Abstract
Thermal non-destructive testing (TNDT) appeared in the 1970s as a mainly experimental technique due to use of primarily-military infrared imagers. In its active mode, TNDT is inherently related to some methods and technical means of thermal stimulation, such as optical, convective, eddy current, microwave and ultrasonic heating. Therefore, the mechanism of defect observation in TNDT is governed by heat conduction laws. Since the 1980s, the author has been intensively introducing some advanced solutions to TNDT theoretical problems. The paper contains the in-depth analysis of potentials provided by both analytical and numerical solutions while calculating temperature signals in defective materials subject to thermal stimulation. The comparison between direct 1D, 2D and 3D models is accomplished to illustrate the accuracy of particular solutions.

Keywords: Thermal non-destructive testing (TNDT), infrared thermography, heat conduction, modelling, numerical models

1. Introduction

Since the 1950s, infrared (IR) thermography, that firstly appeared as a military-oriented technique, has been continuously expanding its civilian application areas. Non-radiometric, or imaging, IR cameras paved the path to radiometric (temperature-measuring) units to be used as a means of passive technical diagnostics. In its active mode, IR thermography, or thermal nondestructive testing (TNDT), was used in the inspection of materials already in the 1960s, merely to mention the excellent works by Beller [1] and Green [2] which forestalled many modern TNDT procedures. However, for many years, the thermal method was considered as a rather exotic technique which was not able to compete with more traditional NDT methods. A new impetus to TNDT was generated when IR technologists started to cooperate with heat transfer experts to bring the theory of heat conduction into practical TNDT problems. Theoretical approaches to material characterization can be traced to the earlier work by Vernotte devoted to the determination of human skin properties [3], while the detection of subsurface defects required, first, the using of classical (‘non-defect’) heat conduction solutions to ‘non-defective’ problems summarized in the ‘bible’ of heat conduction by Carslow and Jaeger [4], and then the solving of specific problems of heat conduction in solids with structural inhomogeneties (defects). The latter approach was used by MacLaughlin and Mirchandani in the USA [5], Balageas et al. in France [6], Carolmaggio and Berardi in Italy [7], Busse et al. in Germany [8] and Vavilov et al. in Russia [9]. These first studies initiated a bunch of later research which was reviewed, for example, in [10].

The author was one of the researchers who implemented numerical method to solve 3D TNDT problems already in the beginning of the 1980s [9]. It is interesting to notice that, at that period of time, the computation of 3D models required up to several minutes of the processor time by using one of the most powerful in the world CDC 7600 computer at Manchester University.
This paper is intended to give a general outlook at the modelling problems in TNDT with the emphasis on the research that is being conducted at Tomsk Polytechnic University, Siberia, Russia.

2. 1D models

If defects are laterally very large, temperature signals are not affected by defect edge heat conduction. Such models can be considered 1D, and the temperature is a function of time and in-depth spatial coordinate. The temperature signals appearing in a homogeneous plate (Fig. 1a) can be evaluated by using the corresponding classical solutions [4]. Surface heat exchange, that is characterized by the heat exchange coefficient \( h \), can be often neglected, thus reducing test problems to adiabatic. For example, the flash adiabatic heating of a plate in Fig. 1a is described by the known expression:

\[
\frac{T^F}{W'} = 1 + 2 \sum_{n=1}^{\infty} e^{-\sigma^2 F o L} \quad \text{and} \quad \frac{T^R}{W'} = 1 + 2 \sum_{n=1}^{\infty} (-1)^n e^{-\sigma^2 F o L},
\]

where the superscripts «\( F \)» and «\( R \)» specify the front and rear surface respectively, \( W' = W \alpha / \lambda L \), \( W \) is the absorbed energy, \( \alpha \) is the thermal diffusivity, \( \lambda \) is the thermal conductivity, \( L \) is the plate thickness, \( F o = \alpha \tau / L^2 \) is the Fourier number, and \( \tau \) is the time.

The front-surface solution for a coating on a semi-infinite substrate is described by the following expression [10]:

\[
T^F(\tau) = \frac{W}{e_c \sqrt{\pi \tau}} \left[ 1 + 2 \sum_{n=1}^{\infty} (-1)^n e^{-\sigma^2 n^2 l_c / \alpha_c} \right],
\]

where \( e_c \) is the thermal effusivity of the coating, \( \sigma^2 = (e_c - e_s) / (e_c + e_s) \) is the reflection coefficient (the subscripts "c" and "s" specify a coating and a substrate respectively), \( \tau^* = n^2 l_c / \alpha_c \) is the specific heat transit time, and \( l_c \) is the coating thickness.

The most convenient for TNDT purposes is the model in Fig. 1c which represents a three-layer model where both adiabatic and non-adiabatic boundary conditions can be implemented, and the central layer simulates either a thermally-resistive or thermally-capacitive defect (Multilayer-3 program, Tomsk Polytech, 1980s). The corresponding mathematical expressions (not reported in this paper) are bulky [10], thus greatly loosing the 'illustrativeness' that is a recognized advantage of classical analytical solutions.

The potentials of the analytical models in Fig. 1 can be illustrated with an example of TNDT of a 180 \( \mu \)m-thick thermal-protection coating on an 8 mm-thick steel substrate (Fig. 2). All model parameters are given in the figure legend, and the results are presented in Log-Log coordinates. The behaviour of Log-Log temperature evolutions in Fig. 2 can be explained in the following way. The temperature vs. time on the steel substrate surface that is adiabatically heated with a Dirac pulse is represented with a straight line 1, as it follows from the known solution:
while expressed in Log-Log coordinates.

\[
\frac{T}{w} = \frac{1}{e^{\sqrt{\pi} \tau}}
\]

(3)

The adiabatic heating of a single thermal-protection layer (curve 2) reveals higher temperature in the beginning due to lower mass of the coating layer and different values of thermal properties, while, at the end of the thermal process, the temperature remains constant and equal to the average temperature of a plate which absorbed $W$ Joules of thermal energy.

The curve 3 is the best presentation of the coating surface temperature if the coating is in ideal contact with the substrate (the reflection coefficient is 0.673 in this case). Finally, the curve 4 shows how the presence of an air gap affects the signal temperature evolution. This curve is to be compared to the curve 3 to exhibit that the differential temperature signal $\Delta T$ typically used in TNDT is positive for a particular period of time, i.e. the defect area is warmer than the surrounding. It is interesting that, at longer times, the defective area is getting colder than non-defect areas due to surface heat exchange (the non-adiabatic case is considered here).

Except purely analytical solutions which become cumbersome in ‘multilayer’ cases, the ‘thermal quadrupole’ method summarized by Maillet et al. has been effectively used to calculate 1D and 2D heat conduction problems [11]. Mathematically, the ‘thermal quadrupole’ method belongs to a class of analytical methods used for solving linear differential equations in simple geometries. It takes advantage of such analytical tools as the integral Laplace transformation (in time), as well as the spatial Fourier and Hankel transformations based on the separation of variables. The corresponding solutions are expressed as linear matrix relationships between vectors of the temperature and heat fluxes on boundaries of a multilayer system. This allows the obtaining of solutions which are independent of boundary conditions.
Figure 2. Heating thermal-protection coating ($\lambda=1$ W/(m·K), $\rho=5000$ kg/m$^3$, $C=460$ J/(kg·K), $\alpha=4.35 \cdot 10^{-7}$ m$^2$/s, $e=1517$ Ws$^{1/2}$/(m$^2$·K), $l_c=180$ μm) on a steel substrate ($\lambda=15$ W/(m·K), $\rho=8000$ kg/m$^3$, $C=500$ J/(kg·K), $\alpha=3.75 \cdot 10^{-6}$ m$^2$/s, $e=7746$ Ws$^{1/2}$/(m$^2$·K), $l_c=8$ mm), reflection coefficient $\Gamma=0.673$, heating energy $W=1500$ J:

1 – adiabatic heating of semi-infinite steel sample by Eq. (3),
2 – adiabatic heating of one-layer (thermal protection) plate by Eq. (1),
3 – adiabatic heating of coating on substrate by Eq. (2),
4 – non-adiabatic square-pulse heating of coating-air defect-substrate structure
   (defect thickness 50 μm, heat pulse duration $\tau_0=3$ ms, heat power density $Q=500$ kW/m$^2$)

3. **2D and 3D ‘classical’ models**

3.1 **2D cylindrical model**

2D and 3D models (see Fig. 3) add one or two more spatial coordinates, thus allowing the analysis of finite-size defects to compare to laterally-infinite defects adhering to the models in Fig. 1. Moreover, since 2D and 3D heat conduction problems are typically being solved numerically, some more subtle phenomena can be analyzed, such as thermal property anisotropy, non-linear surface heat exchange, material semi-transparency, arbitrary heating function, etc. (see below).

The three-layer model in Fig. 3a formulated in the cylindrical coordinates and sometimes called «disk in disk» was numerically realized at Tomsk Polytech in the 1990s as a ThermoCalc-2D program. It is similar to the 1D model in Fig. 1c but involves the defect diameter $D$. For example, the curve in Fig. 3b shows how the differential signal $\Delta T$ over bottom-hole (corrosion-like) defects in AISI 1010 steel decays with smaller $D$. It is clearly seen that only defects which are laterally larger than 100-120 mm can be regarded as 1D. The corrosion model by Fig. 3a is thoroughly discussed in [12]. The calculation accuracy of the models in Fig. 3, being verified by the 1D model in Fig. 1c, has proven to be about few percents (ThermoCalc software).
Figure 3. 2D TNDT models:

a – 2D three-layer TNDT model (ThermoCalc-2D program),
b – $\Delta T$ vs. $D$ (AISI 1010 steel: $L=10$ mm, $\lambda=63.9$ W/(m K), $\alpha=1.9 \times 10^{-5}$ m$^2$/s, $Q=15$ kW/m$^2$, $\tau_0=10$ s; defect: 25% material loss),
c – same as a), but defect-in-shell, e.g. landmine in soil (ThermoCalc-2DM program)

### 3.2 3D Cartesian model

The most practical 3D TNDT model (sometimes called «parallelepiped in parallelepiped») was developed at Tomsk Polytech at the end of the 1990s. It is formulated in the Cartesian coordinates and allows the modelling of a sample consisting of up to 6 layers and containing up to 9 defects (ThermoCalc-6L program). In this model, defects of complicated shapes can be also simulated. The comparison between the 2D and 3D models of corrosion detection is presented in Fig. 4. It is seen that the round-shape defect by the diameter of 50 mm and the 44x44 mm square-shape defect having approximately the same area ($S$ is due to a finite step of the numerical grid used in ThermoCalc-6L) produce very close maximum differential signals $\Delta T_m$ and identical times of their appearance $\tau_m$. However, the extended defect 5x393 mm of the same area is characterized by the much smaller amplitude of $\Delta T_m$ that is clearly seen in the colour thermogram of Fig. 4. Note that, in this case, the numerical grid included 450 thousands nodes with the computation time being about 1 minute on a standard lap-top computer when calculating 150 time steps.

### 4. Advanced 2D and 3D models

#### 4.1 ‘Shell-like’ defects

A more sophisticated version of the above-mentioned 2D program, that is called ThermoCalc-2DM (Tomsk Polytech), allows the analysis of the situations where defects are wrapped in a ‘shell’ (Fig. 3c), such as the detection of landmines (trinitrotoluene in a metallic or plastic case). For example, the introducing of a landmine into soil mat provoke the appearance of air ‘bubbles’ around the landmine case, thus possibly affecting detection conditions [13].

Another ‘shell-like’ test case could take place when analyzing Teflon inserts in composite materials. The classical TNDT theory predicts quite small temperature signals while assuming ideal contact between Teflon inserts and the host composite. However, in practice, such defects are often reliably detected. The explanation of this fact requires the using of a more advanced model.
Such model could include the following phenomena: 1) the presence of thin air gaps around Teflon inserts that might essentially enhance temperature signals, 2) the modification of composite thermal conductivity in the process of fabrication of reference samples. The combination of these phenomena allows the explanation of some peculiarities of experimental temperature patterns over Teflon inserts [10].

Figure 4. Comparison between 2D and 3D models of corrosion

4.2 Arbitrary heating function

In the ThermoCalc-2DM software, the heating can be simulated by an arbitrary function of time. In the case of landmines detection, such approach has allowed the modelling of diurnal cycles of solar radiation with arbitrary cloudy-Sky periods [14].

In TNDT of composites, it was suggested to alternate heating with cooling, even if such stimulation procedure can be hardly implemented experimentally [10].

4.3 Anisotropy analysis

The program version called ThermoCalc-30L enables the modelling of up to 30 anisotropic material layers with coordinate axes being tilted against each other by a chosen spatial angle. Such model simulates well fibre reinforced composites with different orientation of fibres. For example, the scheme in Fig. 7a is related to a 2-ply carbon fibre reinforced plastic (CFRP) of which fibres are tilted by 45°. The thermal conductivity along fibres is $\lambda_\parallel = 6.4$ W/(m K) and in the perpendicular direction $\lambda_\perp = 0.64$ W/(m K). In the development of composite materials, the analysis of thermal diffusivity that is typically determined by applying the flash Parker’s technique is quite important. The thermograms in Fig. 7b-d demonstrate the temperature distributions on the rear surface of the graphite epoxy sample of which the front surface is heated with a laser beam. The composite anisotropy is clearly seen if a composite consists of only a single ply (Fig. 7b). In the case of a 2-ply composite, principal heat conduction occurs under the angle of 45° (Fig. 7c) due to the contribution of the second ply, while a 4-ply composite is characterized by the considerable averaging of conductivity spatial components (Fig. 7d).
4.4 Analyzing material transparency

Most TNDT models assume that tested materials are optically-opaque. However, some simple experiments conducted at Tomsk Polytech, have shown that thin composites and ceramics may reveal a certain grade of transparency, thus introducing volumic heating rather than surface.

The corresponding algorithm implemented in the ThermoCalc-6L Absorb program assumes that the propagation of optical radiation through a medium is governed by Bouguer’s law:

$$w = w_o e^{-\gamma L}, \quad (4)$$

where $w_o$ is the flux power density of incident radiation, $\gamma$ is the medium absorption coefficient which depends on a wavelength and a type of a medium, and $L$ is the distance, or sample thickness.

The experiments on material transparency have been based on the flash heating of a reference aluminium plate fulfilled both directly and through a thin plate of composite. Obtained $w / w_o$ were used to determine $\gamma$ (for example, a 1 mm-thick CFRP sample produced $\gamma = 2506 \text{ 1/m}$).

The calculation results for both opaque and semi-transparent 1 mm-thick CFRP samples are presented in Fig. 8. The samples contain six air-filled defects at three depths (see the legend to Fig. 8a). In the case of the opaque sample, the temperature evolutions look 'classical'; for example, the 50 µm-thick defect produces $\Delta T=0.67^\circ\text{C}$ at $\tau_m=0.14 \text{ s}$, while the twice-thicker defect (100 µm) produces $\Delta T=0.95^\circ\text{C}$ at $\tau_m=0.17 \text{ s}$. The introduction of the
composite semi-transparency ($\gamma = 2506 \text{ l/m}$) results in lower temperature signals ($\Delta T = 0.19 ^\circ \text{C}$ and $\Delta T = 0.37 ^\circ \text{C}$ for the same two defects), as it follows from Fig. 8c. The influence of composite transparency is diminishing for deeper defects (compare the results in Fig. 8b,c for two deepest defects at 0.75 mm depth). However, the example above shows that partial transparency of composites may essentially influence observed temperature signals, thus questioning a decent comparison between experimental data and theoretical results obtained on ‘classical’ TNDT models.

CFRP: $L = 1 \text{ mm}$, $Q = 100 \text{ kW/m}^2$, $\tau_s = 10 \text{ ms}$, all defects 10x10 mm

Defect depth 0.25 mm, defect thickness 50 and 100 $\mu$m

Defect depth 0.50 mm, defect thickness 50 and 100 $\mu$m

Defect depth 0.75 mm, defect thickness 50 and 100 $\mu$m

Figure 8. Influence of CFRP transparency on defect temperature evolutions:

a – temperature distribution at 0.6 s (close to optimum times for the deepest defects),
b – opaque composite;
c – semi-transparent composite ($\gamma = 2506 \text{ l/m}$ for CFRP)
4.5 Thermal properties as functions of coordinates

The ThermoCalc Mine version allows to model linear variation of thermal properties along any of three coordinates. For example, in this way, the landmine detection model can take into account possible in-depth variation of thermal conductivity and diffusivity because of soil moisture. Another example is given below.

Assume that an air-filled 5x5x0.1 mm defect is located at the depth of 0.25 mm in a 1 mm-thick graphite epoxy composite (Fig. 9). 10 ms-long heat pulse has the power of 100 kW/m². Assume for simplicity that only thermal conductivity $\lambda_z$ could change along the $z$-coordinate, i.e. in depth. Consider three cases: 1) constant $\lambda_z = 0.64$ W/(m·K), constant $\lambda_z = 6.4$ W/(m·K), 3) $\lambda_z$ is linearly growing from 0.64 to 6.4 W/(m·K) (see Fig. 9 on the right).

The calculation results obtained with ThermoCalc Mine are as follows:

Case 1: $\Delta T = 0.9538^\circ$C at $\tau = 0.17$ s,
Case 2: $\Delta T = 1.88^\circ$C at $\tau = 0.05$ s,
Case 3: $\Delta T = 1.736^\circ$C at $\tau = 0.06$ s.

The results clearly show that the ‘detectability’ of shallow defects is mainly determined by surface thermal properties, therefore, Cases 2 and 3 produce similar results. However, it is worth noting that there is no simple explanation to the behavior of optimum observation time $\tau$ which is typically shorter with higher thermal diffusivity.

5. Conclusions

This paper describes results of the multi-year research that is being conducted at Tomsk Polytechnic University in the area of pulsed TNMDT. However, by the author’s opinion, the paper also reflects some world trends in the TNMDT theory. It is demonstrated that 1D, 2D and 3D modelling approaches are fairly well-developed to allow the analysis of numerous phenomena that accompany transient thermal processes in bodies with structural inhomogeneities. Analytical 1D models are often used for verification of 2D and 3D numerical models, while the most flexible are numerical models which enable simulation of
both classical TNDT features, such as dependencies of temperature signals and their appearance times on material properties and defect depth and size, as well as some more subtle phenomena, for example, the influence of material anisotropy and optical semi-transparency on detectable signal parameters.

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