

Application of wavelet analysis to signal de-noising in ultrasonic testing of welding flaws

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Abstract

In ultrasonic testing of welding flaws, the reliability and quality of tests are considerably affected by noise and spurious signals. Thus, signal de-noising and increasing of the signal-noise ratio (SNR) are a key to successful application of ultrasonic NDT and NDE. At present, there are many methods for signal de-noising, such as median filtering, non-linear filtering, adaptive filtering, correlation technique, split spectrum processing, artificial neural network, etc. Although these methods are of a practical meaning for increasing SNR of ultrasonic testing, they have some limitations affecting the reliability of testing results.

As a method of time-frequency analysis, wavelet analysis is one of the most effective and promising signal processing techniques with a lot of advantages. In this paper, the theory of wavelet analysis, including wavelet transform (WT), wavelet packet transform (WPT) and lifting wavelet transform (LWT), is introduced. And according to the characteristics of ultrasonic echo-signals of welding flaws, the theory and method of wavelet for signal de-noising are discussed. Finally, the experimental studies of signal de-noising by wavelet analysis are carried out for ultrasonic simulation signals and actual defect echo-signals. Experimental results show that noises can be well suppressed and SNR is improved obviously by wavelet analysis. Comparing LWT with WT, while SNR is similar, LWT is of flexible design and fast computation with a simple programme. In addition, for ultrasonic signal de-noising, WPT is more effective than WT, but its computation is complex. Thus, LWT has a good application prospect for signal de-noising, especially in real-time signal de-noising.

Keywords: Ultrasonic Testing, De-noising, Wavelet Transform, Wavelet Packet, Lifting Wavelet

1. Introduction

Because of the advantages such as exact defect localization, sensitivity to flaw detection and detecting inner flaws of workpieces, ultrasonic testing is widely used in the inspection of weldments^[1]. However, the reliability and quality of tests are considerably affected by noise and spurious signals in ultrasonic testing^[2]. Thus, it is important for de-noising echo-signals in ultrasonic testing and ensuring the reality of defect echo-signals. At present, there are many methods for signal de-noising, such as median filtering, non-linear filtering, adaptive filtering,

correlation technique, split spectrum processing, artificial neural network, etc. Although these methods are of a practical meaning for increasing the signal-noise ratio (SNR) of ultrasonic testing, they have some limitations affecting the reliability of testing results and the accuracy of locational, quantitative analysis and evaluation.

As a method of time-frequency analysis, wavelet analysis is one of the most effective and promising processing techniques with a lot of advantages. In this paper, according to the characteristics of ultrasonic echo-signals of welding flaws, the method of wavelet analysis on signal de-noising is studied. Moreover, the experimental studies of signal de-noising by wavelet analysis are carried out for ultrasonic simulation signals and actual defect echo-signals, and the experimental results and analysis are given.

2. Wavelet Analysis

2.1 Wavelet transform^[3, 4]

Suppose $\psi(t) \in L^2(R)$, its Fourier transform (FT) is $\psi(\omega)$. When $\psi(\omega)$ satisfies the admissibility condition,

$$C_\psi = \int_R \frac{|\psi(\omega)|^2}{|\omega|} d\omega < \infty \quad (1)$$

with $\psi(t)$ is the mother wavelet or basic wavelet. We obtain a family of wavelets from $\psi(t)$ by dilating and translating,

$$\psi_{a,b} = \frac{1}{\sqrt{|a|}} \psi\left(\frac{t-b}{a}\right) \quad a, b \in R, a \neq 0 \quad (2)$$

where a is dilation factor, and b is translation factor.

For the function $f(t) \in L^2(R)$, its continuous wavelet transform (CWT) is

$$W_f(a,b) = \frac{1}{\sqrt{|a|}} \int_R f(t) \psi\left(\frac{t-b}{a}\right) dt = \langle f(t) | \psi_{a,b}(t) \rangle \quad (3)$$

If $f(t)$ is continuous in t , $f(t)$ is reconstructed by the following formula,

$$f(t) = \frac{1}{C_\psi} \iint_{R^2} W_f(a,b) \psi_{a,b}(t) \frac{da}{a^2} db \quad (4)$$

In the practical use, in order that it is easy to realize in the computer, CWT must be discretized. We suppose that $a = a_0^j$ and $b = kb_0 a_0^j$, with $a_0 > 1, b_0 \neq 0, j, k \in Z$, and t is discretized, thus discrete wavelet transform (DWT) is defined as follows

$$Wf_{j,k}(n) = a_0^{-\frac{j}{2}} \sum_k f(n) \psi(a_0^{-j} n - kb_0) \quad (5)$$

French scholar, Mallat puts forward the idea of multiresolution analysis, and gives the fast algorithm of wavelet decomposition and reconstruction, namely Mallat algorithm. According

to this algorithm, if f_k is the discrete sampling data of signal $f(t)$, the $f_k = c_{0,k}$ is true.

And then the decomposition algorithm of the wavelet transform (WT) can be described as

$$\begin{cases} c_{j,k} = \sum_n c_{j-1,n} h_{n-2k} \\ d_{j,k} = \sum_n c_{j-1,n} g_{n-2k} \end{cases} \quad k = 0, 1, \dots, n \quad (6)$$

In addition, the reconstruction algorithm of (WT) is expressed by the formula,

$$c_{j,k} = \sum_n c_{j,n} \tilde{h}_{k-2n} + \sum_n d_{j,n} \tilde{g}_{k-2n} \quad (7)$$

2.2 Wavelet packet transform^[4]

WT only decomposes the low-frequency components of signal, but high-frequency components, namely the detail components, can not be decomposed continuously, so it can not well decompose and express the signals containing a lot of detail information. However, wavelet packet transform (WPT) is a more fine analysis method, and the frequency band is divided into many levels, then the high-frequency components are further decomposed. According to the characteristics of analysis signal, WPT chooses frequency band adaptively, and makes it match with signal spectrum, thus the processing ability of signal is improved greatly. The difference between wavelet and wavelet packet decomposition is shown in Figure 1.

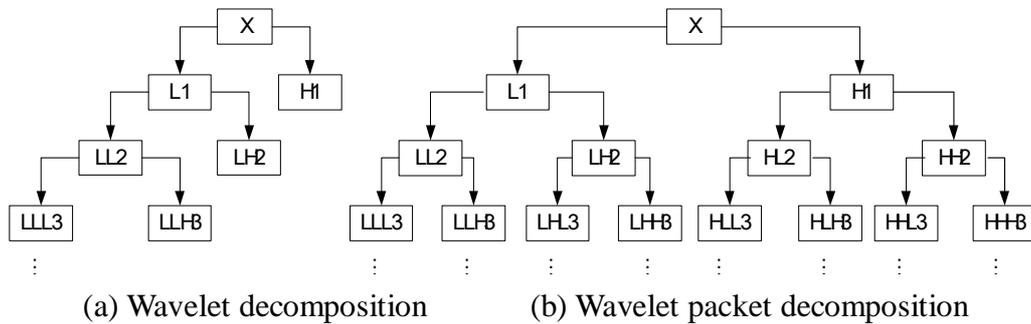


Figure 1. Difference between wavelet and wavelet packet decomposition

The decomposition algorithm of wavelet packet is described as the following formula, namely $\{d_l^{j,2n}\}$ and $\{d_l^{j,2n+1}\}$ can be obtained by $\{d_l^{j+1,n}\}$.

$$\begin{cases} d_l^{j,2n} = \sum_k h_{k-2l} d_k^{j+1,n} \\ d_l^{j,2n+1} = \sum_k g_{k-2l} d_k^{j+1,n} \end{cases} \quad (8)$$

On the other hand, the reconstruction algorithm of wavelet packet is expressed by the following formula, namely $\{d_l^{j+1,n}\}$ can be obtained by $\{d_l^{j,2n+1}\}$ and $\{d_l^{j,2n}\}$.

$$d_l^{j+1,n} = \sum_k (h_{l-2k} d_k^{j,2n} + g_{l-2k} d_k^{j,2n+1}) \quad (9)$$

2.3 Lifting wavelet transform^[5, 6, 7]

It is different from classical WT which analyses problems in frequency domain, lifting wavelet transform (LWT) directly analyses problems in time (spatial) domain. In addition, it can structure all classical wavelet by lifting scheme without depending on FT. The de-noising by LWT has such characteristics of fast computation, no need of extra memory and implementing integer wavelet transform.

The lifting scheme of WT consists of three simple phases^[8]: split, predict and update. In the first step, the data s^j is divided into even and odd subsets: $even_{j-1}$ and odd_{j-1} with

$s^j = \{s_{j,l} | 0 \leq l \leq 2^j\}$, where low frequency signal s^{j-1} and high frequency signal d^{j-1} are obtained by first level WT. The operator can be expressed as

$$even_{j-1} = \{s_{j,2l} | 0 \leq l \leq 2^{j-1} - 1\}, \quad odd_{j-1} = \{s_{j,2l+1} | 0 \leq l \leq 2^{j-1} - 1\}$$

Secondly, due to each sampling point of the signal s^j has certain correlation, odd_{j-1} can be predicted by $even_{j-1}$, namely $odd_{j-1} = P(even_{j-1})$. And then prediction error can be expressed by the following formula, $\gamma = odd_{j-1} - P(even_{j-1})$, where P is prediction operator.

Finally, in order to recover some characteristics of signal which is lost in the process of prediction, $even_{j-1}$ can be updated as $\lambda = even_{j-1} + U(\gamma)$, where U is update operator.

The forward and inverse transforms of the lifting scheme are given in Figure 2.

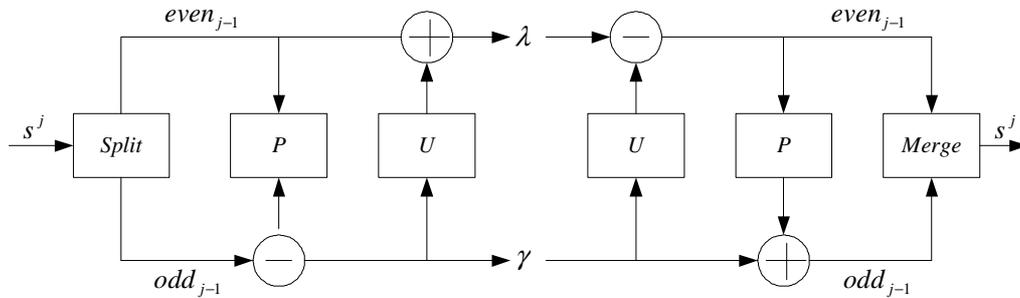


Figure 2. The forward and inverse transforms of the lifting scheme

3 Principle of wavelet for signal de-noising

A noise-contaminated ultrasonic echo-signal can be expressed as^[3]

$$y(t_i) = f(t_i) + n(t_i) \quad i = 1, 2, \dots, N \quad (10)$$

where $f(t_i)$ is original signal, $n(t_i)$ is Gaussian white noise of independent identical distribution, that its expectation is zero and variance is σ^2 . The aim of de-noising is

suppressing $n(t_i)$ and recovering $f(t_i)$. According to the linear property of WT, wavelet coefficients $w_{j,k}$ are composed of two parts: one is the wavelet coefficients $u_{j,k}$ corresponding to signal $f(t_i)$, the other is the wavelet coefficients $v_{j,k}$ corresponding to noise $n(t_i)$. Thus the variance of dyadic wavelet transform for white noise is obtained as

Fehler! Es ist nicht möglich, durch die Bearbeitung von Feldfunktionen Objekte zu erstellen. (11)

From the above formula, we can know that the mean of $|W_{2^j}n(t)|^2$ decreases with the increase of level j , namely white noise has negative singularity. However, the WT modulus maximum of original signal increases as level increases. Therefore, the signal can be distinguished from noise by the different trend of modulus maximum in the multi-scale spatial.

The threshold de-noising algorithm is most effective and widely-used method, and it involves three steps^[9, 10]:

(1) The noise-contained signal is decomposed by wavelet (wavelet packet or lifting wavelet) at N level through the chosen appropriate wavelet. Then corresponding wavelet (wavelet packet or lifting wavelet) decomposition coefficients are obtained.

(2) The decomposition detail coefficients are thresholded by the threshold to obtain a new detail coefficient. At present, there are three frequently-used thresholding methods^[4]: hard-thresholding, soft-thresholding and semi-soft-thresholding functions. The hard-thresholding function is defined as

$$\eta(w) = wI(|w| > T) \quad (12)$$

where T is the threshold. The soft-thresholding function is defined by

$$\eta(w) = (w - \text{sgn}(w)T)I(|w| > T) \quad (13)$$

Generally, the semi-soft-thresholding function is given as

$$\eta(w) = \text{sgn}(w) \frac{T_2(|w| - T_1)}{T_2 - T_1} I(T_2 < |w| < T_1) + wI(|w| > T_2) \quad (14)$$

(3) This new de-noised detail coefficient is then reconstructed with the approximation coefficient to produce a new de-noised signal by wavelet, wavelet packet and lifting wavelet.

The general threshold rule has four methods: (i) the universal threshold, which is obtained as $\lambda = \sigma\sqrt{2\log(n)}$, where n is the signal length and σ is the standard deviation of the noise, (ii) adaptive threshold selection based on principle of Stein's unbiased risk estimate, (iii) heuristic threshold selection, (iv) minimaxi threshold^[4].

4 Experimental results and analysis

In order to check the effectiveness of wavelet analysis on signal de-noising in ultrasonic

testing, the experiments of signal de-noising by WT, WPT and LWT are performed for ultrasonic simulation signals and actual defect echo-signals on the computer using MATLAB7.0.

Figure 3(a) plots the simulation signal with two defects that their amplitudes and central frequencies are different and their central frequencies are 4 and 2.5MHz from left to right. Moreover, the sampling frequency of signal is 40MHz, and the distance of two defects is small. Figure 3(b) gives the signal corrupted by random Gaussian white noise, and is shown that the signal is almost entirely overwhelmed by noise. Figures 3(c)-(e) show the de-noised ultrasonic simulation signal using WT, WPT and LWT respectively.

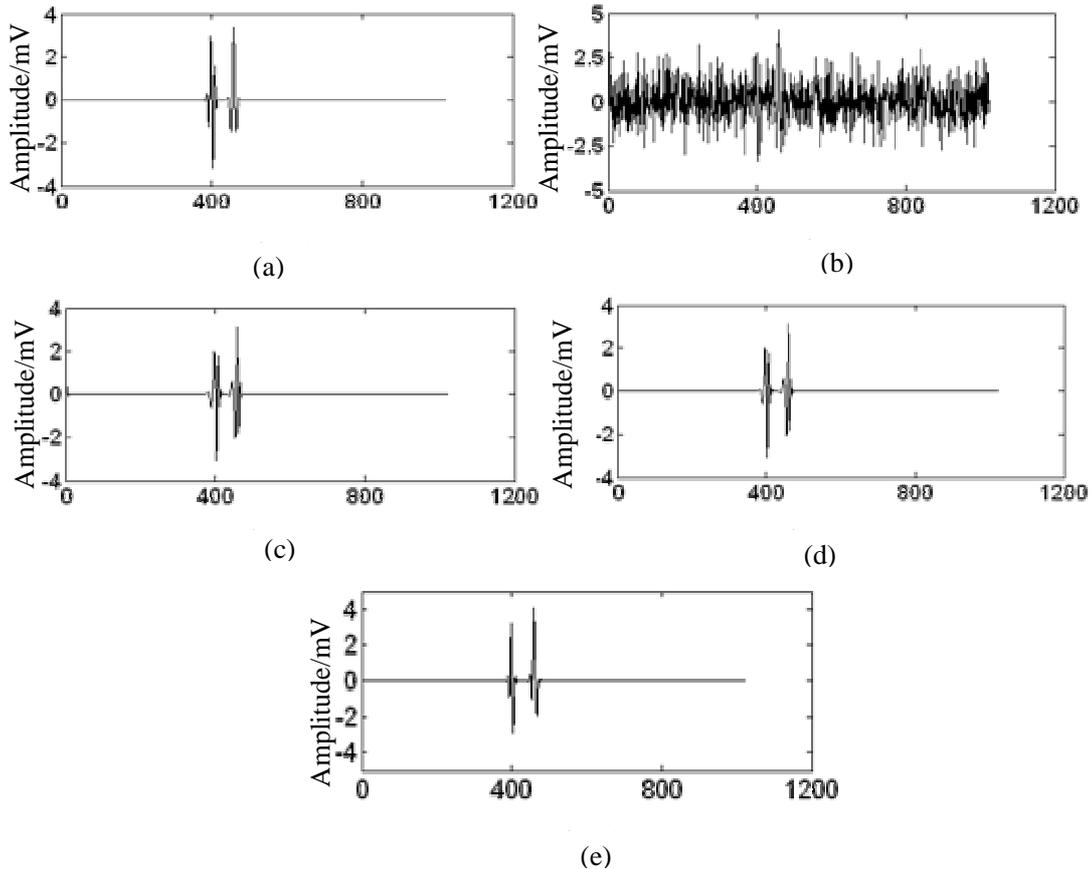


Figure 3. De-noising results of three methods for ultrasonic simulation signal: (a) original signal; (b) the signal corrupted with white noise; (c) the de-noised signal by WT; (d) the de-noised signal by WPT; (e) the de-noised signal by LWT.

It can be seen that all three methods can achieve better de-noising effect. Table 1 lists the SNR increment, standard deviation and time of three methods. The SNR is calculated by means of the expression^[11]:

$$SNR = 10 \log_{10} \left(\frac{\sum_n f^2(t)}{\sum_n [\tilde{f}(t) - f(t)]^2} \right) \quad (15)$$

where $f(t)$ denotes the original signal and $\tilde{f}(t)$ represents the de-noised signal.

Table 1 The de-noising performance of three methods

De-noising method	SNR increment/dB	Standard deviation	Time/s
WT	13.8961	6.6378	0.0131
WPT	13.9433	6.6018	0.3321
LWT	14.0090	5.2051	0.0086

Figure 4 gives the processing results of an actual defect echo-signal. Figure 4(a) illustrates the echo-signal for a traverse cylindrical cavity $\Phi 1\text{mm}$ obtained by straight probe with the frequency of 10MHz, and its sampling frequency is 250MHz. Figure 4(b)-(d) show the de-noised signal resulting from WT, WPT and LWT, respectively. By comparison, all three methods produce satisfactory de-noising effect, and using time is 0.0080s, 0.0238s and 0.0053s, respectively.

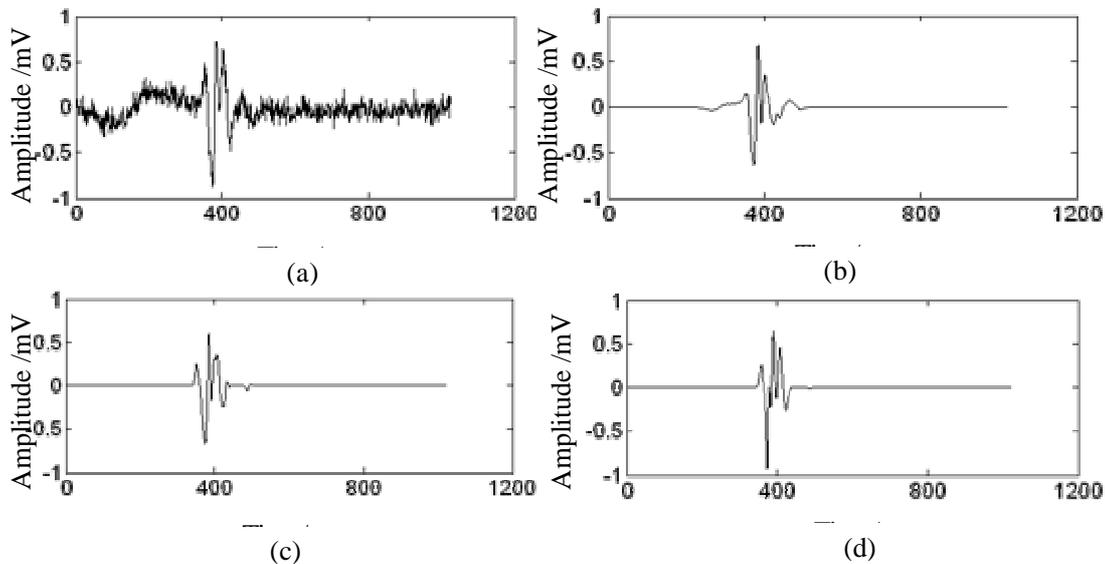


Figure 4 De-noising results of three methods for actual defect echo-signal: (a) original signal; (b) the de-noised signal by WT; (c) the de-noised signal by WPT; (d) the de-noised signal by LWT.

5. Conclusion

Wavelet is an important time-frequency analysis technology, and it is especially suitable for processing ultrasonic signals with unstable and time-varying sharp pulse. In this paper, on the basis of studying the theory of wavelet analysis and discussing the principle of wavelet analysis on de-noising, the de-noising experiments is carried out for ultrasonic simulation signals and actual defect echo-signals. Results show that WT, WPT and LWT all can remove noise well. Comparing with WT, the de-noising effect of WPT is better, but its computation is complex and time-consuming. While SNR is similar, the de-noising time by LWT is less than that by WT, and LWT is of flexible design and fast computation with a simple programme. Hence, LWT has great potential for real-time signal de-noising, with a prospect of application in ultrasonic real-time testing system.

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