

## **Algebraic Reconstruction Technique Class for Linear Scan CT of Long Object**

\*

**Bao-dong LIU<sup>1,2</sup>, Li ZENG<sup>1</sup>, Dong-jiang JI<sup>1,3</sup>**

**1 ICT research center, Key Laboratory of Optoelectronic Technology and System of the Education Ministry of China, Chongqing University, 400044, Chongqing, P. R. China**

**2 College of Optoelectronic Engineering, Chongqing University, China**

**3 Department of Mathematics and Information Science, Tianjin University of Technology and Education, Tianjin 300222, China**

**Phone: +86 02386394834;E-mail:lbd1688@126.com, Lizeng@cta.cq.cn**

### **Abstract**

The nondestructive testing (NDT) of long object (such as pipeline) has significant value. Computed tomography (CT) is able to generate 3D images and display the inner defects of the inspected object. Sometimes the long object fixed outdoors can't be rotated, and the rotating of the X-ray source and detector is also difficult. So the linear scan CT, translating the X-ray source and detector along longitude of long object synchronously, is considered. However, the projection data of linear scan CT is incomplete because of the limit of the ray-beam flare angle. The algebraic reconstruction technique is able to generate higher quality images compared with the filtered back projection (FBP) method when the projection data is noisy or incomplete. In this paper, simultaneous algebraic reconstruction technique (SART) and Ordered Subsets Expectation Maximization (OSEM) are realized for linear scan CT. Further more, some simulation experiments with different translation step and different flare angle are realized. The results show that the algebraic reconstruction technique class can generate approving CT image in linear scan CT.

**Key words:** Nondestructive testing; computed tomography; linear scan; long object; algebraic reconstruction technique

### **1. Introduction**

Computed tomography (CT) can be used to reconstruct the 3D images of long object. In order to scan the long object, they usually are rotated and translated. But sometimes the long object fixed outdoors can't be rotated, and the rotating of the X-ray source and detector is also difficult. So the linear scan CT, translating the X-ray source and detector along longitude of long object synchronously, is considered.

The linear scan CT is quick and simple. When the flare angle of the ray-beam equals to 180°, the projection data is complete. However, because of the limit of the ray-beam flare angle, the projection data of linear scan CT is always incomplete<sup>[1]</sup>. SONG presented a fast quasi-3D imaging method for inspecting moving object. Using cone-beam X-ray and panel detector translation scan (PDTS), quasi-3D data of the object structure can be obtained just by one translation scan. The reconstruction method they used is Gauss-Seidel algorithms<sup>[1]</sup>. CHEN designed a linear scan imaging equipment used for security check. The

---

\* This work supported by the National Natural Science Foundation of China under Grant No. 60672098 and the National 863 Program of China under Grant No. 2006AA04Z104. This research is supported by the National Natural Science Foundation of China under grant No. 60672098, Education Ministry Spring Program under grant No. Z2005 - 2 - 63001 and Chongqing Technology Program under grant No. CSTC2006AB3027.

projection data are rearranged to imitated parallel beam and the images are reconstructed by the progress of filtered back projection <sup>[2]</sup>.

In the field of CT, when projection data is incomplete or noisy, the iterative reconstruction (IR) algorithms are able to generate higher quality CT images compared with the Filtered Back Projection (FBP) algorithm <sup>[3]</sup>. Currently, there are many IR algorithms such as algebraic reconstruction technique (ART), simultaneous iterative reconstruction technique (SIRT), simultaneous algebraic reconstruction technique (SART), Expectation Maximization (EM) and Ordered Subsets Expectation Maximization (OSEM). ART reconstructions usually suffer from salt and pepper noise. The results of SIRT are smoother than those produced by ART but it is at the expense of slower convergence. SART maintains the rapid convergence of ART-type algorithm while at the same time it has the noise suppressing features of SIRT <sup>[4]</sup>. The standard EM algorithm can generate good quality images, but the computation is intensive and convergence slow. OSEM accelerates the convergence by applying the ordered subsets concepts to EM <sup>[5]</sup>. In this paper, we mainly talk about SART and OSEM algorithms of linear scan CT.

## 2. SART and OSEM algorithms of the linear scan CT

### 2.1 Linear Scan CT

The scan geometry used in this paper is cone beam geometry (see Figure 1 and Figure 2).  $S$  is the ray source whose locus is a line.  $S_{start}$  and  $S_{end}$  is the start and end position of the ray source.  $AF$  is the reconstruction field, but only the data of field  $BE$  are displayed. Because the number of projection data of field  $AB$  and field  $EF$  is smaller than that of field  $BE$ , so the images quality of them are not so good as field  $BE$ . The length of field  $AB$  can be calculated as

$$L = 2SG \tan(\alpha / 2) \quad (1)$$

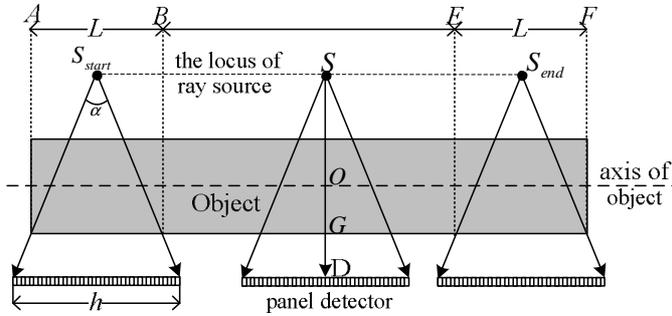


Figure 1. The cutaway view of linear scan geometry

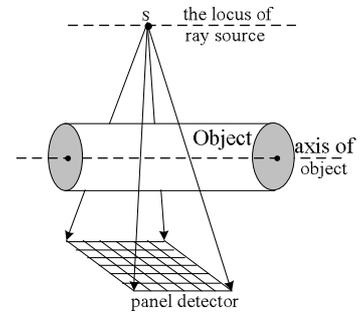


Figure 2. The linear scan geometry

### 2.2 SART and OSEM algorithms

We model a linear imaging system as follows:

$$WF = P \quad (2)$$

where,  $W = (W_{ij})$  denotes an  $M \times N$  matrix,  $F = [f_1, f_2, \dots, f_N]^T \in R^N$  is an underlying image,  $P = [p_1, p_2, \dots, p_M]^T \in R^M$  is observed data. Image reconstruction is to seek  $F$  according to  $P$  and  $W$ .

SART algorithm was first reported in [6]. Its formula can be stated as follows:

$$f_j^{(k)} = f_j^{(k-1)} + \frac{\sum_{i \in \theta_k} \left( \frac{p_i - \beta \phi}{W_{i+}} \cdot w_{ij} \right)}{\sum_{i \in \theta_k} w_{ij}} \quad j = 1, 2, \dots, N. \quad k = 1, 2, \dots \quad (3)$$

where,  $f_j^{(0)} = 0$ ,  $\beta \phi = \sum_{j=1}^N w_{ij} f_j^{(k-1)}$ ,  $W_{i+} = \sum_{j=1}^N w_{ij}$ ,  $\theta_k$  is the  $k$ th projection,

The process of SART is as follows. Assign a value of zero to all the pixels. For the first ray of each projection, we compute the corrections to be made at every pixel and store them in a separate array (namely correction array). Then take up next ray and updates the correction array. And then the next ray, and so on. After we compute all the rays in each projection, add the correction array to the image array. This entire process repeated with every projection<sup>[4]</sup>.

The ordered-subsets expectation-maximization (OSEM) algorithm is an accelerated iterative statistical reconstruction algorithm<sup>[5]</sup>. Its formula used in this paper is given by:

$$f_j^{(k)} = f_j^{(k-1)} \frac{\sum_{i \in \theta_k} \left( \frac{p_i}{\beta \phi} \cdot w_{ij} \right)}{\sum_{i \in \theta_k} w_{ij}} \quad j = 1, 2, \dots, N. \quad k = 1, 2, \dots \quad (4)$$

where,  $f_j^{(0)} = 1$ , and the other parameters in (4) are the same as those in (3). The process of OSEM is similar with SART's. And we must assign a positive value to all the  $f_i^{(k)}$ .

### 3. The results of experimentation and analysis

A long pipeline model is simulated in our experiment. As Figure 3 shows, the pipeline model which contains two layers is composed of 1024 image slices ( $z=1, 2, \dots, 1024$ ). The outside layer contains a small column and the inside one contains six linear cracks, which are symmetrical. The other parameters are given as follows (pixel). Images slice size is  $128 \times 128$ .  $r_1 = 40, r_2 = 48, r_3 = 64$ . The grays of inside layer and out layer are 100 and 200 respectively. The range of crack and small column is from  $z=480$  to  $z=544$ . The width of six cracks is  $w_1 = w_4 = 1$ ,  $w_2 = w_3 = 3$ ,  $w_5 = w_6 = 2$  respectively. The diameter of small column is  $d = 2$ .

Figure 4 denotes the cutaway view of linear scan geometry with two different flare angles. It is assumed that the size and position of panel detector is fixed and the flare angle changes with the distances between ray sources and the center of panel detector. Besides, the axis of object is at the middle of ray source and panel detector.

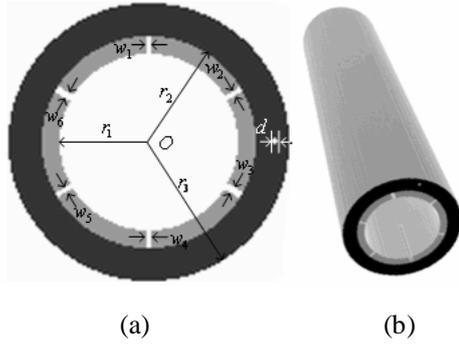


Figure 3. The original images (a) the original slice at position  $z=512$  (b) 3D original image from  $z=480$  to  $z=1024$

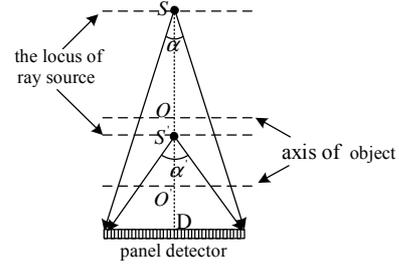


Figure 4. The cutaway view of linear scan geometry with two different flare angles

Some experiments with different flare angles are realized. We find that when the flare angle equals to  $30^\circ$  ( $45^\circ$ ), the qualities of reconstructed slices using SART (OSEM) are satisfied.

Figure 5 shows the reconstruction results for SART (flare angles equal to  $30^\circ$  and 5 iterations) and OSEM (flare angles equal to  $45^\circ$  and 5 iterations). The image is one slice of the 3D model of long object at position  $z=512$ . In this paper, mean square error (MSE) was adopted to evaluate the reconstructed slice at position  $z=512$ . The formula of MSE is given by

$$E = \frac{1}{N} \sum_{i=0}^{N-1} (f_i - \hat{f}_i)^2 \quad (5)$$

where,  $N$  is the number of slice pixels,  $f_i$  is the gray of pixel  $i$  in original slice,  $\hat{f}_i$  is the gray of pixel  $i$  in reconstructed slice. Figure 6 plots MSE versus translation step. From Figure 5 and Figure 6, we can see that when the flare angle is fixed, the MSE is increased with the translated step. It can be explained by the fact that the translation step is smaller the more projection data can be attained.

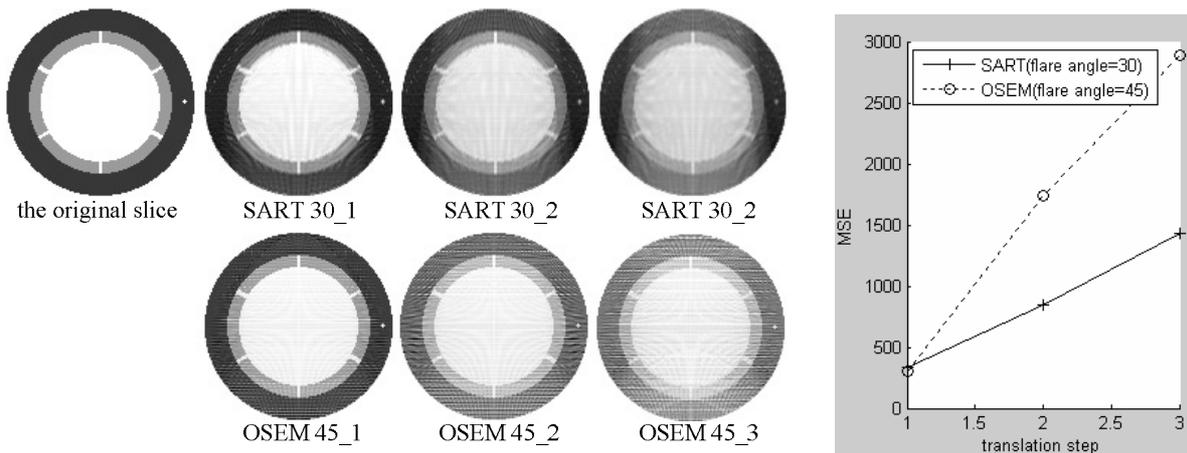


Figure 5. The original and reconstructed slice at position  $z=512$ . (30 and 45 mean flare angle; 1, 2 and 3 denote translation step)

Figure 6. MSE versus translation step

#### 4. Conclusions

Linear scan CT is useful to nondestructive testing of long object. In this paper, linear scan CT is simulated and two kinds of algebraic reconstruction technique (SART and OSEM) are realized. The results of the experiments show that SART and OSEM can generate higher quality CT images. However, reconstruction by SART and OSEM is time consuming. Further research will explore the methods that reduce the calculation time of algebraic reconstruction technique used for linear scan CT.

#### References

- [1] SONG Zheng, WANG Jing-jin, LIN Yong and WANG Jian-guo: X-ray laminography with panel detector translation scan, Atomic Energy Science and Technology, Volume 38 Number 5, Sep. 2004, P444-447.
- [2] CHEN Zhi-qiang etc.: Imaging equipment, China, ZL200520114670.2, Aug. 2007.
- [3] LI Xiang, NI Jun and WANG Ge: Parallel iterative cone beam CT image reconstruction on a PC cluster, Journal of X-Ray Science and Technology, Number 13, 2005, P1-10.
- [4] Avinash C. Kak, Malcolm Slaney: Principles of computerized tomographic. New York: The institute of Electrical and Electronics Engineers, Inc., IEEE Press, 1999, P277-292
- [5] H. Malcolm Hudson and Richard S. Larkin: Accelerated image reconstruction using ordered subsets of projection data, IEEE TRANSMXIONS ON MEDICAL IMAGING, Volume 13, Number 4, DEC. 1994, P601-609.
- [6] A. H. Andersen, A. C. Kak: Simultaneous algebraic reconstruction technique (SART): a superior implementation of the ART algorithm, Ultrasonic Imaging, volume.6, Jan. 1984, P81-94.