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Propagation of Torsional Waves in a Waveguide of Arbitrary Cross-section Immersed in a Fluid

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Abstract

A Semi-Analytical Finite Element (SAFE) method is developed to model accurately the propagation velocity and leakage of guided waves along an immersed waveguide with arbitrary non-circular cross-section. Thus an accurate inverse model is provided to measure the density of the fluid by measuring the change of the torsional wave speed. Experimental results obtained with a rectangular bar in a range of fluids show good agreement with the theoretical predictions.

Keywords: torsional mode, non-circular bar, SAFE, fluid density

1. Introduction

The concept of an ultrasonic 'dipstick' is attractive for use in industry for fluid characterizations. The idea is that an ultrasonic guided wave which propagates in a solid structure can sense the presence and nature of the surrounding fluids. An application of a dipstick sensor for fluid density measurement using guided torsional waves in a bar with a non-circular cross section has been exploited by previous researchers [1]. When such a bar is immersed, the velocity of the wave is affected by the density of the fluid. The previous publications have presented thorough measurements of this velocity change in rectangular bars, but predictive models to simulate these effects have only been approximate, because of the complexity of the wave behaviour in the non-circular cross-sectional shape. Thus the inversion accuracy has been compromised [1, 2].

We propose and demonstrate a Semi-Analytical Finite Element (SAFE) method to model accurately the propagation and leakage of guided waves along a waveguide with arbitrary non-circular cross section that is immersed in a fluid. The model can calculate the velocity change of the torsional wave as a function of the density of the fluid, and can therefore provide an accurate inverse model for the measurements. It can also predict the extent to which the amplitude of the guided waves is attenuated by leakage from the waveguide into the fluid. Experimental results obtained with a rectangular bar in a range of fluids show very good agreement with the theoretical predictions, and substantial improvement with respect to the previous approximate model. Studies using the model are also being used to optimize the sensitivity of such waveguide sensors by selection of the waveguide material and cross-sectional shape.

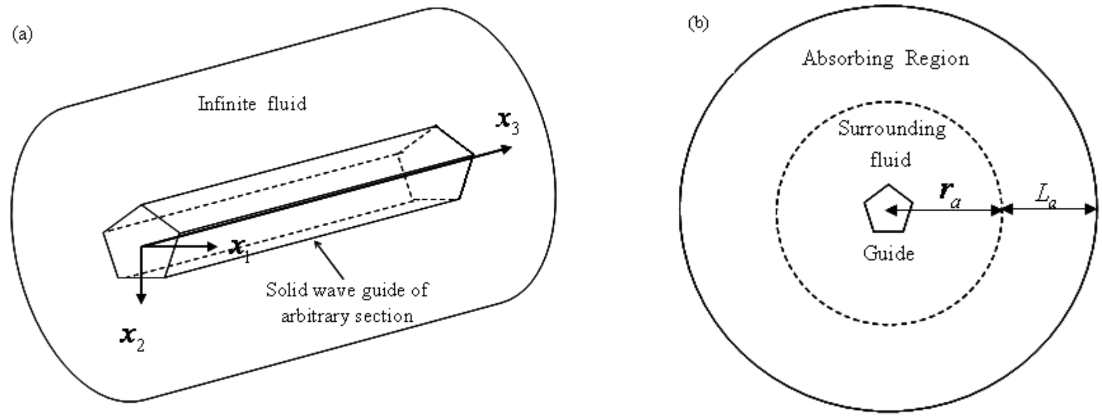

FIGURE 1. Schematics of (a) 3D solid waveguide immersed in a fluid and (b) 2D model with absorbing region used for FE simulation of system shown in (a)

2. Theory

The Semi-Analytical Finite Element (SAFE) method has been well developed to model wave propagation in solid waveguides of arbitrary cross-sections. In this paper, it is extended to analyze the propagation of waves in immersed waveguides. The schematic of the model is shown in figure 1. The main advantage of the SAFE method compared to a full Finite Element model is that only the cross-section (as is shown in figure 1b), which is normal to the direction of the wave propagation, has to be meshed by finite elements.

2.1 Semi-Analytical Finite Element (SAFE) Method in Solids

The mathematical model is based on the three dimensional elasticity approach. The waves propagating along the Ox3 axis are considered to be harmonic. Consequently, the displacement vector in the waveguide can be written:

$$u_i(x_1,x_2,x_3,t)=U_i(x_1,x_2)e^{I(kx_3-\omega t)} \quad ,I=\sqrt{-1} \tag{1}$$

in which k is the wavenumber, $\omega = 2\pi f$ is the angular frequency, f being the frequency, t is the time variable and the subscript i=1,2,3. For general anisotropic material, the equation of dynamic equilibrium can be written in the following form of an eigenvalue problem:

$$C_{ikjl}\frac{\partial^2 U_j}{\partial x_k \partial x_l}+I(C_{i3jk}+C_{ikj3})\frac{\partial(kU_j)}{\partial x_k}-kC_{i3j3}(kU_j)+\rho\omega^2\delta_{ij}U_j=0 \tag{2}$$

with summation over the indices j=1,2,3 and k,l=1,2. The coefficients C_{ikjl} are the stiffness moduli and δ_{ij} is the Kronecker symbol. In the commercial FEM code used in this study [3], the formalism for eigenvalue problems has the general expression:

$$\nabla\cdot(c\nabla U+\alpha U-\gamma)-\beta\nabla U-aU+\lambda d_a U-\lambda^2 e_a U=0 \tag{3}$$

in which all matrix coefficients are given in reference [4].

2.2 Semi-Analytical Finite Element (SAFE) Method in Fluids

We consider a fluid with compressibility coefficient K_f. The equation of dynamic equilibrium in the fluid can be written:

$$\nabla \cdot (K_f \nabla p) + \rho\omega^2 p = 0 \qquad (4)$$

in which p is the pressure of the fluid.

For the boundary condition, when the surface is contacted with a deformable solid, we have at the interface:

$$\vec{n} \cdot (K_f \nabla p) = \rho\omega^2 K_f \vec{n} \cdot \overrightarrow{u^{(solid)}} \qquad (5)$$

where $\vec{n}$ is the outward unit vector of the fluid domain on the interface and $\overrightarrow{u^{(solid)}}$ is the velocity of the interface calculated in the solid domain.

For a wave propagation along the Ox3 direction, the pressure can be written as:

$$p(x_1, x_2, x_3, t) = p(x_1, x_2)e^{I(kx_3 - \omega t)} \qquad (6)$$

By combining equations (4) and (5) and comparing with equation (3), the pressure can be chosen as the finite element variable and the coefficients become:

$$c = K_f,\ a = -\rho\omega^2,\ d_a = \alpha = \beta = \gamma = 0,\ e_a = K_f \qquad (7)$$

2.3 Absorbing Region

In order to solve the problem of a solid waveguide immersed in an infinite fluid, an exterior absorbing region is needed to model the surrounding medium. This region has the same mass density as the fluid but has damping properties which increase with the distance away from the central axis of the system as is shown in Figure 1(b) [5]. To achieve this, the imaginary part of its compressibility coefficient gradually increases according to the following law:

$$K_{fa} = K_f[1 + i\alpha(\frac{|r - r_a|}{L_a})^3] \qquad (8)$$

where K_f represents the compressibility of the liquid, r_a is the inner radius of the absorbing region, L_a is its radial length, and r is the radial position in this absorbing region. α is a coefficient that defines the proportion of the damping at the outer limit of the absorbing region. By introducing the imaginary part of the compressibility, the wave numbers, which are eigen solutions of the system, become complex. The imaginary parts represent the attenuation due to leakage from the bar to the infinite fluid. According to previous studies [5], the length of the absorbing region should be chosen between 2 and 3 times the biggest wavelength of any radiated wave in the whole frequency range, and α can take values comprised between 4 and 1 for small and high frequencies, respectively.

3 Numerical Calculation

3.1 SAFE Method Validation

The first study is a validation case, which considers a steel cylinder bar immersed in water. The example is chosen because it can be fully studied by the software DISPERSE [6], which is based on the global matrix method. The bar is 2mm in diameter and the surrounding water is modeled by a 4mm thick ring having an inner diameter of 2mm. The absorbing region is modeled by a 5mm thick ring having an inner diameter of 10mm. The whole geometry is meshed by 7563 triangular elements of 1st order, which are automatically generated by the software used [3]. The number of degrees of freedom is 14912, and the system can be solved to find values of the wavenumber k at different frequencies. For each frequency, several solutions are obtained; the ones which are dominated by motion of the steel bar are picked up, as they represent modes guided along the bar and radiating at infinity in the water.

Figure 2 presents dispersion curves of wave modes propagating along the steel bar and radiating energy in the infinite water, which are phase velocity, real wave number, group velocity and attenuation respectively. Plain lines are predictions made with the DISPERSE software, while circles represent the SAFE solutions obtained with the model. From the figure it can be seen that the SAFE predictions can reconstruct almost all the curves predicted by DISPERSE. The only disagreement appears at 250 kHz- 400 kHz of the F(1,1) mode on the group velocity and attenuation curves, which is a result of inefficiency of the absorbing region at these frequencies. According to the Snell-Descartes' law [7], the angles of radiation θ_{rad} are determined by the following formula:

$$\sin\theta_{\mathrm{rad}} = \frac{C_{water}}{C_{ph}} \; (C_{water} < C_{ph}) \tag{9}$$

$\theta_{\mathrm{rad}} = 90^{\circ} \; (C_{water} \geq C_{ph}$, one can consider that there is no radiation),

Where C_{water} is the bulk velocity of water and C_{ph} is the phase velocity of the radiating mode. In order to make negligible amplitude of any wave radiated from the guide when it reaches the outer border of the system, the length of the absorbing region should be chosen to be at least twice the projection along the x1 axis of the wavelength of the bulk waves [5]. However, from figure 2(a), it can be seen that the phase velocity of the F(1,1) mode at 250 kHz to 400 kHz is close to the bulk velocity of water, therefore the projection of the radiating wavelength could be very large according to the following formula:

$$\lambda_{x1} = \frac{\lambda_{\mathrm{rad}}}{\cos(\theta_{\mathrm{rad}})}, \tag{10}$$

This would imply the need of a very large absorbing region and of a tremendous number of mesh elements for the cross-section of the system. Therefore it was decided to choose the absorbing region to be 5mm, and accept inaccuracy of the F(1,1) mode at certain frequencies due to the inefficiency of the absorbing regions, while all others will be correct.

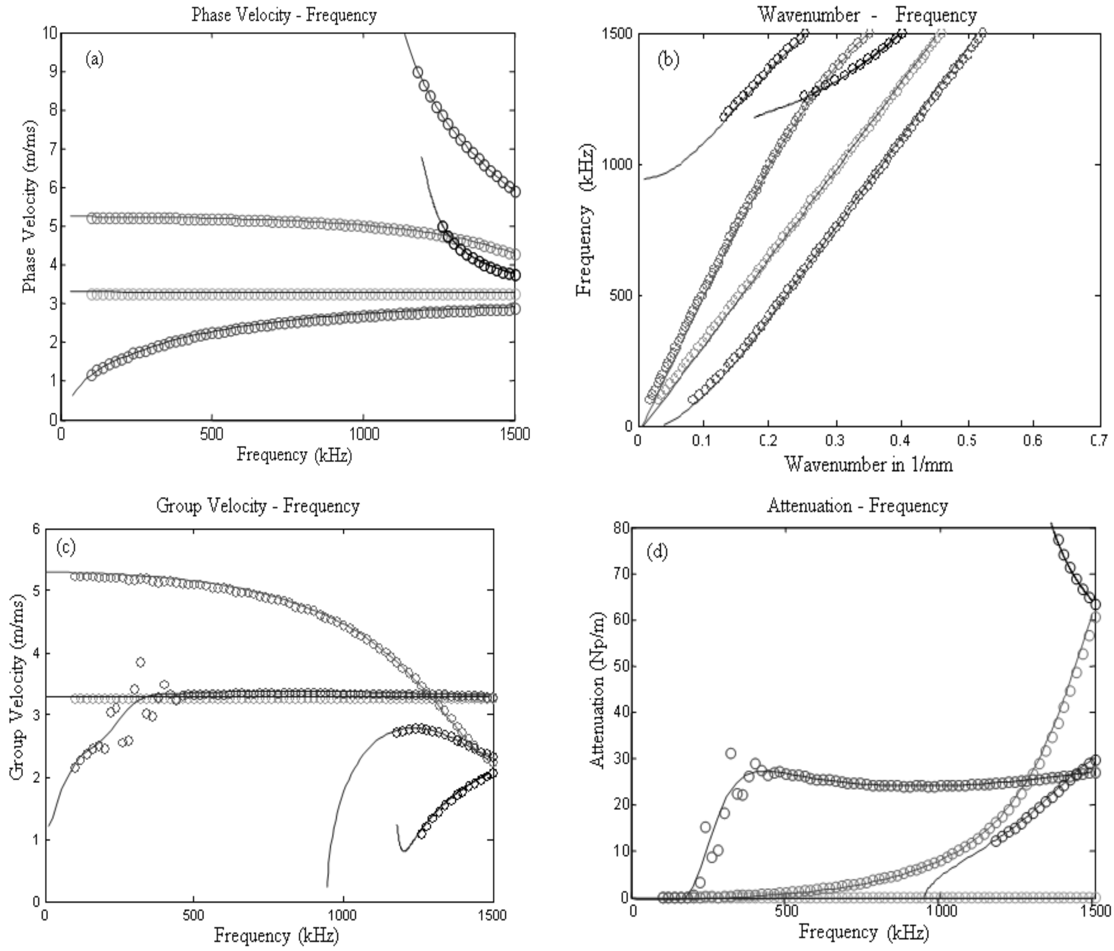

FIGURE 2. Dispersion curves of phase velocity (a), wavenumber (b), group velocity (c) and attenuation (d) of 1mm steel cylinder bar immersed in water predicted by SAFE method (o) and DISPERSE (—).

3.2 SAFE Method for Non-Circular Bar Immersed in Fluids

It is known that the speed of propagation of torsional stress waves in an immersed solid waveguide with noncircular cross-section decreases as the density of the fluid increases. Hence by measuring the speed of propagation of the torsional wave, the density of the fluid can be estimated. Assuming a two-dimensional, inviscid flow field for the fluid, Bau [1] has presented a simple quantitative theory to relate the speed of the torsional wave in a waveguide to the density of the surrounding fluid. However, this theory is only approximate. Some experiments showed [1, 2] the deviation from the theory could be up to 20%.

As it is shown in the previous model, by the SAFE method one can accurately predict the propagation and leakage of guided waves along immersed waveguides. Thus the velocity change of the torsional wave can be calculated as a function of the density of the fluid. Therefore, an accurate inverse model can be provided for the measurements.

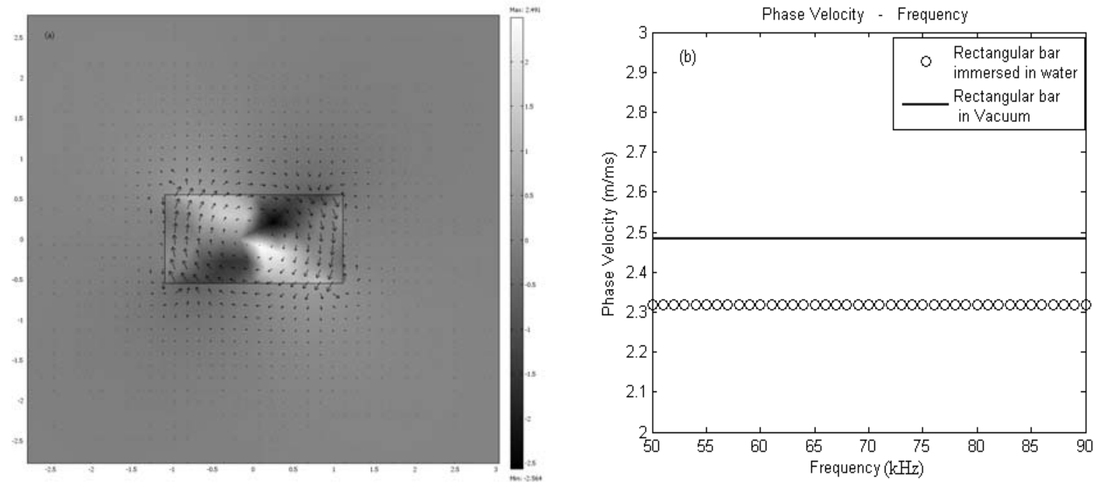

FIGURE 3. Theoretical prediction of mode shape of torsional mode of aluminium bar with rectangular cross-section (1.1mm*2.2mm) immersed in alcohol at 50KHz (a) and dispersion curves of phase velocity (b)

The example model shown here is an aluminium bar with rectangular cross-section (1.1mm*2.2mm) immersed in alcohol ($800kg/m^3$). The predicted mode shape of the bar and fluid due to the torsional mode at 50 kHz is plotted in Figure 3(a). The stress in the steel bar and pressure in the fluid is displayed as a gray scale, and the resulting displacement in the fluid and the cross-section of bar is plotted by arrows. As for the previous model, the dispersion curves over of a chosen frequency range can be generated. Figure 3(b) presents the phase velocity dispersion curves of the torsional mode from 50 kHz to 90 kHz. From the figure, it can be seen the torsional speed of the waveguide which is immersed in alcohol is lower than it is in vacuum. Also, this particular mode has no dispersion at this frequency range, and is therefore useful for measurements.

4. Experiment

4.1 Experimental Setup

An experimental setup was designed to excite the torsional mode on an aluminium bar immersed in a fluid and to measure its group velocity. Figure 4 shows a schematic of the apparatus. The bar was 450mm long with rectangular cross-section (1.1mm*2.2mm). A vessel containing a fluid sample was placed beneath the bar on a table of variable height. By changing the height of the table the bar could conveniently be immersed in the fluid to different depths; the angle between the fluid surface and the bar was 90 degrees. The signal was sent and received by a waveform generator (Macro Design Ltd.), a LeCroy 9400A Storage Oscilloscope was used to store the signal and data was then transferred to a computer for processing. The transducer was used to excite the torsional mode with a 5 cycle Hanning windowed toneburst. The signal was reflected from the end of the bar and traveled back to the transducer, where it was recorded. Measurements were carried out at different centre frequencies and two different immersion depths. The group velocity (C_g) was extracted from the measured signals using:

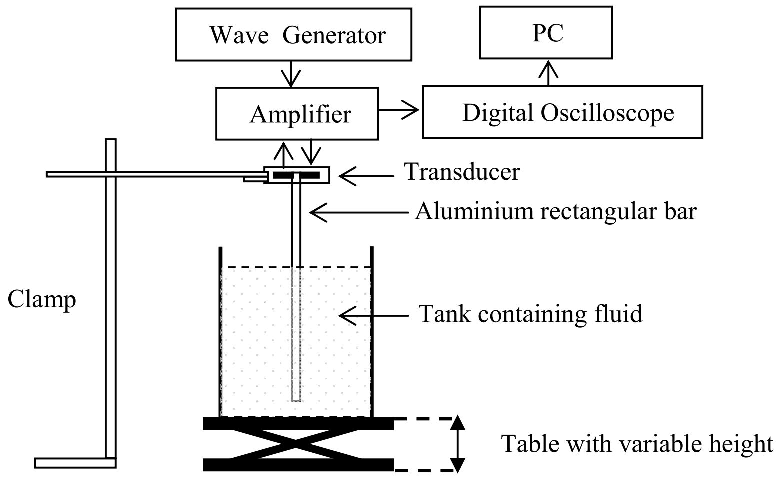

FIGURE 4. Experimental setup

$$C_g = \frac{2(x_2 - x_1)}{\Delta T + \frac{2(x_2 - x_1)}{C_{ga}}} \tag{11}$$

Where $x_2 > x_1$ are two different immersion depths, ΔT is the time difference between the arrival of the wave packages at the two immersion states, C_{ga} is the group velocity of the torsional modes in the free bar.

4.2 Results

The density of a sample of alcohol ($800kg/m^3$) was measured by a conventional measurement and the aluminium bar properties were also evaluated experimentally as $\rho = 2700kg/m^3, C_l = 6320m/s$, $C_s = 3130m/s$.

Figure 5(a) shows the measured group velocity of the torsional mode as a function of frequency and theoretically predicted curves by the SAFE method. The measured results agree well with the theoretical predictions.

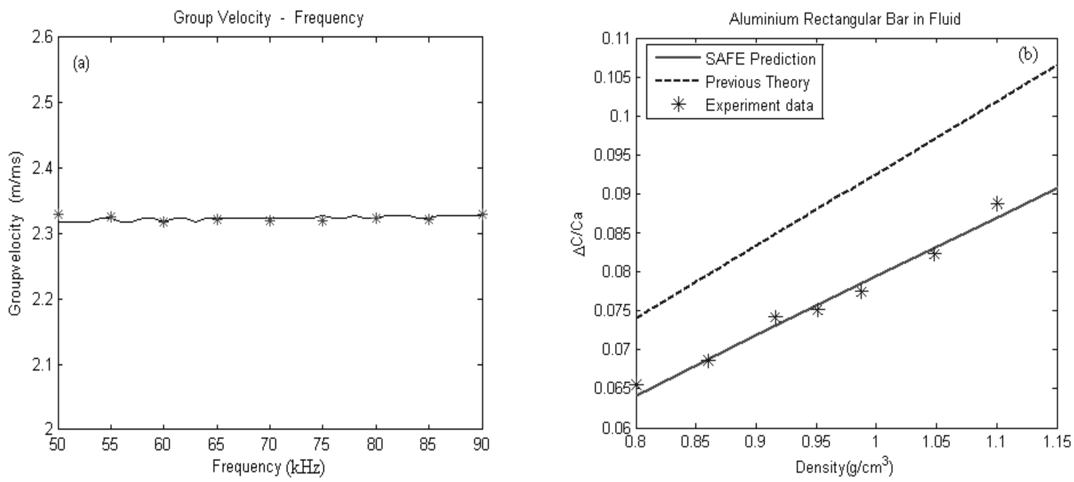

FIGURE 5. Measured (*) and theoretically predicted (—) group velocity of aluminium bar with rectangular cross-section immersed in alcohol (a) and different fluids (b)

In order to verify the theoretical predictions of the fluid density by the SAFE method and compare with the previous approximate theory, a second experiment was carried out. A few fluid samples with density from $800 kg/m^3$ to $1100 kg/m^3$ were chosen, the variation being achieved by changing the concentration of alcohol and salt with water. The centre frequency was selected to be 70 kHz. Figure 5(b) depicts the ratio $(C_{ga} - C_g)/C_g$ as a function of the fluid density. The solid line represents the inverse model by SAFE calculation while the stars are the experiment results. The previous approximate theoretical prediction (dashed lines) [1] is also shown. It can be seen that the SAFE method predictions agree very well with the measurement, and that this represents a substantial improvement with respect to the approximate model.

5. Conclusions

The torsional mode of a non-circular waveguide has previously been employed in fluid density measurements but the accuracy was compromised by the lack of an accurate model. In this paper, the Semi-Analytical Finite Element method was extended to enable the modeling of the propagation and leakage of waves in a non-circular waveguide immersed in a fluid. An accurate model was thus developed to enable velocity measurements to be used to determine the density of the fluid. Experiments were carried out to verify the model on a variety of fluids, showing very good agreement.

Acknowledgements

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Reference:

[1] Bau,H., *Trans. ASME J. Appl. Mech.* **53**, pp. 846-848 (1986).
[2] Shepard, C.L., Burghard, B.J., Friesel, M.A., Hildebrand,B.P., Moua, X., Diaz, A.A and Enderlin, C., *IEEE Trans. Ultrason. Ferroelectr. Frequency Control* **46**, pp. 536-548 (1999).
[3] *COMSOL*, User's Guide and Introduction. Version 3.3 by—COMSOL AB 2006 http://www.comsol.com/
[4] Predoi, M.V., Castaings, M., Hosten, B. and Bacon, C., *J. Acoust. Soc. Am.* **121**, pp. 1935-1944 (2007).
[5] Castaings, M. and Lowe, M.J.S., *Finite Element Model for waves guided along solid systems of arbitrary section coupled to infinite solid media*, submitted to *J. Acoust. Soc. Am.*, May 2007.
[6] *Disperse User's Manual*, Imperial College London, London, UK, 2003 http://www.imperial.ac.uk/ndt/public/productservice/disperse.htm
[7] Auld, B.A., *Acoustic fields and waves in solids*, Robert E Krierger Publishing Company, Florida (1990).