

## **To identify the weaknesses of concrete from GPR images with employing evolution curve model**

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### **Abstract**

GPR systems emit a series of signals in microwave format. The emitted signals will pass through or be reflected at the interface between two different materials. While the emitted signals pass through different materials, the velocities of the emitted signals are changed due to the properties of the materials, like the properties of dielectric permittivity and conductivity permittivity. GPR systems clearly record the changes of the reflected energy of the emitted signals. The weaknesses of concrete usually are caused with the unbalance forces acting during grouting concrete, temperature changes at curing concrete, sea water erosions and etc. The research assumes that the emitted signals will have different behaviors while the signals pass through the weaknesses of concrete with comparing that the signals pass through the health concrete. This paper employs curve evolution model implemented with the level set scheme to isolate the regional boundaries from the image recorded with GPR system. With the proposed approach, the proposed algorithm maintains the numerical stability, and the regional boundaries of the weaknesses of concrete are located such that the elements contained in the same region will have the similar properties in GPR images.

**Key Words: Regional Boundaries, Evolution Curve, GPR**

### **1. Introduction**

This paper focuses on extracting the regional boundaries from the image recorded by ground penetration radar systems in order to identify the weaknesses in concrete. With the assumption that with comparing the emitted signals pass through the health concrete, the signals will have different behaviors while the emitted signals pass through the weaknesses of concrete. The Mumford and Shah function is used to show the energy conditions between the outside and inside of the initially curves <sup>[1]</sup>. Under minimizing the energy, those initial curves are slowly moving toward the true locations for the regional boundaries, and this process is usually named curve evolution. The level set scheme is employed to maintain the numerical stability of the algorithm

employed in the paper. The identified results with the proposed approach are compared with the results got with the impact-echo method to demonstrate the feasibility of the approach employed in this paper.

## 2. Curve Evolution Model

Partitioning a given GPR image  $u_0$  into regions and their boundaries is the main issue in this paper. Let  $\Omega$  be an image space and be shown as  $\Omega \subset \mathbb{R}^2$ .  $\Omega$  is defined as the composition of regions,  $\Omega_i$ , and  $K$  is the closed subset made up with a finite set of smooth curves. The regions can be presented as  $\Omega_i = \Omega \setminus K$ , and the length of  $K$  is defined as  $|K|$ . Mumford and Shah modeled the problem of partitioning an image into regions and their boundaries as follows: given  $u_0: \Omega \rightarrow \mathbb{R}$ , with  $\Omega \subset \mathbb{R}^2$ , find a set of regions  $\Omega_i$  to form  $\Omega$ , and an optimal piecewise smooth approximation  $u$  of  $u_0$ <sup>[1]</sup>. Mumford and Shah proposed the minimization problem and illustrated as follows:

$$\inf_{u, K} \left\{ F^{\text{MS}}(u, K) = \int_{\Omega} |u - u_0|^2 dx + \mu \int_{\Omega \setminus K} |\nabla u|^2 dx + \nu |K| \right\}, \quad (1)$$

where  $\mu > 0, \nu > 0$  are fixed parameters in order to weight the different terms in Eq. (1)<sup>[2]</sup>. To find the solution for the Eq. (1), Mumford and Shah proposed to minimize the following function:

$$F_0^{\text{MS}}(u, K) = \sum_i \int_{\Omega_i} |u_0 - c_i|^2 dx + \nu |K|. \quad (2)$$

The variable  $c_i$  is defined as  $c_i = \text{mean}(u_0)$  in  $\Omega_i$ . For the numerical stability, the curve evolution scheme is employed to find the solution of Eq. (2).

Osher and Sethian proposed an implicit representation for evolving curve<sup>[3]</sup>. The proposed algorithm provides a numerical stable solution which enables the set of initial boundaries  $K$  automatically changed in topology, such as merging and breaking. A curve  $K$  is implicitly represented as the zero level set function  $\phi: \Omega \rightarrow \mathbb{R}$  such that  $\phi(x, y) > 0$  means the point  $(x, y)$  located at the inside of the region,  $\phi(x, y) < 0$  represents the point located at the outside of the region, and  $\phi(x, y) = 0$  means the point located at the boundary. With using the Heaviside function, the length of  $K$  and the area of the region can be shown as follows

$$|K| = \int_{\Omega} |\nabla H(\phi)|, \text{Area} = \int_{\Omega} H(\phi) dx dy \quad (3)$$

where  $H_{\epsilon} = \frac{1}{2} \left[ 1 + \frac{\pi}{2} \tan^{-1} \left( \frac{x}{\epsilon} \right) \right]$ <sup>[4]</sup>.

Vese proposed an algorithm for multiphase object is employed to locate the regional boundaries and generate a smooth approximation  $u$  of  $u_0$ . Four classes and two level set functions are used to implement the image partition. Eq (2) is

rewritten with two level set functions and shown as follows:

$$\begin{aligned}
E(K, \phi) = & \int_{\Omega} (u_0 - c_{11})^2 H(\phi_1) H(\phi_2) dx dy + \int_{\Omega} (u_0 - c_{10})^2 H(\phi_1) (1 - H(\phi_2)) dx dy \\
& + \int_{\Omega} (u_0 - c_{01})^2 (1 - H(\phi_1)) H(\phi_2) dx dy + \int_{\Omega} (u_0 - c_{00})^2 (1 - H(\phi_1)) (1 - \\
& H\phi_2) dx dy + v\Omega \nabla H\phi_1 + v\Omega \nabla H\phi_2
\end{aligned}
\tag{4}$$

where  $c_{11} = \text{mean}(u_0)$  in  $\{(x, y): \phi_1(t, x, y) > 0 \text{ and } \phi_2(t, x, y) > 0\}$ ,  
 $c_{10} = \text{mean}(u_0)$  in  $\{(x, y): \phi_1(t, x, y) > 0 \text{ and } \phi_2(t, x, y) < 0\}$ ,  
 $c_{01} = \text{mean}(u_0)$  in  $\{(x, y): \phi_1(t, x, y) < 0 \text{ and } \phi_2(t, x, y) > 0\}$ ,  
 $c_{00} = \text{mean}(u_0)$  in  $\{(x, y): \phi_1(t, x, y) < 0 \text{ and } \phi_2(t, x, y) < 0\}$ .

Let  $\partial E / \partial \phi$  be the Gateaux derivative, and following evolution equation is formed:

$$\frac{\partial \phi}{\partial t} = - \frac{\partial E}{\partial \phi}.
\tag{5}$$

Therefore, we have

$$\begin{aligned}
\frac{\partial \phi_1}{\partial t} = & \delta_{\varepsilon}(\phi_1) \left\{ v \text{div} \left( \frac{\nabla \phi_1}{|\nabla \phi_1|} \right) - [(u_0 - c_{11})^2 - (u_0 - c_{01})^2] H(\phi_2) \right. \\
& \left. - [(u_0 - c_{10})^2 + (u_0 - c_{00})^2] (1 - H(\phi_2)) \right\}
\end{aligned}
\tag{6}$$

$$\begin{aligned}
\frac{\partial \phi_2}{\partial t} = & \delta_{\varepsilon}(\phi_2) \left\{ v \text{div} \left( \frac{\nabla \phi_2}{|\nabla \phi_2|} \right) - [(u_0 - c_{11})^2 - (u_0 - c_{10})^2] H(\phi_1) \right. \\
& \left. - [(u_0 - c_{01})^2 + (u_0 - c_{00})^2] (1 - H(\phi_1)) \right\}
\end{aligned}$$

where  $\delta_{\varepsilon}(x) = H_{\varepsilon}(x)' = \frac{1}{\pi} \frac{\varepsilon}{\varepsilon^2 + x^2}$ . In doing so, we can have a smooth approximation  $u$  of  $u_0$ , and  $u$  is shown as follows:

$$\begin{aligned}
u = & c_{11} H(\phi_1) H(\phi_2) + c_{10} H(\phi_1) (1 - H(\phi_2)) + \\
& c_{01} (1 - H(\phi_1)) H(\phi_2) + c_{00} (1 - H(\phi_1)) (1 - H(\phi_2))
\end{aligned}
\tag{7}$$

### 3. Experiment and results

In order to evaluate the proposed algorithm, a reinforced concrete block is built and illustrated in Figure 1. In the concrete block, cracks are artificially made and sealed with epoxy resin. Figure 2 (a) shows the condition. The measuring directions are marked on the block as shown in the Figure 2 (b).

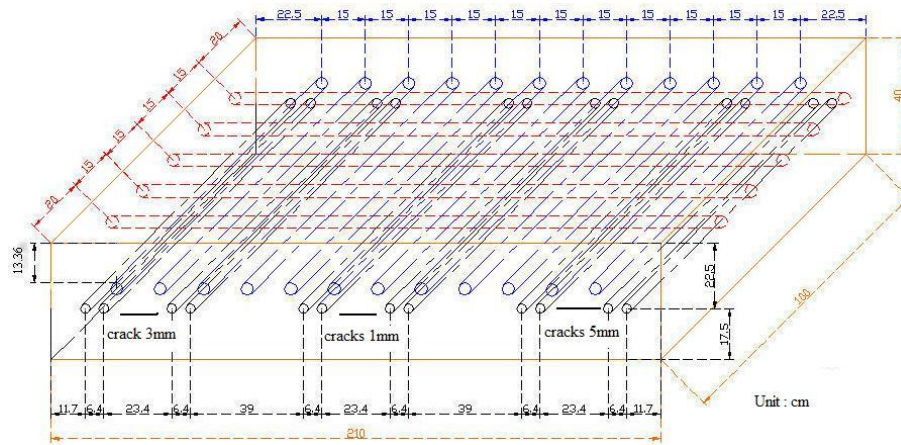
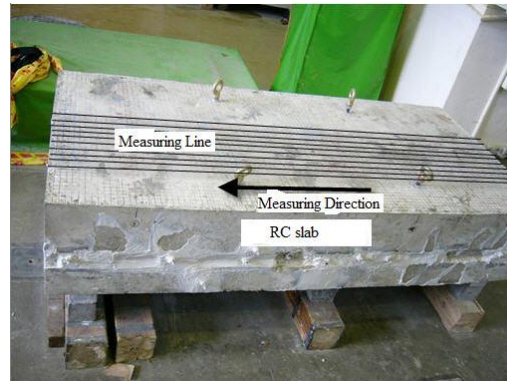


Figure 1. The sketch of the designed concrete block



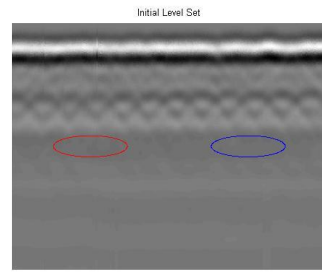
(a). The Cracks are sealed with epoxy resin.



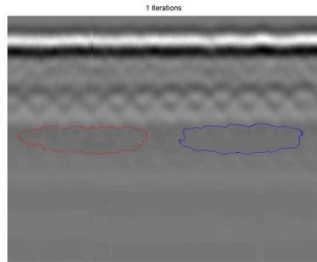
(b). Measuring directions marked on the block.

Figure 2. The designed RC block

The GPR image with two level set functions is illustrated in Figure 3. All the partial derivatives are approximated by the central difference. Two coefficients  $\varepsilon = 1$  and  $\nu = 0.001 \times 255 \times 255$  are chosen and employed to have a series of results. The boundaries extracted with the algorithm and the corresponding optimal piecewise smooth approximation  $u$  of  $u_0$  are illustrated in Figure 3. The relationship between the energy defined in the Eq. (4) and the iterations is shown in Figure 4. The suspicious locations of concrete cracks are marked with the black circle and labeled with A, B and C, and are illustrated in Figure 3 (f) and (g). With examining the cross section of the GPR image with accumulating the all collected GPR data, we found out that at the depth 17.5cm, there are possible concrete cracks founded. Figure 5 shows the situation. The founded position is close to the labeled A.



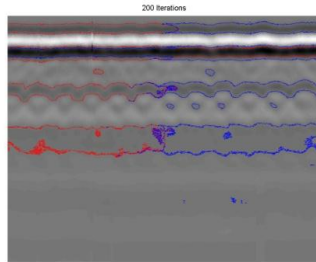
(a). Initial Curve



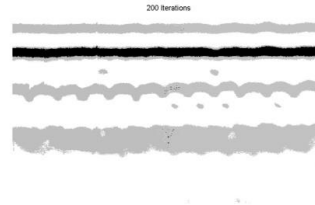
(b). 1 iteration



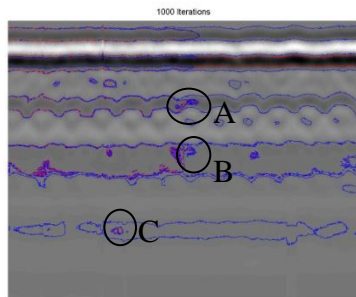
(c). Approximation  $u$  with 1 iteration



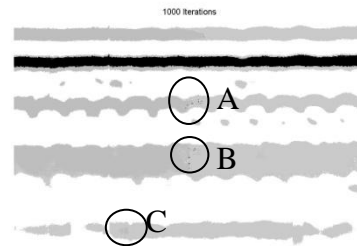
(d). 200 iterations



(e). Approximation  $u$  with 200 iterations



(f). 1000 iterations



(g). Approximation  $u$  with 1000 iterations

Figure 3. The processed results.

#### 4. Conclusion

From the experiment results, we found out that how to identify the weaknesses in concrete can be done with carefully processing the collected GPR images. The proposed approach with the optimal piecewise approximation  $u$  of  $u_0$

provides an efficient way to classify the whole image into different regions such that in each region, it is homogeneous. From the results, the proposed approach can be used to identify the weaknesses of concrete.

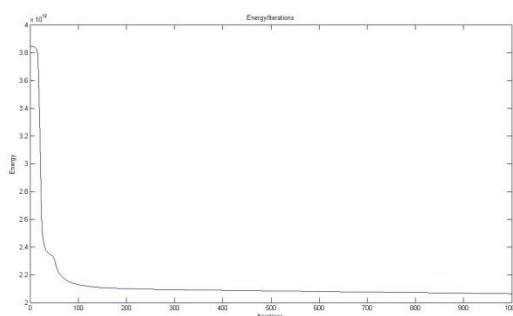


Figure 4. The relationship between energy and iterations

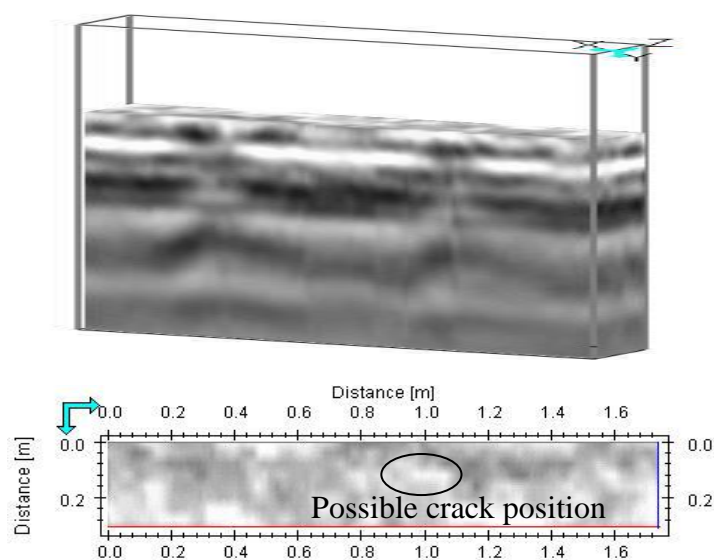


Figure 5 The cross section with possible crack position

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