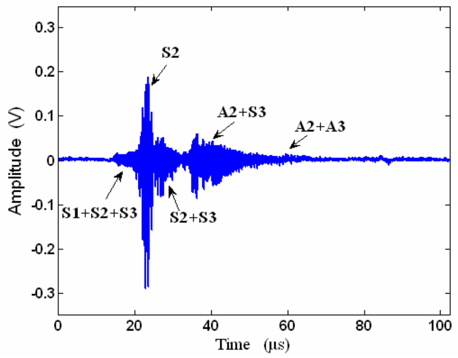
(a) (b)

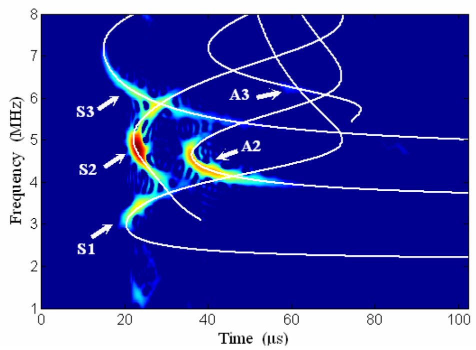
Figure 4. (a) 2-D FFT of time domain signal in 0.82mm thick aluminum plate.

(b) Projection of (a) with removing the part below 50, plus analytical solution (solid line)

For 1.32mm thick aluminium plate, the incidence angle of the probe was 23 degree, and L was 110mm. The time domain response is shown in Figure 5(a). Obviously, there are wave packets more than one, so the Lamb wave in the plate may be multimode. The result of SPWVD in Figure 5(b) shows that the practical energy distribution is accord to the theoretical distribution of S1、S2、S3、A2 and A3 modes, and the corresponding frequency range is about 5±2MHz, which is agreed to the center frequency and bandwidth of the probe. In practical energy distribution, the redder color expresses the more salient mode which has higher amplitude in time domain, such as S2 and A2 mode, so it could be concluded that the Lame wave in the plate had S1、S2、S3、A2 and A3 modes, S2 and A2 mode were more salient. Furthermore, the position of the different Lame modes in time domain could be identified according to the time coordinate in SPWVD, such as shown in Figure 5(a).

For the plate, when using 2-D FFT method, L was changed from 110mm to 78mm with a step of 0.5mm. Figure 6(a) shows the result of 2-D FFT, and Figure 6(b) is the projection of Figure 6(a) onto the f-k plane with removing the part where the value of $H(k,f)$ were less than 50, and it shows that the practical f-k curves are accord to the theoretical curves of S1、S2、S3、A2 and A3 modes. Similarly, the redder color expresses the more salient mode, so the same conclusion could be drawn that there were S1、S2、S3、A2 and A3 modes in the plate, S2 and A2 mode were more salient.

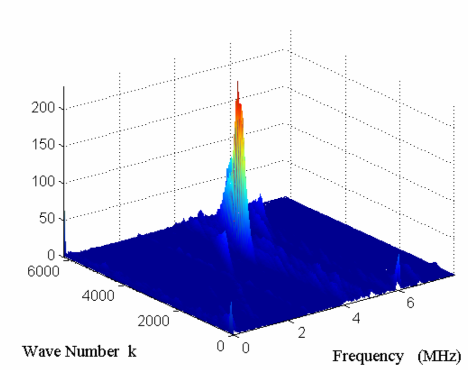


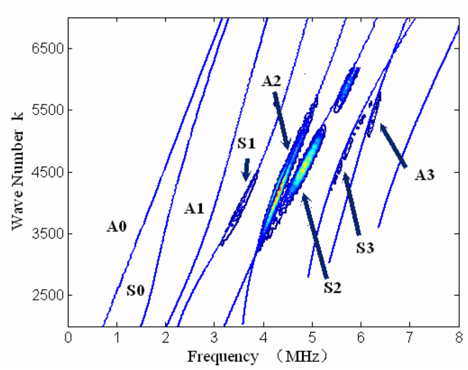


(a) (b)

Figure 5. (a) Time domain signal in 1.32mm thick aluminum plate.

(b) SPWVD of time domain signal, plus analytical solution (solid line)





(a) (b)

Figure 6. (a) 2-D FFT of time domain signal in 1.32mm thick aluminum plate.

(b) Projection of (a) with removing the part below 50, plus analytical solution (solid line)

4. Conclusions

Two time-frequency transform methods of Lamb wave mode identification, SPWVD and 2-D FFT, have been discussed. The experiments were carried out on the aluminium plates with different thickness. The results have shown that all these two methods could be used to identify the Lamb waves in the plate not only single mode but also multimode and the results of identification using these two methods are consistent. The method of SPWVD is need more calculation time for its complex arithmetic, so it is desired to be used in the case where there is no real time required, while the method of 2-D FFT is need to sample data at a lot of points in space, so it better to be used in the case where some automatic scan machines could be used.

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