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Enhanced Spatial Resolution in 2D CT-Reconstruction without Filtered Back Projection: DIRECTT

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Abstract

We report on novel developments of the DIRECTT (Direct Iterative Reconstruction of Computed Tomography Trajectories) algorithm. Details are discussed for model calculations in order to elucidate its universal character for parallel and fan beam geometry. Our approach represents a promising alternative to the conventional filtered back-projection (FP). Instead of Fourier transforming single projections DIRECTT traces single sinoidal-like trajectories in Radon space which are selected from the set of all possible trajectories by appropriate criteria such as their angular averaged (filtered) weight or contrast to adjacent trajectories. The respective reconstruction elements are added to form an array. The projection (Radon transform) of these elements is then subtracted from the original data set. The obtained residual sinogram is treated the same way in the subsequent iteration steps until a pre-selected criterion of convergence is reached. A very precise projection of a gapless reconstruction array is essential for enhanced spatial resolution also taking into account the actual size and shape of its elements (in this case squares).

Abandonment of the spatial filtering overcomes some serious restrictions of the FP: (i) Nyquist's theorem does not refer to the detector pixel sampling (for each single projection!) but holds for the non-equidistant angular oversampling, i.e. the achievable spatial resolution is no longer limited to twice the detector pixel size but can be enhanced considerably (even to sub-pixels in terms of detector elements) by choosing appropriate small increments of rotation, (ii) there is no need for complete data sets, even missing projections are well tolerated, and (iii) region-of-interest reconstructions perform without considerable artifacts.

Keywords: computed tomography, reconstruction algorithm, spatial resolution

Introduction

DIRECTT[1] represents a promising alternative to conventional reconstruction algorithms such as Filtered Back-Projection[2] (FP, the 2D case of the well-known Feldkamp-Davis-Kress algorithm[3]), or Algebraic Reconstruction Techniques (ART).

DIRECTT is of peculiar interest when the focus is on reconstruction of fine structured details or on precise location of reconstructed elements rather than on computing time. (Indeed, current work aims at parallel processing certain details of iteration in order to shorten computing time.) In contrast to FP, DIRECTT does not treat each (detector) projection individually, i.e. it is not deconvoluted globally or (Fourier) filtered, but the entire trajectory of a reconstruction element is considered. In contrast to ART, DIRECTT does not modify the entity of reconstructions elements simultaneously.

The 2D algorithm we introduce here is applicable to parallel as well as fan beam geometry. As to be shown elsewhere it can be extended to 3D cone beam geometry. (The precise projection of cube-shaped reconstruction elements (voxels) is utilized in this case.) It has been shown previously that the algorithm does not only work for sinoidal trajectories exclusively, but yields excellent reconstruction results for laminographic data sets as well.

Reconstruction Principle

Figure 1 schematically displays the algorithm's iterative philosophy.

Subframe 1a (top left) indicates a model volume at the example of a 14 pixel object. Subframe 1b (bottom left) represents the respective density sinogram (Radon transform[4]) which is either achieved by computed projection of model densities (Fig. 1a) or is the initial experimental intensity data converted according to Lambert's law.

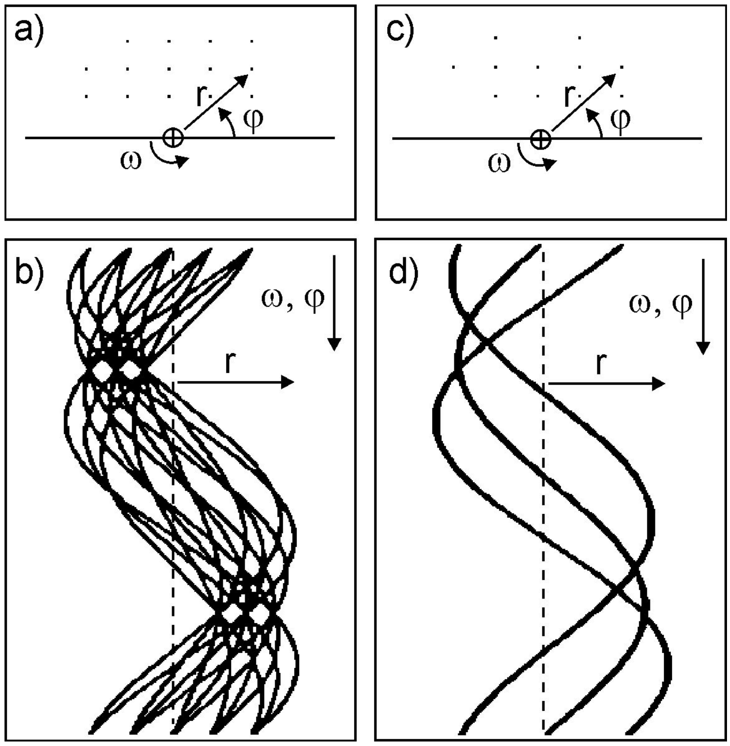

Figure 1. Reconstruction principle of the iterative procedure

DIRECTT chooses those single sinoidal-like trajectories from the respective sinogram (Fig. 1b) which are selected from the set of all possible trajectories by appropriate criteria such as their angular averaged weight or contrast with adjacent trajectories. The respective reconstruction elements are added to an array of pre-selected element size (Fig. 1c, 11 out of the 14 original elements in the example). The projection (Radon transform) of these elements (i.e. a computed sinogram) is then substracted from the original data set. The obtained residual sinogram (Fig. 1d, containing trajectories of 3 remaining elements in the example) is subject to the same procedure in the subsequent iteration steps until a pre-selected criterion of convergence is reached.

In contrast to the FP there is no integral computation along the detector (incl. limited sampling due to its element size) but an optional over-sampling along the numerous projection angles. One of DIRECTT's unique characteristics is its very *precise projection* of reconstruction elements taking into account their actual size and shape which is essential for enhanced spatial resolution as well. That is, reconstruction pixels are considered as a set of dense packed elements of well-defined size and shape instead of being (circular smeared) point functions only. All previous calculations have been performed on base of square entries in a Cartesian matrix.

Enhanced Spatial Resolution – Sub-Pixels – Reconstruction Details

The mentioned abandonment of $1/r$-deconvolution (where r denotes the (projected) radius of the respective reconstruction elements about the rotation axis) or lowpass filtering in Fourier space overcomes some serious restrictions of the FP. On the one hand this holds for the achievable spatial resolution, which is limited to twice the size of two detector elements by means of Nyquist's sampling theorem[5].

Figure 2 visualizes the achievable spatial resolution. As in the subsequent figures we employ the comparison to the FP in order to clarify the advantages and potentials of DIRECTT. As an example we have constructed an ensemble of 5×5 (square) entries within a 128×128 matrix with a vertical 2 pixel grid and 3 pixel grid horizontally. Additionally, their weight (or grey value) modulates as a radial gradient. Projection is computed with a two-fold binning on a (virtual) detector at source-to-axis distance. The sinogram clearly visualizes that trajectories of single entries cannot be resolved. The result of FP is a blurred object which just roughly reproduces the area of the total ensemble at the original center of gravity. Besides the gradient of density the single objects are not detectable at all. For the above mentioned reasons DIRECTT allows for (at least) the four-fold spatial resolution so that reconstruction elements are separated very clearly in horizontal direction (grid width 3). However, judging the resolution in vertical direction there are still some remaining artifacts in between those original entries. Moreover, closer inspection reveals the original gradient of density.

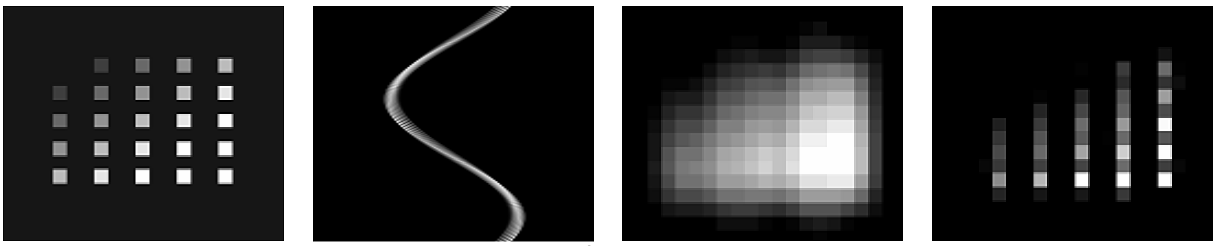

Figure 2. Enhanced spatial resolution: model, two-fold binned sinogram, FP and DIRECTT reconstruction (from left to right)

The cause of gaining spatial resolution is sketched in Figure 3. The projections of three 2×2 sub-structured pixels (of edge length 1) are depicted. Case A (left) represents a pixel of homogeneous density with sub-elements (1, 1, 1, 1) (and therefore homogeneous attenuation coefficient µ,), in case B (center) only one out of the four sub-elements is unequal to zero: (0, 0, 0, 4), whereas case C describes a chess-like substructure: (-1, 1, 1, -1). All these objects are projected on to a detector elements which are of the same size as the unit pixels.

Replacing the four sub-pixels by one unit pixel carrying their average weight (two-fold binning) would yield identical projections in case A and B (with integral weight 1), whereas case C would result in zero weight and therefore no measurable trajectory.

As can be seen from Figure 3 (right) there are substantial differences in the sinograms (Radon transforms) when projecting those sub-structures by means of the precise DIRECTT tools.

Moreover, reconstruction of those single tracks reveals the sub-structured matrix entries without any prior kowledge.

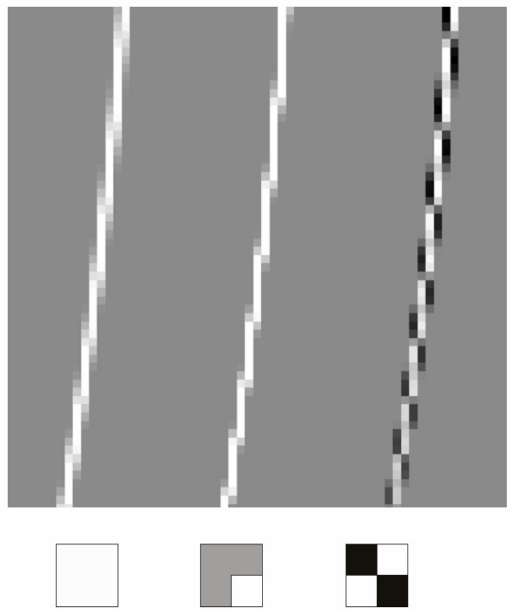

Figure 3. Sinogram sections of sub-structured pixels

Modeling of incomplete data sets

One of the FP's fundamental requirements are complete data sets, i.e. that the entity of volume elements has to be projected for all angular settings of a full rotation. However. in practice, awkward experimental conditions can oppose those ideals: the physical sample size in one or two spatial dimensions exceeds permitted limits so that a full rotation (i) is hindered due to geometrical reasons or (ii) makes no sense with respect to large penetrated paths (take $\mu d>5$ as a rule of thumb, i.e. no measurable intensity). Eventually (iii) the overall detector size limits the achievable fraction of volume which is projected for all angular settings. If the operator restricts the reconstruction of that region of interest (ROI) there may appear artifacts arising from incomplete projected data subsets (i.e. outside the ROI) whose information might distort the inside reconstruction data tremendously.

The advantages of a DIRECTT reconstruction concerning a data set of limited angle (ref. cases (i, ii)) are demonstrated by means of a triangular model of binary density (μ=1 inside the triangle, μ=0 outside) embedded into a 256^2-matrix as depicted in Figure 4. That model is projected in a 90° angular sector (increment 0.5°) as is indicated by the arrows in the left subframe. Inside this subsector the left and right (oblique) edges of the triangle are parallely projected under certain angles, in contrast to the bottom edge. Consequently, the FP results in highly contrasted oblique edges, however, a contrast at the bottom edge is missed at all. The DIRECTT reconstruction taking advantage of a known unique density is visually identical to the original model after 800 cycles of iteration. There is a strong resemblance between the first DIRECTT approximation by one iteration step (no selection up to this stage) and the FP reconstruction, however, the DIRECTT intermediate stages reveals the successive formation of volume elements in the vicinity of the bottom edge.

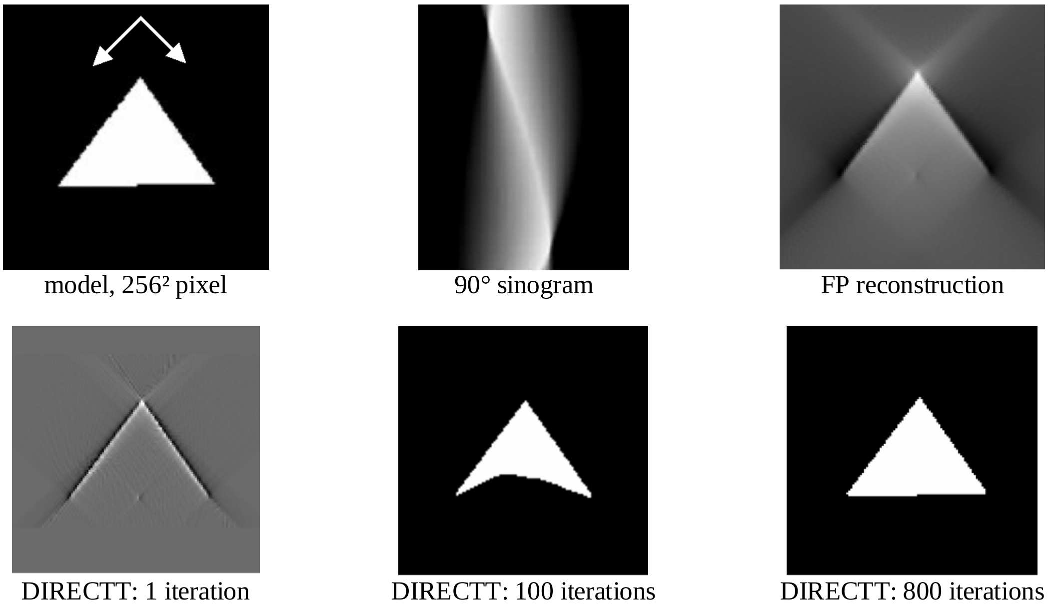

Figure 4. Limited angle reconstructions of discrete model data.

In order to emphasize DIRECTT's advantages concerning ROI reconstructions we employ a 128^2-model consisting of different geometric objects and density levels as well as "DIRECTT" acronym made of dotted letters in a 3×2 grid (see Fig.2 for comparison). The dashed circle in Figure 5 (left) symbolizes the ROI as the steadily irradiated volume fraction. The respective sinogram was computed from 360 projections (increment 1°). The FP generates a blurred image, in particular with respect to the acronym details with an overall error of density of about 40% (compared to the original). In contrast the DIRECTT reconstruction results in a much more detailed image (note the acronym) of integral error of density less than 4% after 300 iterations.

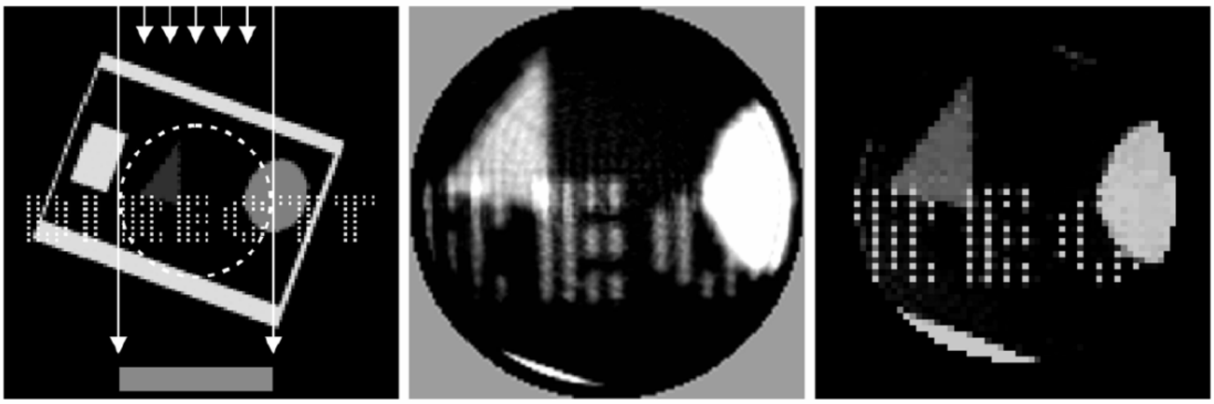

Figure 5. ROI reconstruction: model and selected ROI (left), FP (center), and DIRECTT (right)

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