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Identification of Source Time Function for Train-induced Vibration of Underground Tunnels

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Abstract

The objective of this paper is to identify the dynamic source time function for the train-induced vibration of underground tunnels. Under the consideration of the gaps between adjacent slabs and the constraint from equal-spacing fasteners, the vertical force loaded by the moving train is modelled by a periodical source time function. Consequently, the induced vibration at any specified observation point on the inner wall of the embedded waveguide can be determined on the basis of the transition matrix (T-matrix) method. On the other hand, thanks to the Taipei Rapid Transit Corporation for the permission of an in-situ test, the train-induced vibration on the inner wall of underground tunnels was measured to validate the proposed periodical source time function and further, to identify the associated loading parameters. It can be found that the vibration responses determined numerically by the identified model, both the waveform in time domain and the Fourier spectrum in frequency domain, are in good agreement with the measured signals.

Keywords: train-induced vibration, underground tunnel, T-matrix, dynamic source time function

1. Introduction

The vibrations of underground tunnels or highway bridges caused by the traffic loading will propagate outwardly through the surrounding soil layers, and then, the nearby structures will be affected significantly by the induced vibrations or noises. The analysis of ground motions and the associated structural vibrations due to underground trains is very complicated[1, 2] (Lin and Krylov, 2000; Metrikine and Vrouwenvelder, 2000), however, it can be simplified by three topics: (1) the analysis of the train-induced vibration of tunnels embedded in an infinite domain, (2) the analysis of transient waves scattered from the tunnels in the surrounding soil layers, and (3) time history analysis of buildings on the basis of the input of the train-induced ground accelerations[3] (Gardien and Stuit, 2003).

In this paper, the tunnel is assumed to be an infinite hollow cylindrical waveguide embedded in an infinite elastic domain. Neither the interaction between tunnels nor the boundary conditions between soil layers is considered. Hence, based on the transition matrix (T-matrix) method, the induced vibration of the embedded waveguide and the scattered wave field in the surrounding soil can be solved in terms of the external loading function due to the moving trains. Under the consideration of the gaps between adjacent slabs and the constraint from equal-spacing fasteners, the vertical force loaded by the moving train onto the tunnel can be modeled by a periodic source time function. In order to verify the proposed periodic source time function and to identify the associated loading parameters, the train-induced vibration on the inner wall of underground tunnels was measured by an in-situ test of the Taipei Rapid Transit system. It can be found that the vibration responses determined numerically by the proposed model as well as the identified parameters, both the waveform in time domain and the Fourier spectrum in frequency domain, are in good agreement with the measured signals.

Based on the proposed source time function as well as the identified loading parameters, the train-induced wave field scattered from the tunnel in the surrounding soil can be also determined by the T-matrix method. Then, in the next stage, instead of the infinite surrounding medium, the case of soil layers will be considered and modelled by a layered half-space, and the aforementioned wave field can be applied to define the perturbation source within the upper soil layer. Subsequently, under the boundary conditions of continuity at the interface and traction free on the ground surface, the induced ground vibration can be determined by means of the transmission and reflection coefficient matrices of scattered waves at the interface and free ground. Based on the simulated ground acceleration, the time history analysis can be performed to evaluate the structural vibration caused by the trains moving in underground tunnels.

2. Source Time Function and Tunnel Vibration

2.1 Transition Matrix and Train-induced Tunnel Vibration

As shown in Fig. 1, the underground tunnel is modelled by an infinite hollow cylindrical waveguide surrounded by infinite soil. The 3D cylindrical coordinate system (r-θ-y coordinates) is defined such that y-axis is coincident with the axis of the tunnel and the point y=0 is defined at the middle point of one slab. Based on the radiation conditions at infinity ($r\rightarrow\infty$) as well as the symmetry about the vertical axis, the scalar potentials $\bar{\phi}_m$, $\bar{\psi}_m$ and $\bar{\chi}_m$ with non-negative integer m corresponding to P-, SV- and SH- waves, respectively, can be solved and expressed in the frequency domain as

$$\begin{aligned}
\bar{\phi}_m(r,\theta,y;\omega) &= \frac{1}{2\pi}\sqrt{\varepsilon_m}\cos m\theta \cdot \int_{-\infty}^{\infty} H_m^{(2)}\left(k_p^* r\right) e^{-ik_y y}\, dk_y \\
\bar{\psi}_m(r,\theta,y;\omega) &= \frac{1}{2\pi}\sqrt{\varepsilon_m}\sin m\theta \cdot \int_{-\infty}^{\infty} H_m^{(2)}\left(k_s^* r\right) e^{-ik_y y}\, dk_y \\
\bar{\chi}_m(r,\theta,y;\omega) &= \frac{1}{2\pi}\sqrt{\varepsilon_m}\cos m\theta \cdot \int_{-\infty}^{\infty} H_m^{(2)}\left(k_s^* r\right) e^{-ik_y y}\, dk_y
\end{aligned} \tag{1}$$

Herein, $H_m^{(2)}$ is the m^{th}-order Hankel function of the second kind, ε_m is the Neumann factor (ε_m=1 for m=0 or ε_m=2 for $m\geq1$), k_p^* and k_s^* are defined by

$$k_p^* = -i\sqrt{k_y^2 - k_p^2} \quad ; \quad k_s^* = -i\sqrt{k_y^2 - k_s^2} \tag{2}$$

where $k_p=\omega/C_p$ and $k_s=\omega/C_s$ are the wavenumbers of the longitudinal and shear waves, respectively. Then, based on the scalar potentials, the basis functions of the displacement vectors in the 3D domain $\overline{\mathbf{u}}_{\mathbf{m}}^{\boldsymbol{\alpha}}(r,\theta,y;\omega)$ can be determined by

$$\overline{\mathbf{u}}_{\mathbf{m}}^{(1)} = \nabla\overline{\phi}_m \quad ; \quad \overline{\mathbf{u}}_{\mathbf{m}}^{(2)} = \nabla\times(\overline{\psi}_m\mathbf{e}_{\mathbf{y}}) \quad ; \quad \overline{\mathbf{u}}_{\mathbf{m}}^{(3)} = \frac{1}{k_s}\nabla\times\nabla\times(\overline{\chi}_m\mathbf{e}_{\mathbf{y}}) \tag{3}$$

Furthermore, the m^{th}-order Hankel function of the second kind $H_m^{(2)}$ can be replaced by the m^{th}-order Bessel function J_m to define the regular part of the basis functions $\hat{\overline{\mathbf{u}}}_{\mathbf{m}}^{\boldsymbol{\alpha}}(r,\theta,y;\omega)$. It should be noted that the basis functions represent the outgoing waves and the regular parts represent the standing waves.

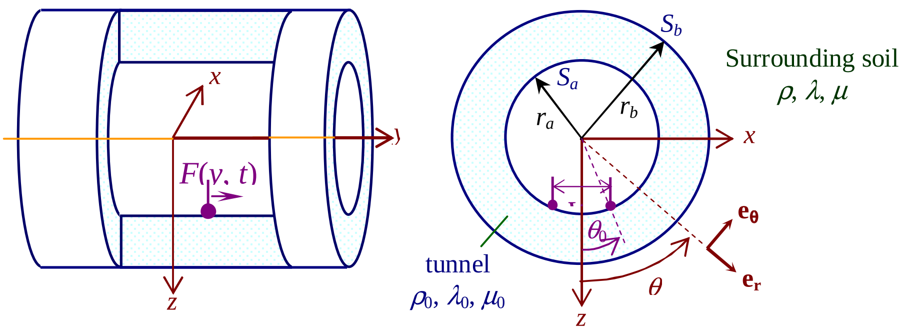

Figure 1. The coordinate systems for the analysis of train-induced vibration of an underground tunnel

As shown in Fig. 1, the cylindrical surface S_a ($r= r_a$) and S_b ($r= r_b$) are defined as the inner wall and outer wall of the tunnel, respectively, and S_b is the interface between the tunnel and the surrounding soil. It should be noted that all of the functions and parameters corresponding to the tunnel are denoted by a subscript ′(0)′. Due to the train-induced external force loaded on the inner wall S_a, the wave field within the tunnel ($r_a \le r \le r_b$) will propagate outwardly and then reflected between the inner and outer walls. Therefore, the wave field within the tunnel ($r_a \le r \le r_b$) can be expanded by the series of the basis functions and the regular parts as

$$\overline{\mathbf{u}}_{(0)} = \sum_{m=0}^{\infty}\sum_{\alpha=1}^{3} f_m^{\alpha}\overline{\mathbf{u}}_{\mathbf{m}(0)}^{\boldsymbol{\alpha}} + \sum_{m=0}^{\infty}\sum_{\alpha=1}^{3} \hat{f}_m^{\alpha}\hat{\overline{\mathbf{u}}}_{\mathbf{m}(0)}^{\boldsymbol{\alpha}} \tag{4}$$

On the other hand, the wave scattered from the tunnel in the surrounding soil ($r \ge r_b$) will propagate outwardly, and hence, it can be expanded by a series of the basis functions as

$$\overline{\mathbf{u}} = \sum_{m=0}^{\infty}\sum_{\alpha=1}^{3} C_m^{\alpha}\overline{\mathbf{u}}_{\mathbf{m}}^{\boldsymbol{\alpha}} \tag{5}$$

Based on the continuity conditions of displacement and traction on the outer wall S_b as well as the boundary condition due to the external force loaded on the inner wall S_a, all of the expansion coefficients f_m^α, $\hat{f}_m^\alpha$ and C_m^α can be determined. Furthermore, it should be noted that the external traction vector loaded on the inner wall S_a can be expressed in frequency domain as

$$\bar{\mathbf{t}}^{\mathbf{a}}(r_a,\theta,y;\omega)=\sum_{m=0}^{\infty}\left(\frac{1}{2\pi}\int_{-\infty}^{\infty}\mathbf{D_m}\tilde{\bar{\mathbf{t}}}_{\mathbf{m}}^{\mathbf{a}}e^{-ik_y y}dk_y\right)\quad \text{with}\quad \mathbf{D_m}=\sqrt{\varepsilon_m}\begin{bmatrix}\cos m\theta & 0 & 0\\ 0 & \sin m\theta & 0\\ 0 & 0 & \cos m\theta\end{bmatrix} \tag{6}$$

Then, all of the expansion coefficients f_m^α, $\hat{f}_m^\alpha$ and C_m^α can be expressed in terms of the loading vector $\tilde{\bar{\mathbf{t}}}_{\mathbf{m}}^{\mathbf{a}}$, and further, the time history of the induced vibration $\mathbf{u}^{\mathbf{a}}(r_a,\theta,y;t)$ on the inner wall S_a can be expressed by

$$\mathbf{u}^{\mathbf{a}}(r_a,\theta,y;t)=\sum_{m=0}^{\infty}\mathbf{u}_{\mathbf{m}}^{\mathbf{a}}(r_a,\theta,y;t)\quad \text{with}\quad \mathbf{u}_{\mathbf{m}}^{\mathbf{a}}(r_a,\theta,y;t)=\frac{1}{4\pi^2}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\mathbf{D_m}\mathbf{T}_{\mathbf{m}}^{\mathbf{a}}\tilde{\bar{\mathbf{t}}}_{\mathbf{m}}^{\mathbf{a}}\,e^{-ik_y y+i\omega t}dk_y d\omega \tag{7}$$

Herein, $\mathbf{T}_{\mathbf{m}}^{\mathbf{a}}$ is the m^{th}-order transition matrix, all of the elements can be determined and expressed explicitly.

2.2 Periodic Source Time Function

In this paper, the train is assumed to move along the positive y-axis with a constant speed c, and the first set of wheels will pass the point y=0 at t=0 coincidently. Furthermore, as shown in Fig. 1, each set of the wheels consists of two wheels with a width of W. Therefore, the r- and θ-components of the external traction vector $\mathbf{t}^{\mathbf{a}}(r_a,\theta,y;t)$ loaded by the first set of wheels onto the inner wall can be expressed by

$$t_r^a=F(y,t)\cdot[\delta(\theta-\theta_0)+\delta(\theta+\theta_0)]\cos\theta_0\quad ;\quad t_\theta^a=F(y,t)\cdot[-\delta(\theta-\theta_0)+\delta(\theta+\theta_0)]\sin\theta_0 \tag{8}$$

Where $\theta_0=\sin^{-1}(W/2a)$ is the angle between the contact point and the vertical axis (see Fig. 1), and $F(y, t)$ is the contact force. Under the consideration of the gaps between adjacent slabs and the constraint from equal-spacing fasteners, the vertical contact force $F(y, t)$ can be defined by

$$F(y,t)=Q(t)\cdot\delta(y-ct)\quad \text{with}\quad Q(t)=Q_0(1+\gamma_1\cos\Omega_1 t+\gamma_2\cos\Omega_2 t) \tag{9}$$

Herein, $\delta(y-ct)$ is to define the location of the wheels along y-axis, and $Q(t)$ is a periodic function representing the amplitude of contact force. Q_0 is the averaged contact pressure, Ω_1 and γ_1 are the angular frequency and amplitude ratio of the force due to the gaps between adjacent slabs, and Ω_2 and γ_2 are corresponding to the fasteners. In addition, the angular frequencies Ω_1 and Ω_2 can be determined by the length of slab L_s and the spacing of fasteners L_f as well as the moving speed c as $\Omega_1=2\pi c/L_s$ and $\Omega_2=2\pi c/L_f$, respectively. It can be found that $[\delta(\theta-\theta_0)+\delta(\theta+\theta_0)]$ and $[-\delta(\theta-\theta_0)+\delta(\theta+\theta_0)]$ are the even and odd functions of θ, respectively, and can be expanded by the series of $\cos m\theta$ and $\sin m\theta$ as

$$\left[\delta(\theta-\theta_0)+\delta(\theta+\theta_0)\right]=\sum_m a_m\sqrt{\varepsilon_m}\cos m\theta \qquad a_m=\frac{\sqrt{\varepsilon_m}}{\pi}\cos m\theta_0$$
$$\left[-\delta(\theta-\theta_0)+\delta(\theta+\theta_0)\right]=\sum_m b_m\sqrt{\varepsilon_m}\sin m\theta \quad \text{with} \quad b_m=-\frac{\sqrt{\varepsilon_m}}{\pi}\sin m\theta_0 \tag{10}$$

Therefore, the external traction vector loaded on the inner wall S_a can be expressed in the frequency domain as

$$\bar{\mathbf{t}}^{\mathbf{a}}(r_a,\theta,y;\omega)=\sum_{m=0}^{\infty}\left(\frac{1}{2\pi}\int_{-\infty}^{\infty}\mathbf{D_m}\,\tilde{\bar{\mathbf{t}}}^{\mathbf{a}}_{\mathbf{m}}e^{-ik_y y}dk_y\right) \quad \text{with} \quad \tilde{\bar{\mathbf{t}}}^{\mathbf{a}}_{\mathbf{m}}=\tilde{\bar{F}}(k_y,\omega)\begin{Bmatrix}a_m\cos\theta_0\\ b_m\sin\theta_0\\ 0\end{Bmatrix} \tag{11}$$

From Eq. (9), we have

$$\tilde{\bar{F}}(k_y,\omega)=\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}F(y,t)\,e^{ik_y y-i\omega t}dydt=2\pi Q_0\left\{\delta(\omega-k_y c)+\sum_{i=1}^{2}\left[\frac{\gamma_i}{2}\delta(\omega-\Omega_i-k_y c)+\frac{\gamma_i}{2}\delta(\omega+\Omega_i-k_y c)\right]\right\} \tag{12}$$

Based on the loading vector $\tilde{\bar{\mathbf{t}}}^{\mathbf{a}}_{\mathbf{m}}$ as expressed by Eqs. (11) and (12), the vibration $\mathbf{u}^{\mathbf{a}}_{\mathbf{W1}}(r_a,\theta,y;t)$ on the inner wall induced by the first set of wheels can be determined by Eq. (7). For the n^{th} set of wheels with a distance L_n from the first set of wheels, the source time function can be defined by $F(y, t-L_n/c)$, and hence, the associated response can be determined from that caused by the first set of wheels with the specific time delay L_n/c and multiplied by a weighting W_n. Therefore, the total response can be defined by

$$\mathbf{u}^{\mathbf{a}}(r_a,\theta,y,t)=\sum_{n=1}^{N}W_n\mathbf{u}^{\mathbf{a}}_{\mathbf{W1}}(r_a,\theta,y,t-L_n/c) \tag{13}$$

3. In-situ Measurement and Identification of Source Time Function

In order to verify the proposed periodic source time function and to identify the loading parameters, an in-situ test was organized to measure the train-induced vibration on the inner wall of an underground tunnel (Taipei Rapid Transit). As shown in Fig. 2, the micro-tremors were setup at the middle points of two adjacent slabs, and they were fixed onto the tunnel ground between the rails with $\theta=0°$.

As the train passes, the train-induced vibrations at the selected two measurement locations can be measured simultaneously, and Figure 3(a) shows the typical velocity signals (vertical component). Because the selected test points are both located at the middle point of one slab, the waveform of the measured signals V_1 and V_2 are much similar to each other and a time delay exists between the two signals V_1 and V_2. This time delay represents the moving time of the train passing through the selected two test points, and can be determined on the basis of the cross correlation function. The cross correlation function is defined by

$$R(\tau)=\int_{-\infty}^{\infty} V_1(t)\cdot V_2(t-\tau)\,dt \tag{14}$$

where V_1 and V_2 are the measured signals, and the value that causes the function $R(\tau)$ to reach its maximum is taken for the traveling time between the two selected points. Therefore, the moving speed of the train can be determined by the traveling distance between the two test points and the calculated moving time. In addition to the time signals, the Fourier amplitudes of V_1 and V_2 are determined and compared in Fig. 3(b), and Figure 3(c) shows the zoomed signals for the low frequency range ($0 \le f \le 4$ Hz). It can be found that the Fourier spectra of V_1 and V_2 for the two selected test points are almost the same. It should be noted that, based on all of the measured signals, the tunnel vibration are almost the same and the moving speed of the train passing through the test locations is a constant of 60 km/h.

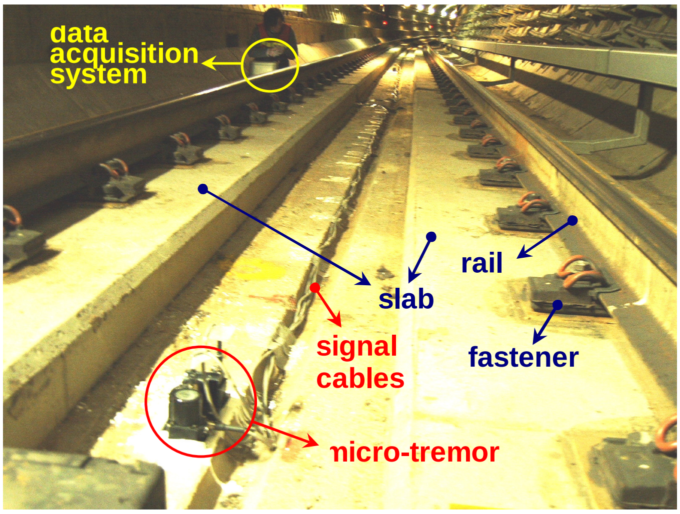

Figure 2. Setup of the micro-tremors in the in-situ test in the underground tunnel

Based on the special case with moving speed (c=60 km/h), length of slab (L_s=14.5 m) and the spacing of fasteners (L_f=0.75 m), the angular frequencies of the proposed periodic source time function can be determined as Ω_1=7.22 rad/s (f_1=1.15 Hz) and Ω_2=139.6 rad/s (f_2=22.2 Hz), and the associated amplitudes Q_0、$Q_1=Q_0\gamma_1$ and $Q_2=Q_0\gamma_2$ will be identified by the measured signal. Based on Eq. (7) as well as the regarded parameters of Taipei Rapid Transit system as shown in Table 1, the r-component of velocity response caused by the first set of wheels on the inner wall with y=0 (middle point of slab) and θ=0° can de determined numerically.

A train consists of 6 carriages, and the length of each carriage is 23.5 m. There are 4 sets of wheels on each carriage, and the distances of the three rear sets from the first one are 4m, 16m, and 20 m, respectively. Therefore, the total response due to the total 24 sets of wheels of a moving train can be carried out by Eq. (13). Based on the comparison of the numerical results with the measured signals, the amplitudes of the periodic loading can be identified as Q_0=0.018, Q_1=0.0004 and Q_2=0.00002, and the weighting of the four sets of wheel on each carriage can be also identified as W_1=1.0, W_2=0.4, W_3=0.8 and W_4=0.1. Figure 4 shows the numerical results which are determined on the basis of the proposed periodic source time function as well as the identified loading parameters. It can be found from Fig. 4 that the vibration response determined numerically by the identified model, both the waveform in time domain and the Fourier spectrum in frequency domain, are in good agreement with the measured signals.

Table 1: Parameters for the analysis of velocity response of underground tunnel

Underground tunnel	Surrounding soil
Inner diameter: a=2800 mm Outer diameter: b=3050 mm S-wave velocity: C_{s0}=2500 m/s P-wave velocity: C_{p0}=4500 m/s Mass density: ρ_0=2400 kg/m3	S-wave velocity: C_s=180 m/s P-wave velocity: C_p=380 m/s Mass density: ρ=2000 kg/m3

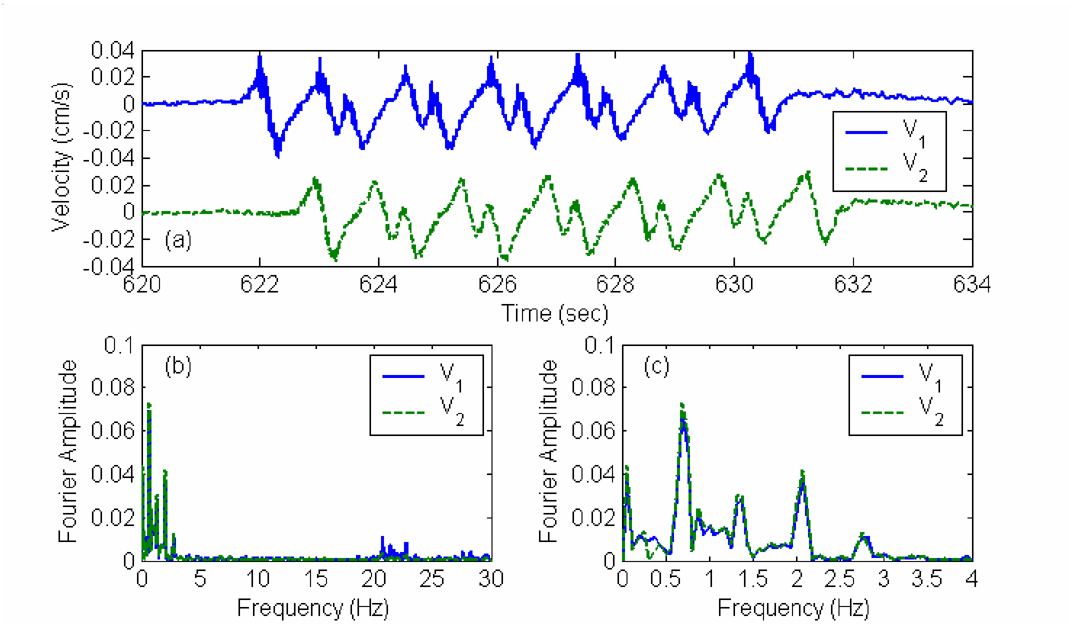

Figure 3. Vibration signals measured on the inner wall of an underground tunnel in the in-situ test: (a) time histories of velocity, (b) Fourier transformation and (c) zoom in low frequency range.

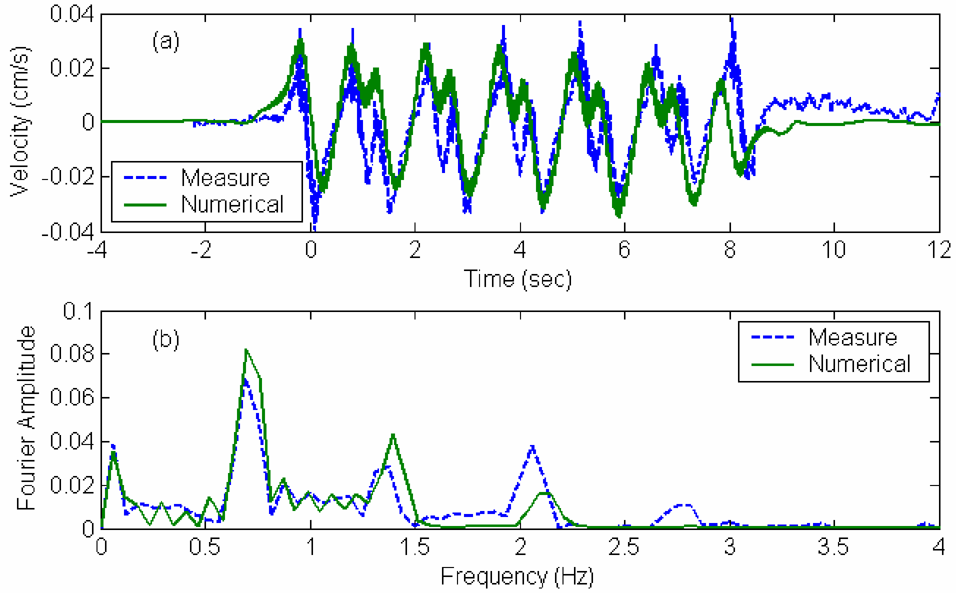

Figure 4. Comparison of the signals determined numerically by the proposed model and the identified loading parameters and the measured signals: (a) time histories of velocity, and (b) Fourier transformation in low frequency range.

4. Conclusion

In this paper, the vibration of an underground tunnel and the associated scattered wave field in the surrounding soil can be solved in terms of the external loading function due to a moving train. Under the consideration of the gaps between adjacent slabs and the constraint from equal-spacing fasteners, the vertical force loaded by the moving train onto the tunnel can be modeled by a periodic source time function. In addition, the proposed periodic source time function and the associated loading parameters can be verified and identified by the measurement of an in-situ test of the Taipei Rapid Transit system. It can be found that the vibration responses determined numerically by the identified model, both the waveform in time domain and the Fourier spectrum in frequency domain, are in good agreement with the measured signals.

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